1. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{n^2 x^n}{2^n}$ converges?

(A) All x except $x = 0$
(B) $|x| = 3$
(C) $-3 \le x \le 3$
(D) $|x| > 3$
(E) The series diverges for all x.

2. Which of the following series converge to 2?

I. $\sum_{n=1}^{\infty} \frac{2n}{n+3}$

II. $\sum_{n=1}^{\infty} \frac{-8}{(-3)^n}$

III. $\sum_{n=0}^{\infty} \frac{1}{2^n}$

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only

3. The third-degree Taylor polynomial about $x = 0$ of $\ln(1 - x)$ is

(A) $-x - \frac{x^2}{2} - \frac{x^3}{3}$

(B) $1 - x + \frac{x^2}{2}$

(C) $x - \frac{x^2}{2} + \frac{x^3}{3}$

(D) $-1 + x - \frac{x^2}{2}$

(E) $-x + \frac{x^2}{2} - \frac{x^3}{3}$

4. The complete interval of convergence of the series $\sum_{k=1}^{\infty} \frac{(x+1)^k}{k^2}$ is

A) $0 < x < 2$ B) $0 \le x \le 2$ C) $-2 < x \le 0$

D) $-2 \le x < 0$ E) $-2 \le x \le 0$

5. For what values of x does the series $1 + 2^x + 3^x + 4^x + \ldots + n^x + \ldots$ converge?

A) No values of x B) $x < -1$ C) $x \ge -1$

D) $x > -1$ E) All values of x

6. For a series S, let

$$S = 1 - \frac{1}{9} + \frac{1}{2} - \frac{1}{25} + \frac{1}{4} - \frac{1}{49} + \frac{1}{8} - \frac{1}{81} + \frac{1}{16} - \frac{1}{121} + \cdots + a_n + \cdots,$$

$$\text{where } a_n = \begin{cases} \frac{1}{2^{(n-1)/2}} & \text{if } n \text{ is odd} \\ \frac{-1}{(n+1)^2} & \text{if } n \text{ is even} \end{cases}$$

Which of the following statements are true?

I. S converges because the terms of S alternate and $\lim_{n \to \infty} a_n = 0$.
II. S diverges because it is not true that $|a_{n+1}| < |a_n|$ for all n.
III. S converges although it is not true that $|a_{n+1}| < |a_n|$ for all n.

(A) None
(B) I only
(C) II only
(D) III only
(E) I and III only

7. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$ is the Taylor series about zero for which of the following functions?

A) $\sin x$ B) $\cos x$ C) e^x

D) e^{-x} E) $\ln(1 + x)$

8. Which is the best of the following polynomial approximations to $\cos(2x)$ near $x = 0$?

A) $1 + \frac{x}{2}$ B) $1 + x$ C) $1 - \frac{x^2}{2}$

D) $1 - 2x^2$ E) $1 - 2x + x^2$