BC SERIES REVIEW

1. n^{TH} TERM $= \frac{1}{2}$ DIVERGES
2. GEOM - $r = \pi/6 < 1$ CONVERGES
3. ROOT TEST $\frac{3}{2} > 1$ DIVERGES
4. $\frac{3}{5}, \frac{1}{3}, \frac{3}{13}, \frac{3}{17}$ $a_{n+1} \le a_n$ $\lim_{n\to\infty} = 0$ ✓ CONVERGES
5. $\frac{(n+1)!}{10^{n+1}} \cdot \frac{10^n}{n!} = \frac{n+1}{10}$ $\lim_{n\to\infty} \frac{n+1}{10} = \infty$ DIVERGE (also by n^{th} term)
6. $u = -n^2$ $du = -2n\,du$ $\frac{-1}{2}\int_1^\infty e^u du$ $\lim_{b\to\infty} -\frac{1}{2}e^{-x^2}\Big|_1^b = \frac{-1}{2e^{b^2}} + \frac{1}{e} = \boxed{\frac{1}{e}}$

   CONVERGES
7. $\lim_{a\to\infty} \int_1^a \frac{x}{5x^2-4}$ $u = 5x^2-4$ $du = 10x\,dx$ $\frac{1}{10}\int_1^a \frac{1}{u} du = \frac{1}{10}\ln a - \frac{1}{10}\ln 1 = \infty$

   DIVERGES
8. P-SERIES $\frac{2}{3} < 1$ DIVERGES
9. $1 = A(n+3) + B(n+2)$ $\sum_{n=1}^{\infty} \frac{1}{n+2} - \frac{1}{n+3} = \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{6}\right) + \cdots = \boxed{\frac{1}{3}}$

   $x = -3$ $B = -1$ $x = -2$ $A = 1$ CONVERGES
10. LCT $\lim_{n\to\infty} \frac{3n-2}{n^3-2n^2+11} \cdot \frac{n^2}{1} = \frac{3n^3}{n^3} = \boxed{3}$ CONVERGES b/c $\frac{1}{n^2}$ DOES
11. COMPARE $\left(\frac{4}{3}\right)^n$ WHICH DIVERGES $\frac{4^n+2}{3^n} > \left(\frac{4}{3}\right)^n$ ∴ DIVERGES
12. COMPARE $\frac{1}{n^4}$ WHICH CONVERGES $\frac{1}{n^4+3} < \frac{1}{n^4}$ ∴ CONVERGES
13. COMPARE $\frac{1}{n}$ DIVERGES $\frac{2n^2+n}{(n+1)^2} = 2$ DIVERGES
14. LIMIT COMPARE $\frac{1}{2^n}$ $\lim_{n\to\infty} \frac{2^n}{2^n-1} = \frac{2^n \ln 2}{2^n \ln 2} = 1$ CONVERGES
15. LCT $\frac{3n^3+2}{n^3 2n} = 3$ ∴ CONVERGES

    $\frac{1}{n^2}$ CONVERGES

16. DIVERGES: compare to $1/n$

17. Converges compare to $\frac{1}{n^{5/3}}$ convergent

$$\lim_{n\to\infty} \frac{\frac{\sqrt[3]{n}}{n^2+1}}{\frac{\sqrt[3]{n}}{n^2}}$$

$$\lim_{n\to\infty} \frac{\sqrt[3]{n}}{n^2+1} \cdot \frac{n^2}{\sqrt[3]{n}} = 1$$

18. Converges

$$\lim_{n\to\infty} \frac{1}{n^2-1} = 0.$$

19. $\lim_{x\to 0} \frac{2e^{2x}}{1} = \boxed{2}$

20. $\lim_{x\to -\infty} \frac{2x}{-e^{-x}} = \lim_{x\to -\infty} \frac{2}{e^{-x}} = \frac{2}{e^{\infty}} = 0$

21. $\lim_{x\to\infty} \frac{x^{1/2}}{e^{x}} = \frac{1/2\, x^{-1/2}}{e^{x}} =$

$$= \lim_{x\to\infty} \frac{1}{2e^{x}\sqrt{x}} = 0$$

22. $\lim_{x\to\infty} \left(1 + \frac{2}{x}\right)^{x}$ 1^{∞}

$$\ln y = \lim_{x\to\infty} x \ln\left(1 + 2/x\right) \quad \infty \cdot 0$$

$$= \lim_{x\to\infty} \frac{\ln\left(1+2/x\right)}{1/x} \quad \frac{0}{0}$$

$$= \lim_{x\to\infty} \frac{\frac{1}{1+2/x} \cdot \frac{-2}{x^2}}{-1/x^2}$$

$$\ln y = \lim_{x\to\infty} \frac{2}{1+2/x}$$

$\ln y = 2$

$y = e^2$

23. $\lim_{x\to 0^+} (\sin x)^x \quad 0^0$

$\ln y = \lim_{x\to 0^+} x \ln \sin x \quad 0 \cdot -\infty$

$= \lim_{x\to 0^+} \dfrac{\ln \sin x}{1/x} \quad \dfrac{-\infty}{\infty}$

$= \lim_{x\to 0^+} \dfrac{\frac{1}{\sin x} \cdot \cos x}{\frac{-1}{x^2}}$

$= \lim_{x\to 0^+} \dfrac{-x^2 \cos x}{\sin x} \quad \dfrac{0}{0}$

$= \lim_{x\to 0^+} \dfrac{-x^2 \cdot -\sin x + \cos x(-2x)}{\cos x}$

$= \lim_{x\to 0^+} \dfrac{x^2 \sin x - 2x\cos x}{\cos x}$

$= \dfrac{0-0}{1}$

$e^{\ln y} = e^0$

$y = 1 \longrightarrow \boxed{\lim_{x\to 0^+} (\sin x)^x = 1}$

1. a) $\dfrac{n!}{(n-2)!} = \dfrac{n(n-1)\cancel{(n-2)}\ldots}{\cancel{(n-2)}\cancel{(n-3)}\cancel{(n-4)}\ldots} = \boxed{n^2 - n}$

b) $\dfrac{2^{n+2} n!}{2^{n+1}(n+1)!} = \dfrac{2n!}{(n+1)!} \quad \dfrac{2\cancel{n}\cancel{(n-1)}\cancel{(n-2)}\ldots}{(n+1)\cancel{(n)}\cancel{(n-1)}\cancel{(n-2)}\ldots} = \boxed{\dfrac{2}{n+1}}$

2. a) $|r| = 2/3 < 1 \Rightarrow$ converges

$a_1 = 1$

$r = -2/3 \qquad S = \frac{1}{1+2/3} = \frac{1}{5/3} = \boxed{\frac{3}{5}}$

b) $\frac{2}{1} + \frac{2}{2} + \frac{2}{3} + \frac{2}{4} + \frac{2}{5}$

$2 + 1 + \frac{2}{3} + \frac{1}{2} + \frac{2}{5}$

$\frac{7}{2} + \frac{2}{3} + \frac{2}{5}$

$\frac{25}{6} + \frac{2}{5}$

$\boxed{\frac{137}{30}}$

c) $\frac{1}{4n^2+8n+3} = \frac{A}{2n+1} + \frac{B}{2n+3}$

$(2n+1)(2n+3)$

$1 = A(2n+3) + B(2n+1)$

$n = -3/2$

$1 = B(2(-\frac{3}{2})+1)$

$1 = B(-3+1)$

$1 = -2B$

$-\frac{1}{2} = B$

$n = -\frac{1}{2}$

$1 = A(2(-\frac{1}{2})+3)$

$1 = A(2)$

$\frac{1}{2} = A$

$$\sum_{n=1}^{\infty} \frac{1}{2(2n+1)} - \frac{1}{2(2n+3)} = \left(\frac{1}{6} - \cancel{\frac{1}{10}}\right) + \left(\cancel{\frac{1}{10}} - \cancel{\frac{1}{14}}\right) + \left(\cancel{\frac{1}{14}} + \frac{1}{18}\right) \ldots$$

$= \boxed{\frac{1}{6}}$

3. a) bounded, not monotonic
b) monotonic (always increasing) not bounded (upper bound $x=3$ but no lower bound)

AP Multiple choice Problems

$\boxed{D}$ 1. $\sum \frac{1}{n^2}$ - convergent - p series $p>1$

$\sum \frac{1}{n}$ - divergent - harmonic

$\sum \frac{(-1)^n}{\sqrt{n}}$ convergent alternating

$\lim_{n\to\infty} \frac{1}{n^{1/2}} = 0$ ✓ $a_{n+1} < a_n$ ✓

$\boxed{C}$ 2. $\sum \frac{1}{n^2}$ - convergent - p series $p>1$

$\sum \frac{1}{n}$ - divergent - harmonic

$\sum \frac{(-1)^{n+1}}{3^{n-1}}$ convergent alternating

$\lim_{n\to\infty} \frac{1}{3^{n-1}} = 0$ ✓ $a_{n-1} < a_n$ ✓

$\boxed{A}$ 3. $\int_2^{\infty} \frac{dx}{x^2}$

$\lim_{a\to\infty} \int_2^a x^{-2}\,dx$

$\lim_{a\to\infty} \left. \frac{-1}{x} \right|_2^a = \lim_{a\to\infty} -\frac{1}{a} + \frac{1}{2} = \boxed{\frac{1}{2}}$

I.

$\boxed{A}$ 4. $\sum_{k=3}^{\infty} \frac{2}{k^2+1}$ compare to $\frac{1}{k^2}$ convergent p-series

$$\lim_{n\to\infty} \frac{2}{k^2+1} \cdot \frac{k^2}{1} = 2$$

$\therefore \sum_{k=3}^{\infty} \frac{2}{k^2+1}$ is convergent by Limit Comparison Test

II. Convergent geometric $r < 1$

III. Convergent alternating

$\boxed{D}$ 5. $f(x) = \sum_{k=1}^{\infty} (\sin^2 x)^k$

$$f(1) = \sum_{k=1}^{\infty} (\sin^2 1)^k$$

$$= \sum_{k=1}^{\infty} (.708)^k \quad \text{convergent geometric}$$

$$S = \frac{.708}{1-.708} = \boxed{2.426}$$

$\boxed{C}$ 6. $\int_1^{\infty} \frac{x}{(1+x^2)^2}\,dx$

$u = 1+x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$\frac{1}{2}\int \frac{1}{u^2}\,du$$

$$= \lim_{a\to\infty} \frac{1}{2}\left(-\frac{1}{(1+x^2)}\right)\Bigg]_1^a$$

$$\lim_{a\to\infty} \frac{1}{2}\left(\frac{-1}{1+a^2} + \frac{1}{2}\right)$$

$$\frac{1}{2}\left(\frac{1}{2}\right)$$

$$\boxed{\frac{1}{4}}$$

$\frac{9}{16} \cdot \frac{2}{2} = \frac{3}{8}$

(4)

$\boxed{C}$ 7. $S = \dfrac{\frac{3}{2}}{1-\frac{3}{8}} = \dfrac{\frac{3}{2}}{\frac{5}{8}} = \frac{3}{2} \cdot \frac{8}{5} = \frac{24}{10} = \boxed{\frac{12}{5}}$

$\boxed{D}$ 8. I. $\lim_{n\to\infty} \frac{5n}{2n-1} = \frac{5}{2}$ ✓

II $\lim_{n\to\infty} \frac{e^n}{n} = \infty$

III. $\lim_{n\to\infty} \frac{e^n}{1+e^n} = 1$ ✓

$\boxed{D}$ 9. $\lim_{x\to 0} \frac{e^{2x}-1}{\tan x} \quad \frac{0}{0}$

$\lim_{x\to 0} \frac{2e^{2x}}{\sec^2 x} = \frac{2}{1} = \boxed{2}$

$\boxed{A}$ 10. I. convergent p-series $p > 1$

II. divergent p-series $p < 1$

III. $\frac{1}{n+4}$ compare to $\frac{1}{n}$ divergent

$$\lim_{n\to\infty} \frac{\frac{1}{n+4}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{1}{n+4} \cdot \frac{n}{1} = \lim_{n\to\infty} \frac{n}{n+4} = 1 \quad \therefore \sum_{n=1}^{\infty} \frac{1}{n+4}$$

diverges by LCT

$\boxed{E}$ 11. A. $\lim_{n\to\infty} \frac{k+3}{k+\sqrt{k}} = 1 \neq 0$ divergent n^{th} term test

B. $\sum \frac{3}{2\sqrt{k}}$ diverges by p-series $p < 1$

C. diverges by LCT to $\frac{1}{k^{1/2}}$

D. diverges by LCT to $\frac{1}{k}$

E. $\sum \frac{3}{k^{3/2}}$ converges by p-series $p > 1$.