KEY
(FRIDAY'S HW)

Series Review (Handwritten)

1. $\sum_{n=0}^{\infty} \frac{1}{2+\sqrt{n}}$ compare to $\frac{1}{\sqrt{n}} \to$ divergent $p<1$

$\lim_{n\to\infty} \frac{1}{2+\sqrt{n}} \cdot \frac{\sqrt{n}}{1}$

$\lim_{n\to\infty} \frac{\sqrt{n}}{2+\sqrt{n}} = 1 > 0 \therefore \sum_{n=0}^{\infty} \frac{1}{2+\sqrt{n}}$ also diverges by LCT

2. $\sum_{n=0}^{\infty} \frac{1}{4n^2-6n+1}$ compare to $\frac{1}{n^2} \to$ convergent p-series

$\lim_{n\to\infty} \frac{1}{4n^2-6n+1} \cdot \frac{n^2}{1}$

$\lim_{n\to\infty} \frac{n^2}{4n^2-6n+1} = \frac{1}{4} > 0 \therefore \sum_{n=0}^{\infty} \frac{1}{4n^2-6n+1}$ also converges by LCT

3. $\sum_{n=0}^{\infty} \left(\frac{2n}{n+1}\right)^n$

$\lim_{n\to\infty} \sqrt[n]{\left|\left(\frac{2n}{n+1}\right)^n\right|}$

$\lim_{n\to\infty} \frac{2n}{n+1} = 2 > 1 \therefore \sum_{n=0}^{\infty} \left(\frac{2n}{n+1}\right)^n$ diverges by root test

4. $\sum_{n=0}^{\infty} \frac{3^n}{(n+1)!}$ $\lim_{n\to\infty} \left|\frac{3^{n+1}}{(n+2)!} \cdot \frac{(n+1)!}{3^n}\right|$

$\lim_{n\to\infty} \left|\frac{3}{n+2}\right| = 0 < 1 \therefore \sum_{n=0}^{\infty} \frac{3^n}{(n+1)!}$ converges by ratio test

5. $\sum_{n=0}^{\infty} \frac{n}{2n^2+1}$ compare to $\frac{1}{n} \rightarrow$ divergent harmonic

$\lim_{n\to\infty} \frac{n}{2n^2+1} \cdot \frac{n}{1}$

$\lim_{n\to\infty} \frac{n^2}{2n^2+1} = \frac{1}{2} > 0$ $\therefore \sum_{n=0}^{\infty} \frac{n}{2n^2+1}$ also diverges by LCT.

6. $\sum_{n=0}^{\infty} \frac{3^{n+1}}{4^n}$ $\lim_{n\to\infty} \left| \frac{3^{n+2}}{4^{n+1}} \cdot \frac{4^n}{3^{n+1}} \right|$

$\lim_{n\to\infty} \frac{3}{4} = \frac{3}{4} < 1$ $\therefore \sum_{n=0}^{\infty} \frac{3^{n+1}}{4^n}$ converges by ratio test

☆7. $\sum_{n=0}^{\infty} n!\, x^n$ $\lim_{n\to\infty} \left| \frac{(n+1)!\, x^{n+1}}{n!\, x^n} \right|$

$\lim_{n\to\infty} |(n+1)(x)| = \infty$

$\boxed{\begin{array}{l}\text{diverges for all } x > 0\text{; converges only at center} \\ R = 0 \\ \text{Interval of convergence: none}\end{array}}$

8. $\sum_{n=0}^{\infty} 3(x-2)^n$ $\lim_{n\to\infty} \left| \frac{3(x-2)^{n+1}}{3(x-2)^n} \right|$

$\lim_{n\to\infty} |x-2|$

$|x-2| < 1$

$x = -1$

$\sum_{n=0}^{\infty} 3(-3)^n$ geometric $|r| \geq 1$ divergent.

$x = 3$

$\sum_{n=0}^{\infty} 3(1)^n$ geometric $|r| \geq 1$ divergent

$\boxed{\begin{array}{l}R = 1 \\ \text{Interval of convergence } (+1, 3)\end{array}}$

9. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ $\qquad \lim_{n\to\infty} \left| \frac{x^{2(n+1)+1}}{(2(n+1)+1)!} \cdot \frac{(2n+1)!}{x^{2n+1}} \right|$

$$\lim_{n\to\infty} \left| \frac{x^{2n+3}}{(2n+3)!} \cdot \frac{(2n+1)!}{x^{2n+1}} \right|$$

$$\lim_{n\to\infty} \left| \frac{x^2}{(2n+3)(2n+2)} \right| = 0 < 1$$

$\boxed{\text{converges for all values of } x \text{; interval of convergence } (-\infty, \infty)}$

$\boxed{A}$ 10. I. convergent p-series $p > 1$

II. divergent p-series $p < 1$

III. compare to $\frac{1}{n}$ $\qquad \lim_{n\to\infty} \frac{1}{n+4} \cdot \frac{n}{1}$

$\lim_{n\to\infty} \frac{n}{n+4} = 1 > 0$. Since $\sum \frac{1}{n}$ is the divergent harmonic, $\sum \frac{1}{n+4}$ must also diverge.

$\nearrow$ about $x = 1$

$\boxed{D}$ 11. $f(x) = \ln x = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4}$

$\boxed{E}$ 12. $x - \frac{x^2}{2^2} + \frac{x^3}{3^2} - \frac{x^4}{4^2} + \dots \frac{(-1)^{n-1} x^n}{n^2}$

$\lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)^2} \cdot \frac{n^2}{x^n} \right|$ $\qquad |x| < 1$ $\qquad x = -1$: $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(-1)^n}{n^2} = \sum_{n=1}^{\infty} \frac{(-1)^{2n-1}}{n^2}$

$R = 1$ $\qquad$ Convergent p series

$\lim_{n\to\infty} \left| \frac{x n^2}{n^2+2n+1} \right|$ $\qquad$ center at 0

$(-1, 1)$ $\qquad x = 1$

~~int of convergence~~ $[-1, 1]$ $\qquad \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(1)^n}{n^2}$ convergent

$\boxed{E}$ 13. I. compare to $\frac{1}{n} \rightarrow$ divergent harmonic $\quad \lim_{n\to\infty} \frac{1}{5n} \cdot \frac{n}{1}$

$\lim_{n\to\infty} \frac{n}{5n} = \frac{1}{5} > 0$ so $\frac{1}{5n}$ diverges

II convergent geometric $r < 1$

III. not telescoping - terms dont cancel

$\sum_{n=1}^{\infty} \frac{1}{5n} - \frac{1}{5n+1}$

$\sum_{n=1}^{\infty} \frac{5n+1-5n}{5n(5n+1)}$

$\sum_{n=1}^{\infty} \frac{1}{25n^2+5n}$ compare to $\frac{1}{n^2} \rightarrow$ convergent p-series

$\lim_{n\to\infty} \frac{1}{25n^2+5n} \cdot \frac{n^2}{1}$

$\lim_{n\to\infty} \frac{n^2}{25n^2+5n} = \frac{1}{25} > 0$ so converges by LCT

$\boxed{E}$ 14. $\sum_{n=0}^{\infty} \frac{(-1)^n (3x^2)^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n 9^n x^{4n}}{(2n)!}$

$\boxed{C}$ 15. $\sum_{n=1}^{\infty} \frac{n3^n}{x^n} \quad \lim_{n\to\infty} \left| \frac{(n+1)3^{(n+1)}}{x^{n+1}} \cdot \frac{x^n}{n3^n} \right|$

$\lim_{n\to\infty} \left| \frac{(n+1)3}{xn} \right|$

$\left| \frac{3}{x} \right| < 1$

$\left| \frac{1}{x} \right| < \frac{1}{3}$

$|x| < 3$ center: 0 $(-3, 3)$

$R = 3$

$\boxed{A}$ 16. $f(x) = \ln(1-x)$

$$\ln x = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}$$

$$\ln(1-x) = (1-x-1) - \frac{(1-x-1)^2}{2} + \frac{(1-x-1)^3}{3}$$

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3}$$

$$\frac{x}{1} < \frac{3}{1}$$