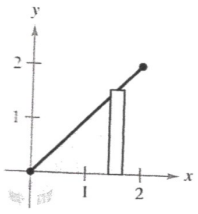
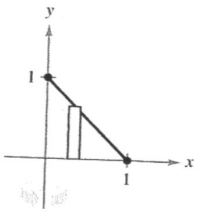
EXERCISES FOR SECTION 6.3

In Exercises 1–12, use the shell method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region about the *y*-axis.

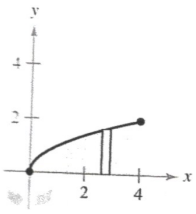
1. $y = x$



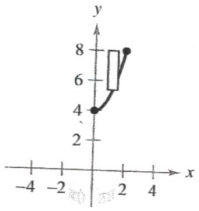
2. $y = 1 - x$



3. $y = \sqrt{x}$



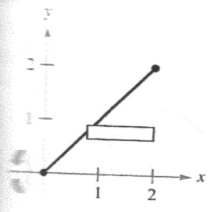
4. $y = x^2 + 4$



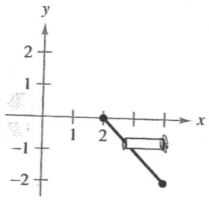
5. $y = x^2,\ y = 0,\ x = 2$
6. $y = \frac{1}{2}x^2,\ y = 0,\ x = 6$
7. $y = x^2,\ y = 4x - x^2$
8. $y = 4 - x^2,\ y = 0$
9. $y = 4x - x^2,\ x = 0,\ y = 4$
10. $y = 2x,\ y = 4,\ x = 0$
11. $y = \dfrac{1}{\sqrt{2\pi}} e^{-x^2/2},\ y = 0,\ x = 0,\ x = 1$
12. $y = \begin{cases} \dfrac{\sin x}{x}, & x > 0 \\ 1, & x = 0 \end{cases},\ y = 0,\ x = 0,\ x = \pi$

In Exercises 13–16, use the shell method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region about the *x*-axis.

13. $y = x$



14. $y = 2 - x$



15. $y = \dfrac{1}{x},\ x = 1,\ x = 2,\ y = 0$
16. $x + y^2 = 16,\ x = 0$

In Exercises 17–20, use the shell method to find the volume of the solid generated by revolving the plane region about the indicated line.

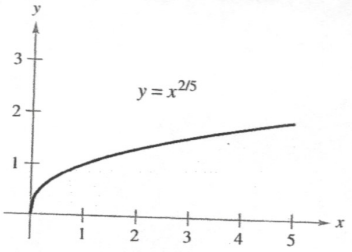
17. $y = x^2,\ y = 4x - x^2$, about the line $x = 4$
18. $y = x^2,\ y = 4x - x^2$, about the line $x = 2$
19. $y = 4x - x^2,\ y = 0$, about the line $x = 5$
20. $y = \sqrt{x},\ y = 0,\ x = 4$, about the line $x = 6$

In Exercises 21–24, use the disk *or* the shell method to find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the indicated line.

21. $y = x^3,\ y = 0,\ x = 2$
    (a) the *x*-axis (b) the *y*-axis (c) the line $x = 4$
22. $y = \dfrac{10}{x^2},\ y = 0,\ x = 1,\ x = 5$
    (a) the *x*-axis (b) the *y*-axis (c) the line $y = 10$
23. $x^{1/2} + y^{1/2} = a^{1/2},\ x = 0,\ y = 0$
    (a) the *x*-axis (b) the *y*-axis (c) the line $x = a$
24. $x^{2/3} + y^{2/3} = a^{2/3},\ a > 0$ (hypocycloid)
    (a) the *x*-axis (b) the *y*-axis

Getting at the Concept

25. Give the integration formula for finding the volume of a solid using the shell method.
26. The region in the figure is revolved about the indicated axes and line. Order the volumes of the resulting solids from least to greatest. Explain your reasoning.
    (a) *x*-axis (b) *y*-axis (c) $x = 5$



In Exercises 27 and 28, give a geometric argument that explains why the integrals have equal values.

27. $\pi \displaystyle\int_1^5 (x - 1)\,dx = 2\pi \int_0^2 y[5 - (y^2 + 1)]\,dy$
28. $\pi \displaystyle\int_0^2 [16 - (2y)^2]\,dy = 2\pi \int_0^4 x\left(\frac{x}{2}\right)dx$