Ratio & Root Test

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15. $\sum_{n=1}^{\infty} n\left(\frac{3}{4}\right)^n$

$$\lim_{n\to\infty} \frac{n+1\left(\frac{3}{4}\right)^{n+1}}{n\left(\frac{3}{4}\right)^n}$$

$$\lim_{n\to\infty} \frac{3n+1}{4n} = \frac{3}{4} < 1 \quad \text{convergence}$$

18. $\sum_{n=1}^{\infty} \frac{n^3}{2^n}$

$$\lim_{n\to\infty} \frac{(n+1)^3}{2^{n+1}} \cdot \frac{2^n}{n^3}$$

$$\lim_{n\to\infty} \frac{(n+1)^3}{2n^3} = \frac{1}{2} < 1 \quad \text{converges}$$

21. $\sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{n!}$

$$\lim_{n\to\infty} \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n}$$

$$\lim_{n\to\infty} \frac{2}{n+1} = 0 < 1 \quad \text{converges}$$

24. $\sum_{n=1}^{\infty} \frac{(2n)!}{n^5}$

$$\lim_{n\to\infty} \frac{(2n+2)!}{(n+1)^5} \cdot \frac{n^5}{2n!} = \frac{(2n+1)(2n+2)n^5}{(n+1)^5}$$

$$= \infty \quad \text{diverges}$$

27. $\sum_{n=0}^{\infty} \frac{3^n}{(n+1)^n}$

$\lim_{n\to\infty} \frac{3^{n+1}}{(n+2)^{n+1}} \cdot \frac{(n+1)^n}{3^n}$

$\lim_{n\to\infty} \frac{3(n+1)^n}{(n+2)^{n+1}}$

Use the Root Test $\lim_{n\to\infty} \sqrt[n]{\frac{3^n}{(n+1)^n}}$

$\lim_{n\to\infty} \frac{3}{n+1} = 0 < 1$ converges

30. $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{4n}}{(2n+1)!}$

$\lim_{n\to\infty} \frac{\frac{2^{4n+4}}{(2(n+1)+1)!}}{\frac{2^{4n}}{(2n+1)!}}$

$\lim_{n\to\infty} \frac{2^{4n+4}}{(2n+3)!} \cdot \frac{(2n+1)!}{2^{4n}}$

$\lim_{n\to\infty} \frac{2^4}{(2n+2)(2n+3)} = 0 < 1$ converges

33. a) $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

$\lim_{n\to\infty} \frac{1}{(n+1)^{3/2}} \cdot \frac{n^{3/2}}{1}$

$\lim_{n\to\infty} \frac{n^{3/2}}{n^{3/2}} = 1$ inconclusive

b) $\lim_{n\to\infty} \frac{1}{(n+1)^{1/2}} \cdot \frac{n^{1/2}}{1}$

$\lim_{n\to\infty} \frac{n^{1/2}}{(n+1)^{1/2}} = 1$

$\lim_{n\to\infty}$

36. $\sum_{n=1}^{\infty} \left(\frac{2n}{n+1}\right)^n$

$\lim_{n\to\infty} \sqrt[\cancel{n}]{\left(\frac{2n}{n+1}\right)^{\cancel{n}}}$

$\lim_{n\to\infty} \frac{2n}{n+1} = 2 > 1$ diverges

39. $\sum_{n=1}^{\infty} \left(2\sqrt[n]{n}+1\right)^n$

$\lim_{n\to\infty} \sqrt[n]{\left(2\sqrt[n]{n}+1\right)^n}$

$\lim_{n\to\infty} 2\sqrt[n]{n}+1$

$2\lim_{n\to\infty} \sqrt[n]{n} + \lim_{n\to\infty} 1$

$\ln y = \lim_{n\to\infty} \frac{1}{n}\ln n$

$\ln y = \lim_{n\to\infty} \frac{\ln n}{n} \quad \frac{\infty}{\infty}$

$\ln y = \lim_{n\to\infty} \frac{1}{n}$

$\ln y = 0$

$y = 1$

$2\lim_{n\to\infty} 1 + \lim_{n\to\infty} 1$

$2+1$

$3 > 1$ diverges

n^n FEFL

42. $1 + \frac{2}{3} + \frac{3}{3^2} + \frac{4}{3^3} + \frac{5}{3^4} + \frac{6}{3^5}$

$$\sum_{n=0}^{\infty} \frac{n+1}{3^n}$$

$$\lim_{n\to\infty} \frac{\sqrt[n]{n+1}}{3}$$

$$\frac{1}{3} \lim_{n\to\infty} \sqrt[n]{n+1}$$

$$\ln y = \lim_{n\to\infty} \frac{1}{n} \ln(n+1) \qquad 0 \cdot \infty$$

$$\ln y = \lim_{n\to\infty} \frac{\ln(n+1)}{n} \qquad \frac{\infty}{\infty}$$

$$\ln y = \lim_{n\to\infty} \frac{1}{n+1}$$

$$\ln y = 0$$

$$y = 1$$

$$\lim_{n\to\infty} \frac{\sqrt[n]{n+1}}{3} = \frac{1}{3} < 1$$ converges

45. $\sum_{n=1}^{\infty} \frac{3}{n^{3/2}}$, $p > 1$ converges by p-series test

48. $\sum_{n=1}^{\infty} \frac{n}{2n^2+1}$ $\quad \lim_{a\to\infty} \int_1^a \frac{n}{2n^2+1}\,dn \qquad u = 2n^2+1$, $du = 4n\,dn$

$$\lim_{a\to\infty} \frac{1}{4} \ln|2n^2+1| \Big]_1^a$$

$$\lim_{a\to\infty} \frac{1}{4} \ln(2a^2+1) - \frac{1}{4}\ln 3$$

∞ diverges by integral test

51. $\sum_{n=1}^{\infty} \frac{10n+3}{n2^n}$ compare to $\frac{1}{2^n}$ convergent geometric

$$\lim_{n\to\infty} \frac{10n+3}{n\cancel{2^n}} \cdot \frac{\cancel{2^n}}{1}$$

$$\lim_{n\to\infty} \frac{10n+3}{n} = 10$$

$\therefore$ converges by limit comparison

54. $\sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln n}$

$$\lim_{n\to\infty} \frac{1}{n\ln n} = 0 \qquad a_{n+1} \le a_n$$

converges by Alt. Series Test