AP Calculus — Series Review

MULTIPLE-CHOICE QUESTIONS

No calculators are to be used for Questions 1–8.

1. Which of the following series is absolutely convergent?

(A) $\sum_{k=0}^{\infty}(-1)^k \frac{k+3}{k+\sqrt{k}}$

(B) $\sum_{k=0}^{\infty}(-1)^k \frac{3}{\sqrt{k}}$

(C) $\sum_{k=0}^{\infty}(-1)^{k+1} \frac{\sqrt{k}}{k+3}$

(D) $\sum_{k=0}^{\infty}(-1)^k \frac{3}{k+3}$

(E) $\sum_{k=0}^{\infty}(-1)^{k+1} \frac{3}{k\sqrt{k}}$

2. The power series $\sum_{k=1}^{\infty}\frac{x^k}{k}$ converges for which values of x?

(A) $x = 0$
(B) $-1 < x < 1$
(C) $-1 \le x < 1$
(D) $-1 \le x \le 1$
(E) x is any real number.

3. Which of the following series is the power series expansion for $f(x) = x(\cos x - 1)$?

(A) $x - \frac{x^3}{2} + \frac{x^5}{24} - \cdots$
(B) $-x^3 + x^5 - x^7 + \cdots$
(C) $\frac{x^3}{2} - \frac{x^5}{24} + \frac{x^7}{720} - \cdots$
(D) $-\frac{x^3}{2} + \frac{x^5}{24} - \frac{x^7}{720} + \cdots$
(E) $1 - \frac{x^2}{2} + \frac{x^4}{24} - \cdots$

4. What are all values of a for which the series $\sum_{k=0}^{\infty}\left(\frac{5}{9-a}\right)^k$ converges?

(A) a < 4
(B) 4 < a < 14
(C) a < 9
(D) a < 9 or a > 9
(E) a < 4 or a > 14

5. The Maclaurin series for $f(x) = \frac{1}{1+x^2}$ is $\sum_{k=0}^{\infty}(-1)^k x^{2k}$. What is the Maclaurin series for $g(x) = \tan^{-1} x$?

(A) $\sum_{k=0}^{\infty}(-1)^k (2k)x^{2k-1}$

(B) $\sum_{k=0}^{\infty}(-1)^k \frac{x^{2k+1}}{2k+1}$

(C) $\sum_{k=0}^{\infty}(-1)^{k+1} \frac{x^{2k+1}}{2k}$

(D) $\sum_{k=0}^{\infty}(-1)^k (2k+1)x^{2k+1}$

(E) $\sum_{k=0}^{\infty}(-1)^{k-1} \frac{x^{2k-1}}{2k}$

6. The Maclaurin series $\sum_{k=0}^{\infty}(-9)^k \frac{x^{4k}}{(2k)!}$ represents which function below?

(A) $\cos(3x^2)$
(B) $\sin(3x^2)$
(C) $\cos(9x^4)$
(D) $\tan^{-1}(3x)$
(E) e^{-3x^2}

7. If the first five terms of the Taylor expansion for $f(x)$ about $x = 0$ are $3 - 7x + \frac{5}{2}x^2 + \frac{3}{4}x^3 - 6x^4$, then $f'''(0) =$

(A) $\frac{1}{8}$
(B) $\frac{3}{4}$
(C) $\frac{9}{2}$
(D) 6
(E) 8

8. Which of the following series diverge?

I. $\sum_{k=0}^{\infty}\frac{k^{\frac{3}{2}}+1}{5k^2+7}$

II. $\sum_{k=2}^{\infty}(-1)^k \frac{1}{\ln(k)}$

III. $\sum_{k=0}^{\infty}(-1)^k \left(\frac{4}{3}\right)^k$

(A) I only
(B) II only
(C) I and II only
(D) I and III only
(E) I, II, and III

You may use a calculator for Questions 9–10.

9. The sixth degree term of the Taylor series expansion for $f(x)=e^{-\frac{1}{2}x^2}$ about $x=0$ has coefficient

(A) $-\frac{1}{48}$

(B) $-\frac{1}{6}$

(C) $\frac{1}{720}$

(D) $\frac{1}{6}$

(E) $-\frac{1}{4608}$

10. Let $T_n(x)$ represent the Taylor Polynomial of degree n about $x=0$ for $f(x)=e^{-x}$. If $T_n(2)$ is used to approximate the value of $f(2)=e^{-2}$, then which of the following expressions is NOT less than $\frac{2^7}{7!}$?

(A) $f(2)-T_6(2)$
(B) $T_6(2)-f(2)$
(C) $f(2)-T_5(2)$
(D) $T_5(2)-f(2)$
(E) $f(2)-T_7(2)$

FREE-RESPONSE QUESTION

You may use a calculator for this question.

Let $f(x)$ be a function that is differentiable for all x. The Taylor expansion for $f(x)$ about $x=0$ is given by $T(x)=\sum_{k=0}^{\infty}(-1)^k\frac{(k+1)x^{3k}}{k!}$. The first four nonzero terms of $T(x)$ are given by $T_4(x)=1-2x^3+\frac{3x^6}{2!}-\frac{4x^9}{3!}$.

a. Show that $T(x)$ converges for all x.
b. Let $W(x)$ be the Taylor expansion for $x^2f'(x)$ about $x=0$. Find the general term for $W(x)$ and find $W_3(x)$.
c. $f(0.5)\approx T_4(0.5)$ and $f(1)\approx T_4(1)$. Find the values of $T_4(0.5)$ and $T_4(1)$.
d. Which value is smaller, $|f(0.5)-T_4(0.5)|$ or $|f(1)-T_4(1)|$? Give a reason for your answer.

10. Let $T_n(x)$ represent the Taylor Polynomial of degree n about $x=0$ for $f(x)=e^{-x}$. If $T_n(2)$ is used to approximate the value of $f(2)=e^{-2}$, then which of the following expressions is NOT less than $\frac{2^7}{7!}$?

(A) $f(2)-T_6(2)$
(B) $T_6(2)-f(2)$
(C) $f(2)-T_5(2)$
(D) $T_5(2)-f(2)$
(E) $f(2)-T_7(2)$

*11. For function $f(x)$, $f(0)=3$, $f'(0)=2$, $f''(0)=5$, *and* $f'''(0)=4$. Using the Taylor series expansion for $f(x)$ about $x=0$, the second degree estimate for $f'(0.1)$ is

(A) 2
(B) 2.500
(C) 2.520
(D) 3.120
(E) 3.225

A calculator may not be used for the following questions.

12. The Maclaurin series $\sum_{k=0}^{\infty}(-5)^k\frac{x^{3k+2}}{k!}$ represents which expression below?

(A) $x^2e^{-5x^3}$
(B) $-5e^{3x+2}$
(C) $x^2\sin(-5x^3)$
(D) $-5\cos(x^3)+2$
(E) $\tan^{-1}(-5x^3+2)$

13. If $f(x)=\sin(x^2)$, the first three terms of the Taylor series expansion about $x=0$ for $f'(x)$ are

(A) $1-2x^2+\frac{2}{3}x^4$.
(B) $2x-x^3+\frac{1}{2}x^5$.
(C) $1-\frac{1}{2}x^4+\frac{1}{16}x^8$.
(D) $2x-x^5+\frac{1}{12}x^9$.
(E) $x^2-\frac{1}{6}x^6+\frac{1}{120}x^{10}$.

14. Which of the following series is conditionally convergent?

(A) $\sum_{k=1}^{\infty}(-1)^k\frac{k^2+1}{k+4}$
(B) $\sum_{k=1}^{\infty}(-1)^ke^{-k}$
(C) $\sum_{k=1}^{\infty}(-1)^{k-1}\frac{2}{k+1}$
(D) $\sum_{k=1}^{\infty}(-1)^{k+1}\ln(k+1)$
(E) $\sum_{k=1}^{\infty}(-1)^k\frac{k+3}{k^3+7}$

15. The radius of convergence of the power series $\sum_{k=0}^{\infty}\frac{x^{2k}}{k\cdot 4^k}$ is

(A) 0
(B) 1
(C) 2
(D) 4
(E) ∞