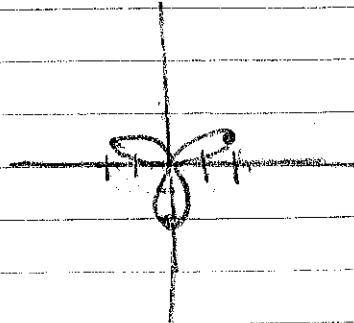


Polar Graphs Review

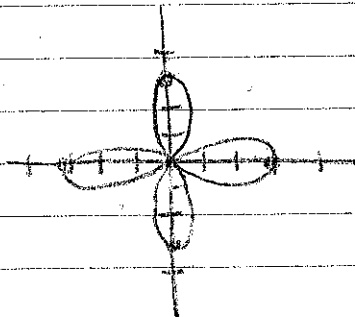
1. $r = 2 \sin 3\theta$ (3 petal rose)

only plug in
what angles
make sense



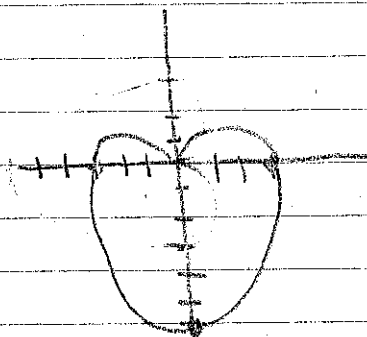
r	3θ	θ
2	$\pi/2$	$\pi/6$
-2	$3\pi/2$	$\pi/2$
2	$5\pi/2$	$5\pi/6$

2. $r = -3 \cos 2\theta$ (4 petal rose)



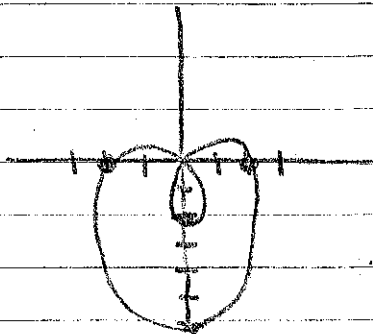
r	2θ	θ
-3	0	0
3	π	$\pi/2$
-3	2π	π
3	3π	$3\pi/2$
-3	4π	2π

3. $r = 3(1 - \sin \theta)$ cardioid



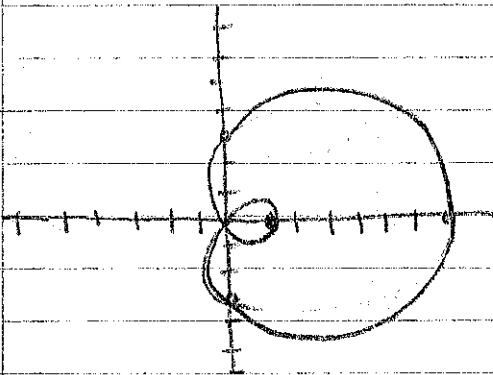
4. $r = 2 - 4\sin\theta$

limacon w/ inner loop



5. $r = 3 + 4\cos\theta$

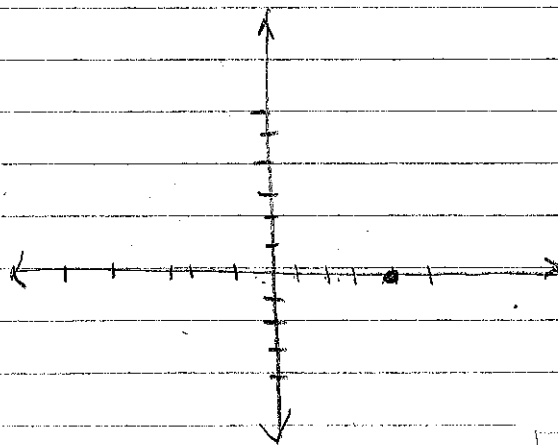
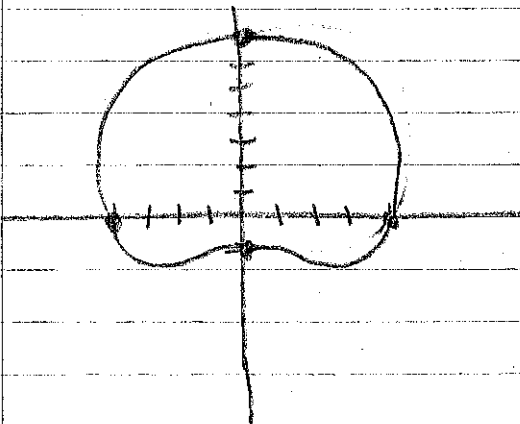
limacon with inner loop



6. $r = 4 + 3\sin\theta$

limacon w/ no loop

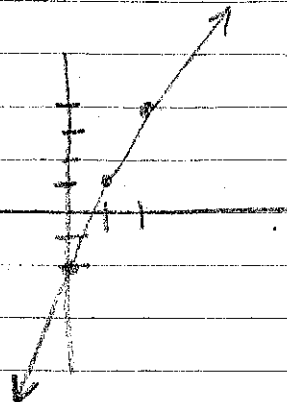
$4 + 6\sin\theta$



$$x = r \cos \theta$$

$$y = r \sin \theta$$

7. $r = \frac{2}{3 \cos \theta - \sin \theta}$ line



$$r = \frac{2}{\frac{3x}{r} - \frac{y}{r}}$$

$$r = \frac{2}{\frac{3x - y}{r}}$$

$$r = \frac{2r}{3x - y}$$

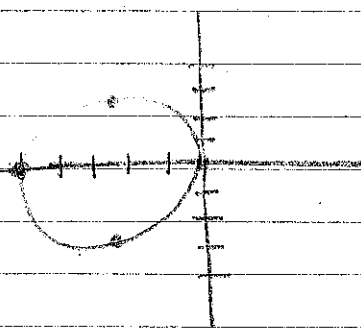
$$r(3x - y) = 2r$$

$$3x - y = 2$$

$$y = -3x + 2$$

$$y = 3x - 2$$

8. $r = -5 \cos \theta$ Circle



r	θ
-5	0
0	$\pi/2$
5	π
	$3\pi/2$
	2π

9. $r = 2 \sin 3\theta$ $x = \pi/4$

$$x = r \cos \theta$$

$$x = 2 \sin 3\theta \cos \theta$$

$$\frac{dx}{d\theta} = -2 \sin 3\theta \sin \theta + \cos \theta \cdot 2 \cos 3\theta \cdot 3$$

$$\frac{dx}{d\theta} = -2 \sin 3\theta \sin \theta + 6 \cos 3\theta \cos \theta$$

$$\frac{dy}{dx} = \frac{2 \sin 3\theta \cos \theta + 6 \sin \theta \cos 3\theta}{-2 \sin 3\theta \sin \theta + 6 \cos 3\theta \cos \theta}$$

$$\frac{dy}{dx} @ \pi/4 = \frac{2 \sin 3\pi/4 \cos \pi/4 + 6 \sin \pi/4 \cos 3\pi/4}{-2 \sin 3\pi/4 \sin \pi/4 + 6 \cos 3\pi/4 \cos \pi/4} = \frac{2 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} + 6 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2}}{-2 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} + 6 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2}}$$

$$y = r \sin \theta$$

$$y = 2 \sin 3\theta \sin \theta$$

$$\frac{dy}{d\theta} = 2 \sin 3\theta \cos \theta + \sin 3\theta \cdot 2 \cos \theta \cdot 3$$

$$\frac{dy}{d\theta} = 2 \sin 3\theta \cos \theta + 6 \sin \theta \cos 3\theta$$

$$9. \text{cont} = \frac{1-3}{-1-3}$$

$$= -2$$

$$= \boxed{\frac{1}{2}}$$

$$10. r = 3 - 3\cos\theta \quad x = \pi/2$$

$$x = r\cos\theta$$

$$x = (3 - 3\cos\theta)\cos\theta$$

$$\frac{dx}{d\theta} = (3 - 3\cos\theta)(-\sin\theta) + \cos\theta(3\sin\theta)$$

$$\frac{dx}{d\theta} = -3\sin\theta + 3\cos\theta + 3\sin\theta\cos\theta$$

$$\frac{dy}{dx} = \frac{-3(\sin\theta - \cos\theta - \sin\theta\cos\theta)}{3(\cos\theta - \cos^2\theta + \sin^2\theta)}$$

$$y = r\sin\theta$$

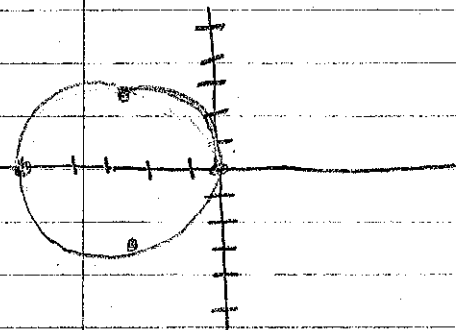
$$y = (3 - 3\cos\theta)\sin\theta$$

$$\frac{dy}{d\theta} = (3 - 3\cos\theta)\cos\theta + \sin\theta(3\sin\theta)$$

$$\frac{dy}{d\theta} = 3\cos\theta - 3\cos^2\theta + 3\sin^2\theta$$

$$\begin{aligned} @ \pi/2 \quad \frac{dy}{dx} &= \frac{\sin \pi/2 - \cos \pi/2 - 3\sin \pi/2 \cos \pi/2}{\cos \pi/2 - \cos^2 \pi/2 + \sin^2 \pi/2} \\ &= \frac{1 - 0 - 0}{0 - 0 + 1} \\ &= \boxed{-1} \end{aligned}$$

$$11. r = -5\cos\theta \quad (\text{circle})$$



$$A = \frac{1}{2} \int_0^\pi (-5\cos\theta)^2 d\theta$$

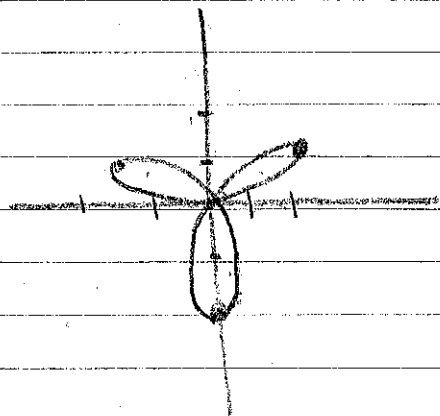
$$A = \frac{25\pi}{4} \approx 19.635$$

$$(A = \pi r^2$$

$$A = \pi(2.5)^2 = \frac{25\pi}{4})$$

12. $r = 2 \sin(3\theta)$ 3 petal rose.

r	3θ	θ
2	$\frac{\pi}{6}$	$\frac{\pi}{6}$
-2	$\frac{5\pi}{6}$	$\frac{5\pi}{12}$
2	$\frac{5\pi}{6}$	$\frac{5\pi}{12}$



$$A = \frac{1}{2} \int_0^{\pi} (2 \sin(3\theta))^2 d\theta$$

$$A = \pi$$

13. $r = 4 \cos(6\theta)$ 12 petal rose

$$A = \frac{1}{2} \int_0^{2\pi} (4 \cos(6\theta))^2 d\theta$$

$$= 8\pi \approx 25.133$$

14. $r = -5 \cos \theta$ circle w/ diameter 5

$$C = 2\pi r$$

$$C = 2\pi(2.5)$$

$$C = 5\pi$$

$$\text{OR } s = \int_0^{\pi} \sqrt{(-5 \cos \theta)^2 + (5 \sin \theta)^2} d\theta$$

$$= 5\pi \approx 15.708$$

15. $r = 2 \sin 3\theta$ (3 petal rose)

$$s = \int_0^{\pi} \sqrt{(2 \sin 3\theta)^2 + (6 \cos 3\theta)^2} d\theta$$

$$= 13.365$$

16. $r = 4 \cos 6\theta$ (12 petal rose)

$$s = \int_0^{2\pi} \sqrt{(4 \cos 6\theta)^2 + (-24 \sin 6\theta)^2} d\theta$$
$$= 99.600$$

The area of the closed region bounded by the polar graph of $r = \sqrt{3 + \cos \theta}$ is given by the integral

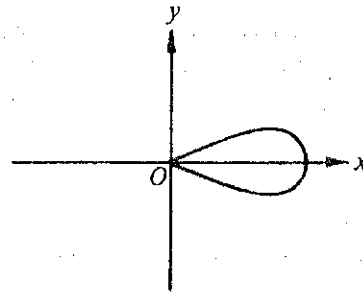
- (17) (A) $\int_0^{2\pi} \sqrt{3 + \cos \theta} d\theta$ (B) $\int_0^{\pi} \sqrt{3 + \cos \theta} d\theta$ (C) $2 \int_0^{\pi/2} (3 + \cos \theta) d\theta$
 (D) $\int_0^{\pi} (3 + \cos \theta) d\theta$ (E) $2 \int_0^{\pi/2} \sqrt{3 + \cos \theta} d\theta$

The area of the region enclosed by the polar curve $r = 1 - \cos \theta$ is

- (18) (A) $\frac{3}{4}\pi$ (B) π (C) $\frac{3}{2}\pi$ (D) 2π (E) 3π

The area of the region enclosed by the polar curve $r = \sin(2\theta)$ for $0 \leq \theta \leq \frac{\pi}{2}$ is

- (19) (A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{\pi}{8}$ (E) $\frac{\pi}{4}$



Which of the following gives the area of the region enclosed by the loop of the graph of the polar curve $r = 4 \cos(3\theta)$ shown in the figure above?

- (21) (A) $16 \int_{\frac{\pi}{3}}^{\frac{\pi}{6}} \cos(3\theta) d\theta$ (B) $8 \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos(3\theta) d\theta$ (C) $8 \int_{\frac{\pi}{3}}^{\frac{\pi}{6}} \cos^2(3\theta) d\theta$
 (D) $16 \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos^2(3\theta) d\theta$ (E) $8 \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos^2(3\theta) d\theta$

Which of the following is equal to the area of the region inside the polar curve $r = 2 \cos \theta$ and outside the polar curve $r = \cos \theta$?

- (20) (A) $3 \int_0^{\pi} \cos^2 \theta d\theta$ (B) $3 \int_0^{\pi} \cos^2 \theta d\theta$ (C) $\frac{3}{2} \int_0^{\pi} \cos^2 \theta d\theta$ (D) $3 \int_0^{\pi} \cos \theta d\theta$ (E) $3 \int_0^{\pi} \cos \theta d\theta$

The area of the region inside the polar curve $r = 4 \sin \theta$ and outside the polar curve $r = 2$ is given by

- (22) (A) $\frac{1}{2} \int_0^{\pi} (4 \sin \theta - 2)^2 d\theta$ (B) $\frac{1}{2} \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (4 \sin \theta - 2)^2 d\theta$ (C) $\frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (4 \sin \theta - 2)^2 d\theta$
 (D) $\frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (16 \sin^2 \theta - 4) d\theta$ (E) $\frac{1}{2} \int_0^{\pi} (16 \sin^2 \theta - 4) d\theta$

FREE RESPONSE

$$1. \quad x^2 + y^2 = 2$$

$$(x-1)^2 + y^2 = 1$$

$$y^2 = 2 - x^2$$

$$y = \pm \sqrt{-x^2 + 2}$$

$$y = \sqrt{-x^2 + 2}$$

$$x^2 + 0^2 = 2$$

$$x = \sqrt{2}$$

$$y^2 = 1 - (x-1)^2$$

$$y = \pm \sqrt{1 - (x-1)^2}$$

$$y = \sqrt{1 - (x^2 - 2x + 1)}$$

$$y = \sqrt{-x^2 + 2x}$$

$$a) \quad A = \int_0^1 \sqrt{-x^2 + 2x} \, dx + \int_1^{\sqrt{2}} \sqrt{-x^2 + 2} \, dx$$

$$x^2 + y^2 = 2$$

$$x^2 = 2 - y^2$$

$$x = \sqrt{2 - y^2}$$

$$(x-1)^2 + y^2 = 1$$

$$(x-1)^2 = 1 - y^2$$

$$x-1 = -\sqrt{1 - y^2}$$

$$x = 1 - \sqrt{1 - y^2}$$

$$b) \quad \int_0^1 \sqrt{2 - y^2} - (1 - \sqrt{1 - y^2}) \, dy$$

$$c) \quad r = \sqrt{2} \quad r = 2 \cos \theta$$

$$\frac{1}{2} \int_0^{\pi/4} (\sqrt{2})^2 \, d\theta + \frac{1}{2} \int_{\pi/4}^{\pi/2} (2 \cos \theta)^2 \, d\theta$$

$$\frac{1}{2} \int_0^{\pi/6} (\sqrt{2})^2 \, d\theta + \frac{1}{2} \int_{\pi/6}^{\pi/2} (2 \cos \theta)^2 \, d\theta$$

$$\frac{1}{2} \int_0^{\pi/6} (2 \cos \theta)^2 \, d\theta + \frac{1}{2} \int_{\pi/6}^{\pi/2} (2 \cos \theta)^2 \, d\theta$$

$$2. a. A = \frac{1}{2} \int_0^{\pi} (\theta + \sin 2\theta)^2 d\theta$$

$$= 4.382$$

$$* b. r = \theta + \sin 2\theta$$

$$v = r \cos \theta$$

$$-2 = r \cos \theta$$

$$-2 = (\theta + \sin 2\theta) \cos \theta$$

$$-2 = (\theta + 2\sin \theta \cos \theta) \cos \theta$$

$$-2 = \theta \cos \theta + 2\sin \theta \cos^2 \theta$$

$$0 = \theta \cos \theta + 2\sin \theta \cos^2 \theta - 2$$

$$\theta = 2.786$$

c. since $\frac{dr}{d\theta} < 0$, the radius r is decreasing, and the curve is moving closer to the pole, $(0,0)$.

d. greatest distance \Rightarrow maximum

$$\text{wait } \frac{dr}{d\theta} = 0$$

$$\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$$

$$0 = 1 + 2\cos(2\theta)$$

$$\frac{-1}{2} = \cos 2\theta$$

$$\frac{4\pi}{3}, \frac{2\pi}{3} = 2\theta$$

$$\frac{2\pi}{3}, \frac{\pi}{3} = \theta$$

$$0 < \theta < \pi/2 \text{ so } \theta = \pi/3$$

check endpoints on closed interval

	r	θ	
	0	0	
max \leftarrow	1.913	$\frac{\pi}{3}$	$\frac{\pi}{3} + \sin 2(\frac{\pi}{3})$
	1.571	$\frac{\pi}{2}$	$\frac{\pi}{2} + \sin 2(\frac{\pi}{2})$

The maximum distance occurs when $\theta = \pi/3$.

$$3. a. \theta = \frac{2\pi}{3}$$

$$2 \cdot \frac{1}{2} \int_0^{2\pi/3} 4 d\theta + 2 \cdot \frac{1}{2} \int_{2\pi/3}^{\pi} (3+2\cos\theta)^2 d\theta$$

$$\int_0^{2\pi/3} 4 d\theta + \int_{2\pi/3}^{\pi} (3+2\cos\theta)^2 d\theta$$

$$8.378 + 1.992$$

$$\boxed{10.370}$$

$$b. r = 3 + 2\cos\theta \quad \frac{dr}{dt} = \frac{dr}{d\theta}$$

$$\frac{dr}{dt} = \frac{dr}{d\theta} = -2\sin\theta$$

$$\begin{aligned} \text{at } \frac{\pi}{3} &= -2\sin\frac{\pi}{3} \\ &= -2\frac{\sqrt{3}}{2} \end{aligned}$$

$$\frac{dr}{dt} = -\sqrt{3}$$

$$\begin{aligned} \text{at } \pi/3 \quad r &= 3 + 2\cos\pi/3 \\ &= 3 + 1 \\ &= 4 \end{aligned}$$

Since $r > 0$ and $\frac{dr}{dt} < 0$, the particle is moving closer to the pole, (0,0) when $r = \pi/3$

$$c. r = 3 + 2\cos\theta \quad \frac{dy}{dt} = \frac{dy}{d\theta}$$

$$y = r \sin\theta$$

$$y = (3 + 2\cos\theta) \sin\theta$$

$$\text{at } \frac{\pi}{6}$$

$$y = (3 + 2 \cos \theta) \sin \theta$$

$$\frac{dy}{dt} = \frac{dy}{d\theta} = (3 + 2 \cos \theta) \cos \theta + (\sin \theta)(-2 \sin \theta)$$

$$= (3 + 2 \cos \theta) \cos \theta - 2 \sin^2 \theta$$

$$\text{at } \frac{\pi}{3} = \left(3 + 2 \cos \frac{\pi}{3} \right) \cos \frac{\pi}{3} - 2 \sin^2 \frac{\pi}{3}$$

$$= (3 + 1) \left(\frac{1}{2} \right) - 2 \left(\frac{\sqrt{3}}{2} \right)^2$$

$$= 2 - 2 \left(\frac{3}{4} \right)$$

$$= 2 - \frac{3}{2}$$

$$\boxed{\frac{dy}{dt} = \frac{1}{2}}$$

$$y = (3 + 2 \cos \theta) \sin \theta$$

$$= (3 + 2 \cos \frac{\pi}{3}) \sin \frac{\pi}{3}$$

$$= 4 \cdot \frac{\sqrt{3}}{2}$$

$$= 2\sqrt{3}$$

Since $y > 0$ and $\frac{dy}{dt} > 0$, the particle is moving away from

the x-axis when $\theta = \frac{\pi}{3}$.