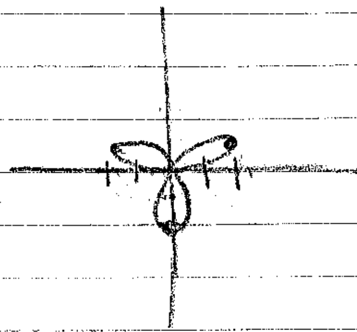
Polar Graphs Review

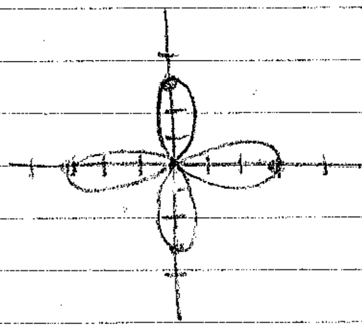
1. $r = 2\sin 3\theta$ (3 petal rose)

only plug in what gives max/min



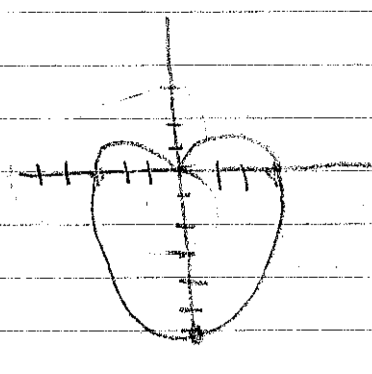
r	3θ	θ
2	$\pi/2$	$\pi/6$
-2	$3\pi/2$	$\pi/2$
2	$5\pi/2$	$5\pi/6$

2. $r = -3\cos 2\theta$ (4 petal rose)

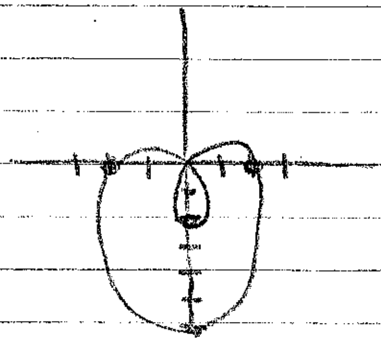


r	2θ	θ
-3	0	0
3	π	$\pi/2$
-3	2π	π
3	3π	$3\pi/2$
-3	4π	2π

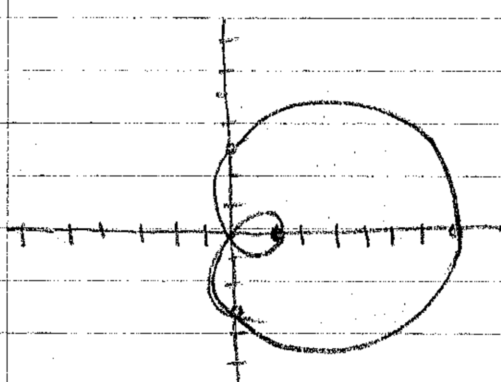
3. $r = 3(1 - \sin\theta)$ cardiod



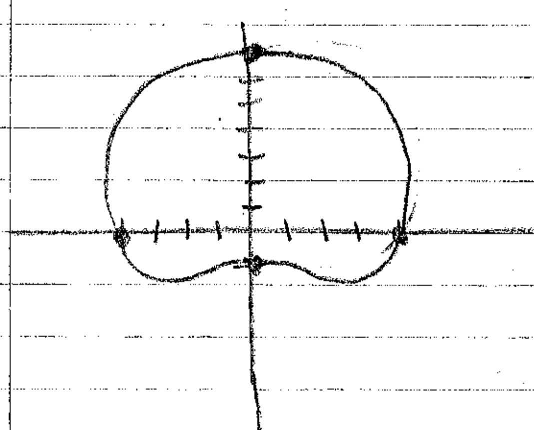
4. $r = 2 - 4\sin\theta$ limacon w/ inner loop



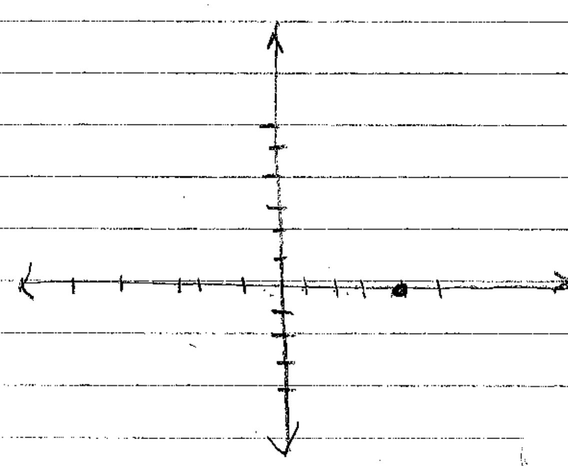
5. $r = 3 + 4\cos\theta$ limacon with inner loop



6. $r = 4 + 3\sin\theta$ limacon w/ no loop



$4 + 6\sin\theta$



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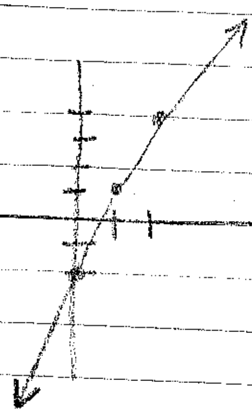
$x = r\cos\theta$

$y = r\sin\theta$

7. $r = \dfrac{2}{3\cos\theta - \sin\theta}$ line



$r = \dfrac{2}{3\frac{x}{r} - \frac{y}{r}}$

$r = \dfrac{2}{\frac{3x-y}{r}}$

$r = \dfrac{2r}{3x-y}$

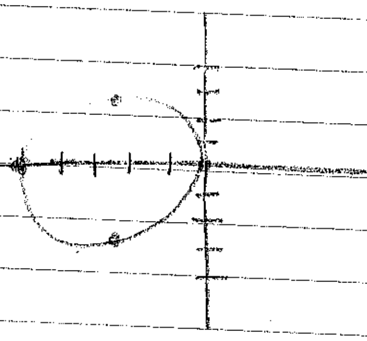
$r(3x-y) = 2r$

$3x - y = 2$

$-y = -3x + 2$

$y = 3x - 2$

8. $r = -5\cos\theta$ circle



r	θ
-5	0
0	$\pi/2$
5	π
	$3\pi/2$
	2π

9. $r = 2\sin 3\theta \quad x = \pi/4$

$x = r\cos\theta$

$x = 2\sin 3\theta \cos\theta$

$\frac{dx}{d\theta} = -2\sin 3\theta \sin\theta + \cos\theta \cdot 2\cos 3\theta \cdot 3$

$\frac{dx}{d\theta} = -2\sin 3\theta \sin\theta + 6\cos 3\theta \cos\theta$

$y = r\sin\theta$

$y = 2\sin 3\theta \sin\theta$

$\frac{dy}{d\theta} = 2\sin 3\theta \cdot \cos\theta + \sin\theta \cdot 2\cos 3\theta \cdot 3$

$\frac{dy}{d\theta} = 2\sin 3\theta\cos\theta + 6\sin\theta\cos 3\theta$

$$\frac{dy}{dx} = \frac{2\sin 3\theta\cos\theta + 6\sin\theta\cos 3\theta}{-2\sin 3\theta\sin\theta + 6\cos 3\theta\cos\theta}$$

$$\frac{dy}{dx} @ \pi/4 = \frac{2\sin\frac{3\pi}{4}\cos\frac{\pi}{4} + 6\sin\frac{\pi}{4}\cos\frac{3\pi}{4}}{-2\sin\frac{3\pi}{4}\sin\frac{\pi}{4} + 6\cos\frac{3\pi}{4}\cos\frac{\pi}{4}} = \frac{2\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} + 6\frac{\sqrt{2}}{2}\cdot -\frac{\sqrt{2}}{2}}{-2\,\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} + 6\cdot -\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}}$$

9. cont $= \frac{1-3}{-1-3}$

$= \frac{-2}{-4}$

$= \boxed{\frac{1}{2}}$

10. $r = 3 - 3\cos\theta \qquad x = \pi/2$

$x = r\cos\theta$ $\qquad$ $y = r\sin\theta$

$x = (3-3\cos\theta)\cos\theta$ $\qquad$ $y = (3-3\cos\theta)\sin\theta$

$\frac{dx}{d\theta} = (3-3\cos\theta)(-\sin\theta) + \cos\theta(3\sin\theta)$ $\qquad$ $\frac{dy}{d\theta} = (3-3\cos\theta)\cos\theta + \sin\theta(3\sin\theta)$

$\frac{dx}{d\theta} = -3\sin\theta + 3\cos\theta + 3\sin\theta\cos\theta$ $\qquad$ $\frac{dy}{d\theta} = 3\cos\theta - 3\cos^2\theta + 3\sin^2\theta$

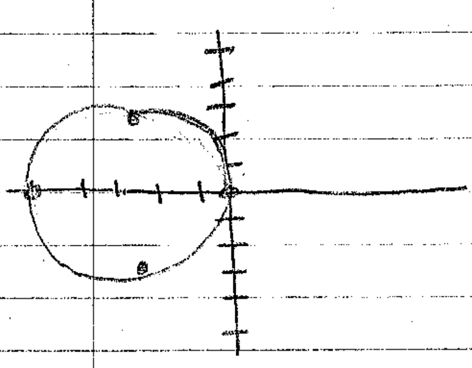
$\frac{dy}{dx} = \frac{-3(\sin\theta - \cos\theta - 3\sin\theta\cos\theta)}{3(\cos\theta - \cos^2\theta + \sin^2\theta)}$ switch

@ $\pi/2$ $\frac{dy}{dx} = \frac{\sin\pi/2 - \cos\pi/2 - 3\sin\pi/2\cos\pi/2}{\cos\pi/2 - \cos^2\pi/2 + \sin^2\pi/2}$

$= \frac{1-0-0}{0-0+1}$

$= \boxed{1}$

11. $r = -5\cos\theta$ (circle)



$A = \frac{1}{2}\int_0^{\pi} (-5\cos\theta)^2 d\theta$

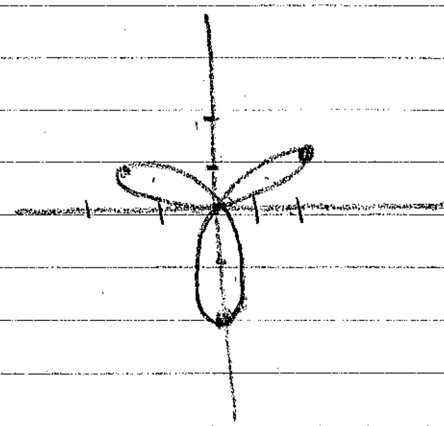
$A = \frac{25\pi}{4} \approx 19.635$

$\left(A = \pi r^2 \quad A = \pi(2.5)^2 = \frac{25\pi}{4}\right)$

[redacted]

12. $r = 2\sin(3\theta)$ 3 petal rose

r	3θ	θ
2	$\frac{\pi}{2}$	$\pi/6$
-2	$3\pi/2$	$\pi/2$
2	$5\pi/2$	$5\pi/6$



$$A = \frac{1}{2}\int_0^{\pi} (2\sin(3\theta))^2 \, d\theta$$

$$A = \pi$$

13. $r = 4\cos(6\theta)$ 12 petal rose

$$A = \frac{1}{2}\int_0^{2\pi} (4\cos 6\theta)^2 \, d\theta$$

$$= 8\pi \approx 25.133$$

14. $r = -5\cos\theta$ circle w/ diameter 5

$C = 2\pi r$

$C = 2\pi(2.5)$ or $s = \int_0^{\pi} \sqrt{(-5\cos\theta)^2 + (5\sin\theta)^2} \, d\theta$

$C = 5\pi$

$= 5\pi \approx 15.708$

15. $r = 2\sin 3\theta$ (3 petal rose)

$$s = \int_0^{\pi} \sqrt{(2\sin 3\theta)^2 + (6\cos 3\theta)^2} \, d\theta$$

$$= 13.365$$

16. $r = 4\cos 6\theta$ (12 petal rose)

$$S = \int_0^{2\pi} \sqrt{(4\cos 6\theta)^2 + (-24\sin 6\theta)^2}\, d\theta$$

$$= 99.600$$

AREA OF POLAR CURVES

17 The area of the closed region bounded by the polar graph of $r=\sqrt{3+\cos\theta}$ is given by the integral

(A) $\int_0^{2\pi}\sqrt{3+\cos\theta}\,d\theta$ (B) $\int_0^{\pi}\sqrt{3+\cos\theta}\,d\theta$ (C) $2\int_0^{\pi/2}(3+\cos\theta)\,d\theta$

(D) $\int_0^{\pi}(3+\cos\theta)\,d\theta$ (E) $2\int_0^{\pi/2}\sqrt{3+\cos\theta}\,d\theta$

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18 The area of the region enclosed by the polar curve $r=1-\cos\theta$ is

(A) $\frac{3}{4}\pi$ (B) π (C) $\frac{3}{2}\pi$ (D) 2π (E) 3π

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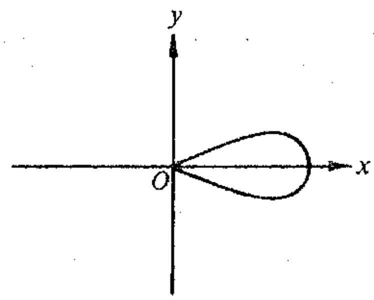
19 The area of the region enclosed by the polar curve $r=\sin(2\theta)$ for $0\le\theta\le\frac{\pi}{2}$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{\pi}{8}$ (E) $\frac{\pi}{4}$



21 Which of the following gives the area of the region enclosed by the loop of the graph of the polar curve $r=4\cos(3\theta)$ shown in the figure above?

(A) $16\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}\cos(3\theta)\,d\theta$ (B) $8\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\cos(3\theta)\,d\theta$ (C) $8\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}\cos^2(3\theta)\,d\theta$

(D) $16\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\cos^2(3\theta)\,d\theta$ (E) $8\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\cos^2(3\theta)\,d\theta$

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20 Which of the following is equal to the area of the region inside the polar curve $r=2\cos\theta$ and outside the polar curve $r=\cos\theta$?

(A) $3\int_0^{\frac{\pi}{2}}\cos^2\theta\,d\theta$ (B) $3\int_0^{\pi}\cos^2\theta\,d\theta$ (C) $\frac{3}{2}\int_0^{\frac{\pi}{2}}\cos^2\theta\,d\theta$ (D) $3\int_0^{\frac{\pi}{2}}\cos\theta\,d\theta$ (E) $3\int_0^{\pi}\cos\theta\,d\theta$

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22 The area of the region inside the polar curve $r=4\sin\theta$ and outside the polar curve $r=2$ is given by

(A) $\frac{1}{2}\int_0^{\pi}(4\sin\theta-2)^2\,d\theta$ (B) $\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}(4\sin\theta-2)^2\,d\theta$ (C) $\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}(4\sin\theta-2)^2\,d\theta$

(D) $\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}\left(16\sin^2\theta-4\right)d\theta$ (E) $\frac{1}{2}\int_0^{\pi}\left(16\sin^2\theta-4\right)d\theta$

[redacted]

FREE RESPONSE

1. $x^2+y^2=2$ $\qquad$ $(x-1)^2+y^2=1$

$y^2=2-x^2$ $\qquad$ $y^2=1-(x-1)^2$

$y=\pm\sqrt{-x^2+2}$

$y=\pm\sqrt{1-(x-1)^2}$

$y=\sqrt{-x^2+2}$

$y=\sqrt{1-(x^2-2x+1)}$

$x^2+0^2=2$

$x=\sqrt{2}$ $\qquad$ $y=\sqrt{-x^2+2x}$

a) $A=\int_0^1 \sqrt{-x^2+2x}\,dx+\int_1^{\sqrt{2}}\sqrt{-x^2+2}\,dx$

$x^2+y^2=2$ $\qquad$ $(x-1)^2+y^2=1$

$x^2=2-y^2$ $\qquad$ $(x-1)^2=1-y^2$

$x=\sqrt{2-y^2}$ $\qquad$ $x-1=-\sqrt{1-y^2}$

$x=1-\sqrt{1-y^2}$

b) $\int_0^1 \sqrt{2-y^2}-\left(1-\sqrt{1-y^2}\right)dy$

c) $r=\sqrt{2}$ $\quad$ $r=2\cos\theta$

$\frac{1}{2}\int_0^{\pi/4}(\sqrt{2})^2\,d\theta+\frac{1}{2}\int_{\frac{\pi}{4}}^{\pi/2}(2\cos\theta)^2\,d\theta$

2. a. $A = \frac{1}{2}\int_0^{\pi} (\theta + \sin 2\theta)^2 \, d\theta$

$= \underline{4.382}$

* b. $r = \theta + \sin 2\theta$ $\qquad$ $x = r\cos\theta$

$\theta = 2.786$

$-2 = r\cos\theta$

$-2 = (\theta + \sin 2\theta)\cos\theta$

$-2 = (\theta + 2\sin\theta\cos\theta)\cos\theta$

$-2 = \theta\cos\theta + 2\sin\theta\cos^2\theta$

$0 = \theta\cos\theta + 2\sin\theta\cos^2\theta + 2$

c. since $\frac{dr}{d\theta} < 0$, the radius r is decreasing, and the curve is moving closer to the pole, (0,0).

d. greatest distance $\Rightarrow$ maximum

want $\frac{dr}{d\theta} = 0$ $\qquad$ $\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$

$0 = 1 + 2\cos(2\theta)$

$-\frac{1}{2} = \cos 2\theta$

$\frac{4\pi}{3}, \frac{2\pi}{3} = 2\theta$

$\frac{2\pi}{3}, \frac{\pi}{3} = \theta$

$0 < \theta < \pi/2$ so $\theta = \pi/3$

check endpoints on closed interval

	r	θ	
	0	0	
max $\leftarrow$	1.913	$\pi/3$	$\pi/3 + \sin 2\pi/3$
	1.571	$\pi/2$	$\pi/2 + \sin 2(\pi/2)$

The maximum distance occurs when $\theta = \pi/3$.

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3. a. $\theta = \dfrac{2\pi}{3}$

$$2 \cdot \frac{1}{2}\int_0^{2\pi/3} 4\, d\theta \;+\; 2 \cdot \frac{1}{2}\int_{\frac{2\pi}{3}}^{\pi} (3+2\cos\theta)^2\, d\theta$$

$$\int_0^{\frac{2\pi}{3}} 4\, d\theta + \int_{2\pi/3}^{\pi} (3+2\cos\theta)^2\, d\theta$$

$8.378 + 1.992$

$\boxed{10.370}$

b. $r = 3+2\cos\theta$ $\qquad \dfrac{dr}{dt} = \dfrac{dr}{d\theta}$

$\dfrac{dr}{dt} = \dfrac{dr}{d\theta} = -2\sin\theta$

$@\ \dfrac{\pi}{3} = -2\sin \pi/3$

$= -2\dfrac{\sqrt{3}}{2}$

$\dfrac{dr}{dt} = -\sqrt{3}$

at $\pi/3$ $\quad r = 3+2\cos \pi/3$

$= 3+1$

$= 4$

Since $r > 0$ and $\dfrac{dr}{dt} < 0$, the particle is moving closer to the pole, $(0,0)$ when $r = \pi/3$

c. $r = 3+2\cos\theta$ $\qquad \dfrac{dy}{dt} = \dfrac{dy}{d\theta}$

$y = r\sin\theta$

$y = (3+2\cos\theta)\sin\theta$

(D) $\frac{1}{2}\int_{\frac{\pi}{6}}^{5\pi/6} (10\sin^2\theta - 4)\, d\theta$ $\qquad$ (E) $\frac{1}{2}\int_0 (10\sin^2\theta - 4)\, d\theta$

$y = (3 + 2\cos\theta)\sin\theta$

$\frac{dy}{dt} = \frac{dy}{d\theta} = (3 + 2\cos\theta)\cos\theta + (\sin\theta)(-2\sin\theta)$

$= (3 + 2\cos\theta)\cos\theta - 2\sin^2\theta$

at $\frac{\pi}{3} = \left(3 + 2\cos\frac{\pi}{3}\right)\cos\frac{\pi}{3} - 2\sin^2\frac{\pi}{3}$

$= (3 + 1)\left(\frac{1}{2}\right) - 2\left(\frac{\sqrt{3}}{2}\right)^2$

$= 2 - 2\left(\frac{3}{4}\right)$

$= 2 - \frac{3}{2}$

$\boxed{\frac{dy}{dt} = \frac{1}{2}}$

$y = (3 + 2\cos\theta)\sin\theta$

$= (3 + 2\cos\pi/3)\sin\pi/3$

$= 4 \cdot \frac{\sqrt{3}}{2}$

$= 2\sqrt{3}$

Since $y > 0$ and $\frac{dy}{dt} > 0$, the particle is moving away from the x-axis when $\theta = \frac{\pi}{3}$.