CALCULUS BC
SECTION I, Part A
Time—55 minutes
Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and place the letter of your choice in the corresponding box on the student answer sheet. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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HUMPHREY

1. If $f(x) = \frac{3x-2}{2x+3}$, then $f'(x) =$

(A) $-\frac{13}{(2x+3)^2}$

(B) $\frac{3}{(2x+3)^2}$

(C) $\frac{5}{(2x+3)^2}$

(D) $\frac{13}{(2x+3)^2}$

(E) $\frac{12x+5}{(2x+3)^2}$

$$f'(x) = \frac{(2x+3)(3) - (3x-2)(2)}{(2x+3)^2}$$

$$= \frac{\cancel{6x}+9-\cancel{6x}+4}{(2x+3)^2}$$

$$\frac{13}{(2x+3)^2}$$

2. A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = \sin(2t)$. If the position of the particle at time $t = \frac{\pi}{2}$ is $x = 4$, what is the particle's position at time $t = 0$?

(A) $-\frac{1}{2}$ (B) 2 (C) 3 (D) 5 (E) 8

$v(t) = \sin(2t)$

$s(t) = \int_0^{\pi/2} \sin(2t)\,dt$

$u = 2t$
$du = 2dt$
$\frac{1}{2}du = dt$

$s(t) = \frac{1}{2}\int \sin u\,du$

$-\frac{1}{2}\cos(2t)\Big]_0^{\pi/2}$

$-\frac{1}{2}\cos\pi + \frac{1}{2}\cos 0$

$\frac{1}{2} + \frac{1}{2}$

!

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$t=0$ $t=\frac{\pi}{2}$
$x=3$ $x=4$

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3. What is the value of $\sum_{n=0}^{\infty}\left(\frac{-2}{3}\right)^n$?

(A) -2 (B) $-\frac{2}{5}$ (C) $\frac{3}{5}$ (D) 3 (E) The series diverges.

$\frac{1}{1+2/3}$

4. For values of h very close to 0, which of the following functions best approximates $f(x) = \frac{\tan(x+h) - \tan x}{h}$?

(A) $\sin x$

(B) $\frac{\sin x}{x}$

(C) $\frac{\tan x}{x}$

(D) $\sec x$

(E) $\sec^2 x$

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5. The length of the curve $y = x^4$ from $x = 1$ to $x = 5$ is given by

(A) $\int_1^5 \sqrt{1+4x^3}\,dx$

(B) $\int_1^5 \sqrt{1+x^4}\,dx$

(C) $\int_1^5 \sqrt{1+4x^6}\,dx$

(D) $\int_1^5 \sqrt{1+16x^6}\,dx$

(E) $\int_1^5 \sqrt{1+x^8}\,dx$

6. $\int \frac{e^x}{1+e^x}\,dx =$

$u = 1+e^x$

$du = e^x dx$

(A) $\ln\left(\frac{1}{e^x}+1\right)+C$

(B) $\ln\left(1+e^x\right)+C$

(C) $x - \ln\left(1+e^x\right)+C$

(D) $e^x + x + C$

(E) $\tan^{-1}\left(e^x\right)+C$

$\int \frac{1}{u}\,du$

$\ln|1+e^x| + C$

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7. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x - y - 1$ with the initial condition $f(1) = -2$. What is the approximation for $f(1.4)$ if Euler's method is used, starting at $x = 1$ with two steps of equal size?

(A) -2 (B) -1.24 (C) -1.2 (D) -0.64 (E) 0.2

$\frac{dy}{dx} = x - y - 1$

$dy = (1+2-1)(.2)$

$= .4$

$(1, -2)$

$(1.2, -1.6)$

$(1.4, -1.24)$

$dy = (1.2 + 1.6 - 1)(.2)$

$= (1.8)(.2)$

$= .36$

x	0	2	4	6
$f(x)$	4	k	8	12

8. The function f is continuous on the closed interval $[0, 6]$ and has the values given in the table above. The trapezoidal approximation for $\int_0^6 f(x)\,dx$ found with 3 subintervals of equal length is 52. What is the value of k?

(A) 2 (B) 6 (C) 7 (D) 10 (E) 14

$52 = \frac{1}{2}\left[(4+k)2 + (k+8)(2) + (8+12)2\right]$

$104 = 8 + 2k + 2k + 16 + 40$

$104 = 4k + 64$

$-64 \quad -64$

$40 = 4k$

$10 = k$

32
24
56

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9. The function f is twice differentiable, and the graph of f has no points of inflection. If $f(6) = 3$, $f'(6) = -\frac{1}{2}$, and $f''(6) = -2$, which of the following could be the value of $f(7)$?

(A) 2 (B) 2.5 (C) 2.9 (D) 3 (E) 4

must be less

tangent line

approximation

$y - 3 = -\frac{1}{2}(x - 6)$

$y - 3 = -\frac{1}{2}(7 - 6)$

$y - 3 = -\frac{1}{2}$

$y = 2.5$

10. A function f has Maclaurin series given by $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots + \frac{x^{2n}}{(2n)!} + \cdots$. Which of the following is an expression for $f(x)$?

(A) $\cos x$

(B) $e^x - \sin x$

(C) $e^x + \sin x$

(D) $\frac{1}{2}(e^x + e^{-x})$

(E) e^{x^2}

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$

$e^{-x} = 1 - x + \frac{(-x)^2}{2!} + \frac{(-x)^3}{3!} + \frac{(-x)^4}{4!}$

$\frac{1}{2}\left(2 + x^2 + \frac{2x^4}{4!}\right)$

$1 + \frac{x^2}{2} + \frac{x^4}{4!}$

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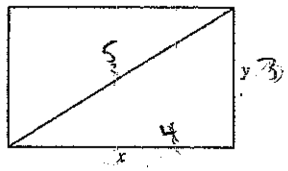

11. The sides and diagonal of the rectangle above are strictly increasing with time. At the instant when $x = 4$ and $y = 3$, $\frac{dx}{dt} = \frac{dz}{dt}$ and $\frac{dy}{dt} = k\frac{dz}{dt}$. What is the value of k at that instant?

(A) $\frac{1}{4}$

(B) $\frac{1}{3}$

(C) 3

(D) 4

(E) It cannot be determined from the information given.

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$2(4)\frac{dz}{dt} + 2(3)k\frac{dz}{dt} = 2(5)\frac{dz}{dt}$

$8\frac{dz}{dt} + 6k\frac{dz}{dt} = 10\frac{dz}{dt}$

$6k\frac{dz}{dt} = 2\frac{dz}{dt}$

$k = \frac{1}{3}$

12. If $f'(x) = \frac{2}{x}$ and $f(\sqrt{e}) = 5$, then $f(e) =$

(A) 2 (B) ln 25 (C) $5 + \frac{2}{e} - \frac{2}{e^2}$ (D) 6 (E) 25

$f'(x) = \frac{2}{x}$ $f(\sqrt{e}) = 5$

$\int \frac{2}{x}$

$2\ln|x| + C$ $f(x) = 2\ln|x| + 4$

$5 = 2\ln\sqrt{e} + C$ $f(e) =$

$5 = 1 + C$

$4 = C$

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13. For time $t > 0$, the position of a particle moving in the xy-plane is given by the parametric equations $x = 4t + t^2$ and $y = \frac{1}{3t+1}$. What is the acceleration vector of the particle at time $t = 1$?

(A) $\left(2, \frac{1}{32}\right)$

(B) $\left(2, \frac{9}{32}\right)$

(C) $\left(5, \frac{1}{4}\right)$

(D) $\left(6, -\frac{3}{16}\right)$

(E) $\left(6, -\frac{1}{16}\right)$

$\langle 4t + t^2, \frac{1}{3t+1} \rangle$

$v(t) = \langle 4 + 2t, \frac{-3}{(3t+1)^2} \rangle$

$a(t) = \langle 2, \frac{-3(2(3t+1)\cdot 3)}{(3t+1)^4} \rangle$

$= \langle 2, \frac{18}{(3t+1)^3} \rangle$

$= \langle 2, \frac{18}{64} \rangle$

14. $\int \frac{8}{x^2 - 4}\,dx =$

(A) $4\tan^{-1}\left(\frac{x}{2}\right) + C$

(B) $8\ln|x^2 - 4| + C$

(C) $2\ln\left|\frac{x-2}{x+2}\right| + C$

(D) $2\ln\left|\frac{x+2}{x-2}\right| + C$

(E) $2\ln|x+2| + 2\ln|x-2| + C$

$\int \frac{8}{(x-2)(x+2)}$ $\frac{A}{x-2} + \frac{B}{x+2}$

$8 = (x+2)A + B(x-2)$

$x = -2$ $x = 2$

$8 = -4B$ $8 = 4A$

$-2 = B$ $2 = A$

$\int \frac{2}{x-2} - \frac{2}{x+2}\,dx$

$2\ln|x-2| - 2\ln|x+2| + C$

$2\ln\frac{|x-2|}{|x+2|} + C$

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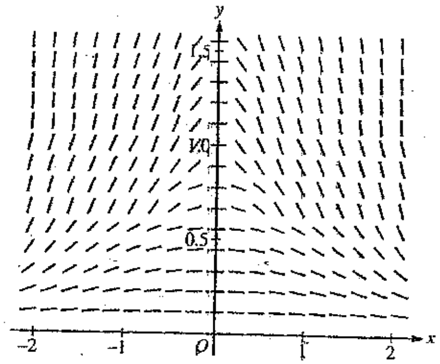

15. The slope field for a certain differential equation is shown above. Which of the following could be a solution to the differential equation with the initial condition $y(0) = 1$?

(A) $y = \cos x$

(B) $y = 1 - x^2$

(C) $y = e^x$

(D) $y = \sqrt{1 - x^2}$

(E) $y = \dfrac{1}{1 + x^2}$

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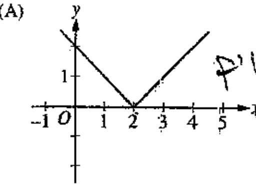
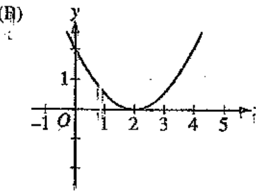
16. If $f'(x) = |x - 2|$, which of the following could be the graph of $y = f(x)$?

(A)

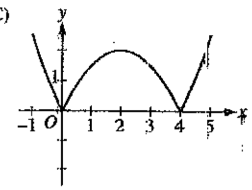


f'(x)

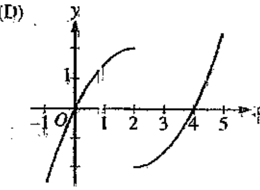
(B)



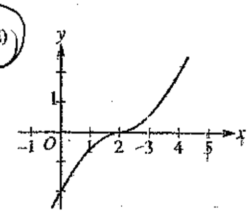
(C)



(D)



(E)



f'(x) = 0 @ x = 2

f'(x) decreasing where x<2

f'(x) increasing where x>2

f'(x) always +

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17. The radius of convergence for the power series $\sum_{n=1}^{\infty} \frac{(x-3)^{2n}}{n}$ is equal to 1. What is the interval of convergence?

~~(A) $-4 \le x < -2$~~
~~(B) $-1 < x < 1$~~
~~(C) $-1 \le x < 1$~~
(D) $2 < x < 4$
(E) $2 \le x < 4$

2, 4

$\sum_{n=1}^{\infty} \frac{(2-3)^{2n}}{n}$

$\sum_{n=1}^{\infty} \frac{(-1)^{2n}}{n}$

Harmonic ⇒ diverges

18. If $f(x) = \arccos(x^2)$, then $f'(x) =$

(A) $\frac{1}{\sqrt{1-x^4}}$

(B) $\frac{-2x}{\sqrt{1-x^4}}$

(C) $\frac{2x}{\sqrt{1-x^4}}$

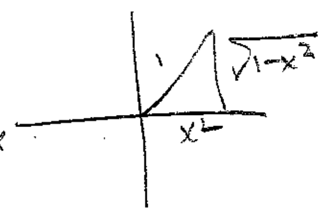
(D) $\frac{-4x^3}{\sqrt{1-x^4}}$

(E) $\frac{4x^3}{\sqrt{1-x^4}}$

$f(x) = \arccos(x)^2$

$\cos y = x^2$

$-\sin y \frac{dy}{dx} = 2x$



$\frac{dy}{dx} = \frac{2x}{\sin y}$

$\frac{dy}{dx} = \frac{2x}{\sqrt{1-x^4}}$

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19. What is the slope of the line tangent to the curve $y + 2 = \frac{x^2}{2} - 2\sin y$ at the point $(2, 0)$?

(A) -2 (B) 0 (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) 2

20. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

II. $\sum_{n=1}^{\infty} \frac{3^n}{n!}$

III. $\sum_{n=1}^{\infty} \left(\frac{e}{\pi}\right)^n$

(A) None
(B) II only
(C) III only
(D) I and II only
(E) II and III only

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21. The function f given by $f(x) = 9x^{2/3} + 3x - 6$ has a relative minimum at $x =$

(A) -8 (B) $-\sqrt[3]{2}$ (C) -1 (D) $-\frac{1}{8}$ (E) 0

$f'(x) = \frac{2}{3} \cdot 9 \cdot x^{-1/3} + 3$

$0 = \frac{6}{x^{1/3}} + 3$

$-3x^{1/3} = 6$

$x^{1/3} = -2$

$-\frac{1}{8} = x$

-8 0

x	$f(x)$	$f'(x)$
0	2	5
4	−3	11

22. The function f has a continuous derivative. The table above gives values of f and its derivative for $x = 0$ and $x = 4$. If $\int_0^4 f(x)\,dx = 8$, what is the value of $\int_0^4 x f'(x)\,dx$?

(A) −20 (B) −13 (C) −12 (D) −7 (E) 36

$x = u$ $\quad f'(x) = dv$

$du = dx$ $\quad f(x) = v$

$\int_0^4 x f'(x)\,dx = x f(x) - \int_0^4 f(x)\,dx$

8

$\Big]_0^4$

$-12 - 8$

-20

$\frac{2}{3} \cdot 9x^{-1/3} + 3$

$0 = \frac{6}{x^{1/3}} + 3$

$-3 = \frac{6}{x^{1/3}}$

$x^{1/3} = -2$

$x = -8$

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23. What is the slope of the line tangent to the polar curve $r = 2\theta$ at the point $\theta = \frac{\pi}{2}$?

(A) $-\frac{\pi}{2}$ (B) $-\frac{2}{\pi}$ (C) 0 (D) $\frac{\pi}{2}$ (E) 2

$x = r\cos\theta$ $\quad y = r\sin\theta$

$x = 2\theta\cos\theta$ $\quad y = 2\theta\sin\theta$

$\frac{dx}{d\theta} = 2\theta \cdot \sin\theta + 2\cos\theta$ $\quad \frac{dy}{d\theta} = 2\theta\cos\theta + 2\sin\theta$

$\frac{dy}{dx} = -\frac{\cancel{2}(\theta\cos\theta + \sin\theta)}{\cancel{2}(\theta\sin\theta - \cos\theta)}$

$= -\frac{\left(\frac{\pi}{2}\cos\frac{\pi}{2} + \sin\frac{\pi}{2}\right)}{\frac{\pi}{2}\sin\frac{\pi}{2} - \cos\frac{\pi}{2}}$

$= \frac{\frac{\pi}{2}(0) + 1}{\frac{\pi}{2} - 0}$

$-\frac{2}{\pi}$

24. The radius of a circle is increasing. At a certain instant, the rate of increase in the area of the circle is numerically equal to twice the rate of increase in its circumference. What is the radius of the circle at that instant?

(A) $\frac{1}{2}$ (B) 1 (C) $\sqrt{2}$ (D) 2 (E) 4

$A = \pi r^2$ $\quad C = 2\pi r$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ $\quad \frac{dC}{dt} = 2\pi \frac{dr}{dt}$

$2\pi r \frac{dr}{dt} = 2\left(2\pi \frac{dr}{dt}\right)$

$2\cancel{\pi} r \cancel{\frac{dr}{dt}} = 4\cancel{\pi} \cancel{\frac{dr}{dt}}$

$2r = 4$

$r = 2$

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n	$\sum_{k=1}^{n}\left(\frac{1}{x_k}\right)\left(\frac{1}{n}\right)$
100	5.19
200	5.88
300	6.28
400	6.57
500	6.79

25. The table above shows several Riemann sum approximations to $\int_0^1 \frac{1}{x}\,dx$ using right-hand endpoints of n subintervals of equal length of the interval $[0, 1]$. Which of the following statements best describes the limit of the Riemann sums as n approaches infinity?

(A) The limit of the Riemann sums is a finite number less than 10.

(B) The limit of the Riemann sums is a finite number greater than 10.

(C) The limit of the Riemann sums does not exist because $\left(\frac{1}{x_n}\right)\left(\frac{1}{n}\right)$ does not approach 0.

(D) The limit of the Riemann sums does not exist because it is a sum of infinitely many positive numbers.

(E) The limit of the Riemann sums does not exist because $\int_0^1 \frac{1}{x}\,dx$ does not exist.

$\lim_{a \to 0^+} \int_a^1 \frac{1}{x}$

$\lim_{a \to 0^+} \ln|x| \Big]_a^1$

$\lim_{a \to 0^+} \ln|1| - \ln a$

$0 + \infty$

∞

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26. The coefficients of the power series $\sum_{n=0}^{\infty} a_n(x-2)^n$ satisfy $a_0 = 5$ and $a_n = \left(\frac{2n+1}{3n-1}\right)a_{n-1}$ for all $n \geq 1$. The radius of convergence of the series is

(A) 0 (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) 2 (E) infinite

$\frac{2}{3}|x-2| < 1$

$|x-2| < \frac{3}{2}$

27. If f is the function given by $f(x) = \int_4^{2x} \sqrt{t^2 - t}\,dt$, then $f'(2) =$

(A) 0 (B) $\frac{7}{2\sqrt{12}}$ (C) $\sqrt{2}$ (D) $\sqrt{12}$ (E) $2\sqrt{12}$

$2\left(\sqrt{(2x)^2 - 2x}\right)$

$2\sqrt{(2(2))^2 - 4}$

$2\sqrt{16-4}$

$2\sqrt{12}$

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$\frac{\sin x}{x}$

$\lim_{x \to \infty} \sin\left(\frac{x+1}{x^2}\right) = 0$

$\sin \frac{1}{0}$

28. The function f is given by $f(x) = \sin\left(\frac{x+1}{x^2}\right)$. Which of the following statements are true?

I. The graph of f has a horizontal asymptote at $y = 0$.

II. The graph of f has a horizontal asymptote at $y = 1$.

III. The graph of f has a vertical asymptote at $x = 0$.

(A) I only

~~(B) II only~~

~~(C) III only~~

~~(D) I and III only~~

~~(E) II and III only~~

$$f'(x) = \cos\left(\frac{x+1}{x^2}\right) \cdot \frac{x^2(1) - (x+1)(2x)}{x^4}$$

$$= \cos\left(\frac{x+1}{x^2}\right) \cdot \frac{\cancel{x^2} - \cancel{x^2} - 2x}{x^4}$$

$$= -\frac{2}{x^3} \cos\left(\frac{x+1}{x^2}\right)$$

END OF PART A OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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CALCULUS BC
SECTION I, Part B
Time—50 minutes
Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and place the letter of your choice in the corresponding box on the student answer sheet. Do not spend too much time on any one problem.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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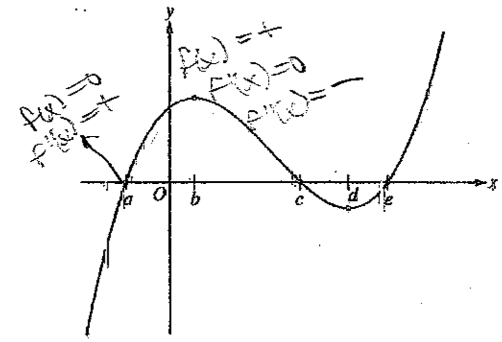
29) 76. If $\int_0^1 f(x)\,dx = 2$ and $\int_0^4 f(x)\,dx = -3$, then $\int_1^4 (3f(x)+2)\,dx =$

(A) -13 (B) -9 ~~(C) -7~~ (D) 3 (E) 21

$-9-6+6$

$3\int_1^4 f(x) + \int_1^4 2$

$3\int_0^4 f(x) - 3\int_0^1 f(x) + 6$



Graph of f

30) 77. The figure above shows the graph of the polynomial function f. For which value of x is it true that $f''(x) < f'(x) < f(x)$?

(A) a (B) b (C) c (D) d (E) e

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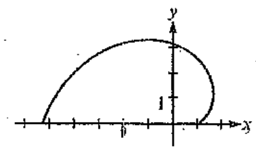
31)



78. The graph above shows the polar curve $r = 2\theta + \cos\theta$ for $0 \le \theta \le \pi$. What is the area of the region bounded by the curve and the x-axis?

(A) 3.069 (B) 4.935 (C) 9.870 (D) 17.456 (E) 34.912

$\frac{1}{2}\int_0^{\pi} (2\theta + \cos\theta)^2 \, d\theta$

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32) 79. If f is a differentiable function such that $f(3) = 8$ and $f'(3) = 5$, which of the following statements could be false?

(A) $\lim_{x \to 3} f(x) = 8$ ✓ must be cont.

(B) $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^-} f(x)$ ✓

(C) $\lim_{x \to 3} \frac{f(x) - 8}{x - 3} = 5$ ✓

(D) $\lim_{h \to 0} \frac{f(3+h) - 8}{h} = 5$ def der. @ 3

(E) $\lim_{x \to 3} f'(x) = 5$

$f(3) = 8$

$f'(3) = 5$

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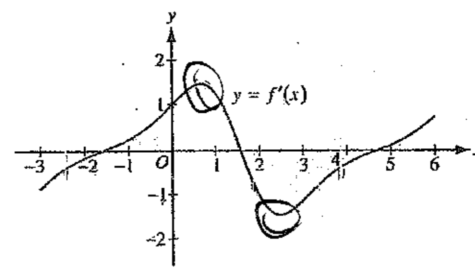
B B B B B B B B B

33)



80. The figure above shows the graph of f', the derivative of the function f, on the interval $[-3, 6]$. If the derivative of the function h is given by $h'(x) = 2f'(x)$, how many points of inflection does the graph of h have on the interval $[-3, 6]$?

(A) One (B) Two (C) Three (D) Four (E) Five

---

34) 81. Let R be the region in the first quadrant bounded by the y-axis, the x-axis, the graph of $y = e^{-x^2/2}$, and the line $x = 3$. The region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. What is the volume of the solid?

(A) 0.886 (B) 0.906 (C) 1.078 (D) 1.245 (E) 2.784

$y = e^{-x^2/2}$

$\int_0^3 \left(e^{-x^2/2}\right)^2 dx = .886$

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35)

82. If f is a continuous function on the closed interval $[a, b]$, which of the following must be true?

(A) There is a number c in the open interval (a, b) such that $f(c) = 0$.

(B) There is a number c in the open interval (a, b) such that $f(a) < f(c) < f(b)$.

(C) There is a number c in the closed interval $[a, b]$ such that $f(c) \geq f(x)$ for all x in $[a, b]$.

(D) There is a number c in the open interval (a, b) such that $f'(c) = 0$. could be a cusp

(E) There is a number c in the open interval (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

not necessarily differentiable!

MVT

36)

x	2.5	2.8	3.0	3.1
$f(x)$	31.25	39.20	45	48.05

83. The function f is differentiable and has values as shown in the table above. Both f and f' are strictly increasing on the interval $0 \leq x \leq 5$. Which of the following could be the value of $f'(3)$?

(A) 20 (B) 27.5 (C) 29 (D) 30 (E) 30.5

$(2.8, 39.20)$ $(3.1, 48.55)$ $\frac{\Delta y}{\Delta x} = \frac{8.85}{.3} = 29.5$

$(2.8, 39.20)$ $(3.0, 45)$ $\frac{\Delta y}{\Delta x} = \frac{5.8}{.2} = 29$

$\therefore f'(3)$ could = 30

$(3.0, 45)$ $(3.1, 48.05)$ $\frac{\Delta y}{\Delta x} = \frac{3.05}{.1} = 30.5$

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37)

84. The rate of change, $\frac{dP}{dt}$, of the number of people on an ocean beach is modeled by a logistic differential equation. The maximum number of people allowed on the beach is 1200. At 10 A.M., the number of people on the beach is 200 and is increasing at the rate of 400 people per hour. Which of the following differential equations describes the situation?

(A) $\frac{dP}{dt} = \frac{1}{400}(1200 - P) + 200$

(B) $\frac{dP}{dt} = \frac{2}{5}(1200 - P)$

(C) $\frac{dP}{dt} = \frac{1}{500}P(1200 - P)$

(D) $\frac{dP}{dt} = \frac{1}{400}P(1200 - P)$

(E) $\frac{dP}{dt} = 400P(1200 - P)$

$\frac{dP}{dt} = kP(L-P)$

$400 = k(200)(1200-200)$

$400 = 200{,}000k$

$\frac{1}{500} = k$

$\frac{dP}{dt} = \frac{1}{500}P(1200-P)$

38)

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$
2	0	0	5	7

85. The third derivative of the function f is continuous on the interval $(0, 4)$. Values for f and its first three derivatives at $x = 2$ are given in the table above. What is $\lim_{x \to 2} \frac{f(x)}{(x-2)^2}$?

(A) 0 (B) $\frac{5}{2}$ (C) 5 (D) 7 (E) The limit does not exist.

$\lim_{x \to 2} \frac{f(x)}{(x-2)^2} = \frac{0}{0}$

$\lim_{x \to 2} \frac{f'(x)}{2(x-2)} = \frac{0}{0}$

$\lim_{x \to 2} \frac{f''(x)}{2} = \frac{5}{2}$

L'Hopital's

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39)

86. The Taylor polynomial of degree 100 for the function f about $x = 3$ is given by

$$P(x) = (x-3)^2 - \frac{(x-3)^4}{2!} + \frac{(x-3)^6}{3!} + \cdots + (-1)^{n+1}\frac{(x-3)^{2n}}{n!} + \cdots - \frac{(x-3)^{100}}{50!}.$$ What is the value of $f^{(30)}(3)$?

(A) $-\frac{30!}{15!}$ (B) $-\frac{1}{30!}$ (C) $\frac{1}{30!}$ (D) $\frac{1}{15!}$ (E) $\frac{30!}{15!}$

$\frac{(x-3)^{30}}{15!}$ $\frac{1}{15!} = \frac{f^{30}(3)}{30!}$ $\frac{30!}{15!} = f^{(30)}(3)$

$\frac{30!}{15!} = \frac{f^{(30)}(3)(15)!}{15!}$

40)

87. The function f has derivatives of all orders for all real numbers, and $f^{(4)}(x) = e^{\sin x}$. If the third-degree Taylor polynomial for f about $x = 0$ is used to approximate f on the interval $[0, 1]$, what is the Lagrange error bound for the maximum error on the interval $[0, 1]$?

(A) 0.019 (B) 0.097 (C) 0.113 (D) 0.399 (E) 0.417

$\frac{\max e^{\sin x}}{4!} x^4$ on $[0,1]$

$\frac{e^{\sin 1}}{4!}$

$\frac{2.3198}{4!}$

.0966

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41)

88. The rate at which water is sprayed on a field of vegetables is given by $R(t) = 2\sqrt{1+5t^3}$, where t is in minutes and $R(t)$ is in gallons per minute. During the time interval $0 \le t \le 4$, what is the average rate of water flow, in gallons per minute?

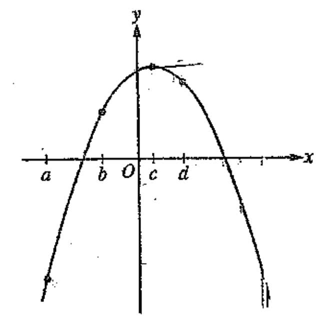
(A) 8.458 (B) 13.395 (C) 14.691 (D) 18.916 (E) 35.833

$R(t) = 2\sqrt{1+5t^3}$

Avg. value $= \frac{1}{4-0}\int_0^4 2\sqrt{1+5t^3}\,dt$

$= \frac{2}{4}\int_0^4 \sqrt{1+5t^3}\,dt$

42)



Graph of f

89. The figure above shows the graph of a function f. Which of the following has the greatest value?

(A) $f(a)$ (B) $f'(a)$ (C) $f'(c)$ (D) $f(d) - f(c)$ (E) $\frac{f(b)-f(a)}{b-a}$

−

+
slope of tangent @ a

0

$(-)-0$
−

+
avg. rate of change b/t a & b

GO ON TO THE NEXT PAGE.

43)

90. The nth derivative of a function f at $x = 0$ is given by $f^{(n)}(0) = (-1)^n \dfrac{n+1}{(n+2)2^n}$ for all $n \geq 0$. Which of the following is the Maclaurin series for f?

(A) $-\frac{1}{2} + \frac{1}{3}x - \frac{3}{32}x^2 + \frac{1}{60}x^3 - \cdots$

(B) $\frac{1}{2} - \frac{1}{3}x + \frac{3}{16}x^2 - \frac{1}{10}x^3 + \cdots$

(C) $\frac{1}{2} + \frac{1}{3}x + \frac{3}{32}x^2 + \frac{1}{60}x^3 + \cdots$

(D) $\frac{1}{2} - \frac{1}{3}x + \frac{3}{32}x^2 - \frac{1}{60}x^3 + \cdots$

(E) $\frac{1}{2} - 3x + \frac{32}{3}x^2 - 60x^3 + \cdots$

alternating starts + b/c $(-1)^0$

$f^0(0) = \dfrac{(-1)^0(0+1)}{(0+2)2^0} = \dfrac{\frac{1}{2}}{0!}$

$f'(0) = \dfrac{(-1)^1(1+1)}{(1+2)(2)^1} = \dfrac{-2}{6} = \dfrac{-\frac{1}{3}}{1!}$

$f''(x) = \dfrac{(-1)^2(2+1)}{(2+2)(2)^2} = \dfrac{\frac{3}{16}}{2!}$

44)

x	$f(x)$	$f'(x)$
0	1	1
1	3	4
2	11	13

91. The table above gives selected values for a differentiable and increasing function f and its derivative. If g is the inverse function of f, what is the value of $g'(3)$?

(A) $\frac{1}{13}$ (B) $\frac{1}{4}$ (C) 1 (D) 4 (E) 13

$f(1) = 3$ $f'(1) = 4$

$g'(3)$ $g(3) = 1$ $g'(3) = \frac{1}{4}$

45)

92. Let f be the function with first derivative defined by $f'(x) = \sin(x^3)$ for $\boxed{0 \leq x \leq 2}$. At what value of x does f attain its maximum value on the closed interval $0 \leq x \leq 2$?

(A) 0 (B) 1.162 (C) 1.465 (D) 1.845 (E) 2

$f'(x) = \sin(x^3)$

1.464

graph $\sin x^3 = 0$ on $[0,2]$

$f'(x) \Rightarrow +$ to $-$ @ $x = 1.464$

END OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART B ONLY.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.