EXERCISES FOR SECTION 9.4

In Exercises 1–6, plot the point in polar coordinates and find the corresponding rectangular coordinates for the point.

1. $(4, 3\pi/6)$
2. $(-2, 7\pi/4)$
3. $(-4, -\pi/3)$
4. $(0, -7\pi/6)$
5. $(\sqrt{2}, 2.36)$
6. $(-3, -1.57)$

In Exercises 7–10, use the *angle* feature of a graphing utility to find the rectangular coordinates for the point given in polar coordinates. Plot the point.

7. $(5, 3\pi/4)$
8. $(-2, 11\pi/6)$
9. $(-3.5, 2.5)$
10. $(8.25, 1.3)$

In Exercises 11–14, the rectangular coordinates of a point are given. Plot the point and find *two* sets of polar coordinates for the point for $0 \le \theta < 2\pi$.

11. $(1, 1)$
12. $(0, -5)$
13. $(-3, 4)$
14. $(4, -2)$

In Exercises 15–18, use the *angle* feature of a graphing utility to find one set of polar coordinates for the point given in rectangular coordinates.

15. $(3, -2)$
16. $(3\sqrt{2}, 3\sqrt{2})$
17. $\left(\frac{5}{2}, \frac{4}{3}\right)$
18. $(0, -5)$

19. Plot the point $(4, 3.5)$ if the point is given in (a) rectangular coordinates and (b) polar coordinates.

20. *Graphical Reasoning*

(a) Set the window format of a graphing utility to rectangular coordinates and locate the cursor at any position off the coordinate axes. Move the cursor horizontally and describe any changes in the displayed coordinates of the points. Repeat the process moving the cursor vertically.

(b) Set the window format of a graphing utility to polar coordinates and locate the cursor at any position off the coordinate axes. Move the cursor horizontally and describe any changes in the displayed coordinates of the points. Repeat the process moving the cursor vertically.

(c) Why are the results in parts (a) and (b) different?

In Exercises 21–28, convert the rectangular equation to polar form and sketch its graph.

21. $x^2 + y^2 = a^2$
22. $x^2 + y^2 - 2ax = 0$
23. $y = 4$
24. $x = 10$
25. $3x - y + 2 = 0$
26. $xy = 4$
27. $y^2 = 9x$
28. $(x^2 + y^2)^2 - 9(x^2 - y^2) = 0$

In Exercises 29–36, convert the polar equation to rectangular form and sketch its graph.

29. $r = 3$
30. $r = -2$
31. $r = \sin\theta$
32. $r = 5\cos\theta$
33. $r = \theta$
34. $\theta = \dfrac{5\pi}{6}$
35. $r = 3\sec\theta$
36. $r = 2\csc\theta$

In Exercises 37–46, use a graphing utility to graph the polar equation. Find an interval for θ over which the graph is traced *only once*.

37. $r = 3 - 4\cos\theta$
38. $r = 5(1 - 2\sin\theta)$
39. $r = 2 + \sin\theta$
40. $r = 4 + 3\cos\theta$
41. $r = \dfrac{2}{1 + \cos\theta}$
42. $r = \dfrac{2}{4 - 3\sin\theta}$
43. $r = 2\cos\left(\dfrac{3\theta}{2}\right)$
44. $r = 3\sin\left(\dfrac{5\theta}{2}\right)$
45. $r^2 = 4\sin 2\theta$
46. $r^2 = \dfrac{1}{\theta}$

47. Convert the equation

$$r = 2(h\cos\theta + k\sin\theta)$$

to rectangular form and verify that it is the equation of a circle. Find the radius and the rectangular coordinates of the center of the circle.

48. *Distance Formula*

(a) Verify that the Distance Formula for the distance between the two points (r_1, θ_1) and (r_2, θ_2) in polar coordinates is

$$d = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_1 - \theta_2)}.$$

(b) Describe the position of the points relative to each other if $\theta_1 = \theta_2$. Simplify the Distance Formula for this case. Is the simplification what you expected? Explain.

(c) Simplify the Distance Formula if $\theta_1 - \theta_2 = 90°$. Is the simplification what you expected? Explain.

(d) Choose two points on the polar coordinate system and find the distance between them. Then choose different polar representations of the same two points and apply the Distance Formula again. Discuss the result.

In Exercises 49–52, use the result of Exercise 48 to approximate the distance between the two points in polar coordinates.

49. $\left(4, \dfrac{2\pi}{3}\right), \left(2, \dfrac{\pi}{6}\right)$
50. $\left(10, \dfrac{7\pi}{6}\right), (3, \pi)$
51. $(2, 0.5), (7, 1.2)$
52. $(4, 2.5), (12, 1)$