EXERCISES FOR SECTION 8.8

In Exercises 1–4, state where the power series is centered.

1. $\sum_{n=0}^{\infty} nx^n$
2. $\sum_{n=1}^{\infty} \frac{(-1)^n 1 \cdot 3 \cdot \cdots (2n-1)}{2^n n!} x^n$
3. $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^3}$
4. $\sum_{n=0}^{\infty} \frac{(-1)^n (x-\pi)^{2n}}{(2n)!}$

In Exercises 5–10, find the radius of convergence of the power series.

5. $\sum_{n=0}^{\infty} (-1)^n \frac{x^n}{n+1}$
6. $\sum_{n=0}^{\infty} (2x)^n$
7. $\sum_{n=1}^{\infty} \frac{(2x)^n}{n^2}$
8. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2^n}$
9. $\sum_{n=0}^{\infty} \frac{(2x)^{2n}}{(2n)!}$
10. $\sum_{n=0}^{\infty} \frac{(2n)! x^{2n}}{n!}$

In Exercises 11–34, find the interval of convergence of the power series. (Be sure to include a check for convergence at the endpoints of the interval.)

11. $\sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n$
12. $\sum_{n=0}^{\infty} \left(\frac{x}{k}\right)^n, \quad k > 0$
13. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n}$
14. $\sum_{n=0}^{\infty} (-1)^{n+1}(n+1)x^n$
15. $\sum_{n=0}^{\infty} \frac{x^n}{n!}$
16. $\sum_{n=0}^{\infty} \frac{(3x)^n}{(2n)!}$
17. $\sum_{n=0}^{\infty} (2n)! \left(\frac{x}{2}\right)^n$
18. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(n+1)(n+2)}$
19. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{4^n}$
20. $\sum_{n=0}^{\infty} \frac{(-1)^n n!(x-4)^n}{3^n}$
21. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-5)^n}{n5^n}$
22. $\sum_{n=0}^{\infty} \frac{(x-2)^{n+1}}{(n+1)4^{n+1}}$
23. $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x-1)^{n+1}}{n+1}$
24. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-c)^n}{nc^n}$
25. $\sum_{n=1}^{\infty} \frac{(x-c)^{n-1}}{c^{n-1}}, \quad c > 0$
26. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$
27. $\sum_{n=1}^{\infty} \frac{n}{n+1}(-2x)^{n-1}$
28. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$
29. $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$
30. $\sum_{n=1}^{\infty} \frac{n! x^n}{(2n)!}$
31. $\sum_{n=1}^{\infty} \frac{k(k+1)(k+2)\cdots(k+n-1)x^n}{n!}, \quad k \geq 1$
32. $\sum_{n=1}^{\infty} \left[\frac{2 \cdot 4 \cdot 6 \cdot \cdots 2n}{3 \cdot 5 \cdot 7 \cdot \cdots (2n+1)}\right] x^{2n+1}$
33. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 3 \cdot 7 \cdot 11 \cdot \cdots (4n-1)(x-3)^n}{4^n}$
34. $\sum_{n=1}^{\infty} \frac{n!(x-c)^n}{1 \cdot 3 \cdot 5 \cdot \cdots (2n-1)}$

In Exercises 35–38, find the intervals of convergence of (a) $f(x)$, (b) $f'(x)$, (c) $f''(x)$, and (d) $\int f(x)\,dx$. Include a check for convergence at the endpoints.

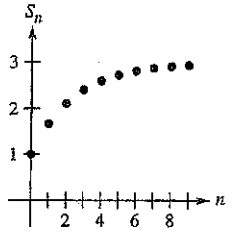
35. $f(x) = \sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n$
36. $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-5)^n}{n5^n}$
37. $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x-1)^{n+1}}{n+1}$
38. $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-2)^n}{n}$

Writing **In Exercises 39–42, match the graph of the first ten terms of the sequence of partial sums of the series**

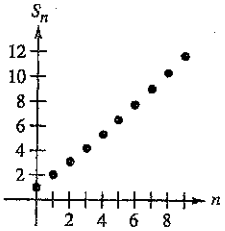
$$g(x) = \sum_{n=0}^{\infty} \left(\frac{x}{3}\right)^n$$

with the indicated value of the function. [The graphs are labeled (a), (b), (c), and (d).] Explain how you made your choice.

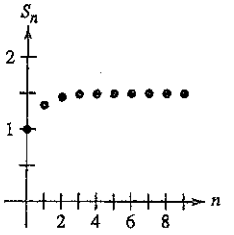
(a)



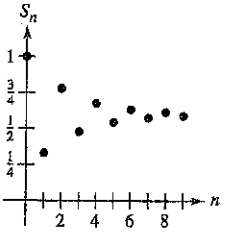
(b)



(c)



(d)



39. $g(1)$
40. $g(2)$
41. $g(3.1)$
42. $g(-2)$

Getting at the Concept

43. Define a power series centered at c.
44. What is the radius of convergence of a power series? What is the interval of convergence of a power series?
45. What are the three basic forms of the domain of a power series?
46. Describe how to differentiate and integrate a power series with a radius of convergence R. Will the series resulting from the operations of differentiation and integration have a different radius of convergence? Explain.