EXERCISES FOR SECTION 9.5

In Exercises 1 and 2, find the area of the region bounded by the graph of the polar equation using (a) a geometric formula, and (b) integration.

1. $r = 8 \sin \theta$
2. $r = 3 \cos \theta$

In Exercises 3–8, find the area of the region.

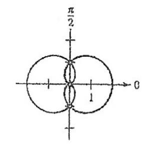
3. One petal of $r = 2 \cos 3\theta$
4. One petal of $r = 6 \sin 2\theta$
5. One petal of $r = \cos 2\theta$
6. One petal of $r = \cos 5\theta$
7. Interior of $r = 1 - \sin \theta$
8. Interior of $r = 1 - \sin \theta$ (above the polar axis)

In Exercises 9–12, use a graphing utility to graph the polar equation and find the area of the indicated region.

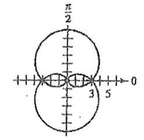
9. Inner loop of $r = 1 + 2 \cos \theta$
10. Inner loop of $r = 4 - 6 \sin \theta$
11. Between the loops of $r = 1 + 2 \cos \theta$
12. Between the loops of $r = 2(1 + 2 \sin \theta)$

In Exercises 13–22, find the points of intersection of the graphs of the equations.

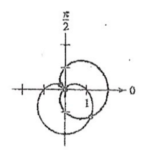
13. $r = 1 + \cos \theta$
$r = 1 - \cos \theta$



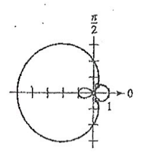
14. $r = 3(1 + \sin \theta)$
$r = 3(1 - \sin \theta)$



15. $r = 1 + \cos \theta$
$r = 1 - \sin \theta$



16. $r = 2 - 3 \cos \theta$
$r = \cos \theta$



17. $r = 4 - 5 \sin \theta$
$r = 3 \sin \theta$
18. $r = 1 + \cos \theta$
$r = 3 \cos \theta$
19. $r = \frac{\theta}{2}$
$r = 2$
20. $\theta = \frac{\pi}{4}$
$r = 2$
21. $r = 4 \sin 2\theta$
$r = 2$
22. $r = 3 + \sin \theta$
$r = 2 \csc \theta$

In Exercises 23 and 24, use a graphing utility to approximate the points of intersection of the graphs of the polar equations. Confirm your results analytically.

23. $r = 2 + 3 \cos \theta$
$r = \frac{\sec \theta}{2}$
24. $r = 3(1 - \cos \theta)$
$r = \frac{6}{1 - \cos \theta}$

Writing **In Exercises 25 and 26, use a graphing utility to find the points of intersection of the graphs of the polar equations. Watch the graphs as they are traced in the viewing window. Explain why the pole is not a point of intersection obtained by solving the equations simultaneously.**

25. $r = \cos \theta$
$r = 2 - 3 \sin \theta$
26. $r = 4 \sin \theta$
$r = 2(1 + \sin \theta)$

In Exercises 27–32, use a graphing utility to graph the polar equations and find the area of the indicated region.

27. Common interior of $r = 4 \sin 2\theta$ and $r = 2$
28. Common interior of $r = 3(1 + \sin \theta)$ and $r = 3(1 - \sin \theta)$
29. Common interior of $r = 3 - 2 \sin \theta$ and $r = -3 + 2 \sin \theta$
30. Common interior of $r = 5 - 3 \sin \theta$ and $r = 5 - 3 \cos \theta$
31. Common interior of $r = 4 \sin \theta$ and $r = 2$
32. Inside $r = 3 \sin \theta$ and outside $r = 2 - \sin \theta$

In Exercises 33–36, find the area of the region.

33. Inside $r = a(1 + \cos \theta)$ and outside $r = a \cos \theta$
34. Inside $r = 2a \cos \theta$ and outside $r = a$
35. Common interior of $r = a(1 + \cos \theta)$ and $r = a \sin \theta$
36. Common interior of $r = a \cos \theta$ and $r = a \sin \theta$ where $a > 0$

37. ***Antenna Radiation*** The radiation from a transmitting antenna is not uniform in all directions. The intensity from a particular antenna is modeled by

$$r = a \cos^2 \theta.$$

(a) Convert the polar equation to rectangular form.
(b) Use a graphing utility to graph the model for $a = 4$ and $a = 6$.
(c) Find the area of the geographical region between the two curves in part (b).

38. ***Area*** The area inside one or more of the three interlocking circles

$$r = 2a \cos \theta, \quad r = 2a \sin \theta, \quad \text{and} \quad r = a$$

is divided into seven regions. Find the area of each region.

39. ***Conjecture*** Find the area of the region enclosed by $r = a \cos(n\theta)$ for $n = 1, 2, 3, \ldots$. Use the results to make a conjecture about the area enclosed by the function if n is even and if n is odd.

40. ***Area*** Sketch the strophoid

$$r = \sec \theta - 2 \cos \theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}.$$

Convert this equation to rectangular coordinates. Find the area enclosed by the loop.

In Exercises 41–44, find the length of the curve over the indicated interval.

Polar Equation	*Interval*
41. $r = a$	$0 \le \theta \le 2\pi$
42. $r = 2a \cos \theta$	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$
43. $r = 1 + \sin \theta$	$0 \le \theta \le 2\pi$
44. $r = 8(1 + \cos \theta)$	$0 \le \theta \le 2\pi$

In Exercises 45–50, use a graphing utility to graph the polar equation over the indicated interval. Use the integration capabilities of the graphing utility to approximate the length of the curve accurate to two decimal places.

Polar Equation	*Interval*
45. $r = 2\theta$	$0 \le \theta \le \frac{\pi}{2}$
46. $r = \sec \theta$	$0 \le \theta \le \frac{\pi}{3}$
47. $r = \frac{1}{\theta}$	$\pi \le \theta \le 2\pi$
48. $r = e^{\theta}$	$0 \le \theta \le \pi$
49. $r = \sin(3 \cos \theta)$	$0 \le \theta \le \pi$
50. $r = 2 \sin(2 \cos \theta)$	$0 \le \theta \le \pi$

In Exercises 51–54, find the area of the surface formed by revolving the curve about the given line.

Polar Equation	*Interval*	*Axis of Revolution*
51. $r = 6 \cos \theta$	$0 \le \theta \le \frac{\pi}{2}$	Polar axis
52. $r = a \cos \theta$	$0 \le \theta \le \frac{\pi}{2}$	$\theta = \frac{\pi}{2}$
53. $r = e^{a\theta}$	$0 \le \theta \le \frac{\pi}{2}$	$\theta = \frac{\pi}{2}$
54. $r = a(1 + \cos \theta)$	$0 \le \theta \le \pi$	Polar axis

In Exercises 55 and 56, use the integration capabilities of a graphing utility to approximate to two decimal places the area of the surface formed by revolving the curve about the polar axis.

Polar Equation	*Interval*
55. $r = 4 \cos 2\theta$	$0 \le \theta \le \frac{\pi}{4}$
56. $r = \theta$	$0 \le \theta \le \pi$

Getting at the Concept

57. Give the integral formulas for area and arc length in polar coordinates.
58. Explain why finding points of intersection of polar graphs may require further analysis beyond solving two equations simultaneously.
59. Which integral yields the arc length of $r = 3(1 - \cos 2\theta)$? State why the other integrals are incorrect.

(a) $3\int_0^{2\pi} \sqrt{(1 - \cos 2\theta)^2 + 4 \sin^2 2\theta}\, d\theta$

(b) $12\int_0^{\pi/4} \sqrt{(1 - \cos 2\theta)^2 + 4 \sin^2 2\theta}\, d\theta$

(c) $3\int_0^{\pi} \sqrt{(1 - \cos 2\theta)^2 + 4 \sin^2 2\theta}\, d\theta$

(d) $6\int_0^{\pi/2} \sqrt{(1 - \cos 2\theta)^2 + 4 \sin^2 2\theta}\, d\theta$

60. Give the integral formulas for the area of the surface of revolution formed when the graph of $r = f(\theta)$ is revolved about (a) the x-axis and (b) the y-axis.

61. ***Surface Area of a Torus*** Find the surface area of the torus generated by revolving the circle given by $r = a$ about the line $r = b \sec \theta$, where $0 < a < b$.

62. ***Approximating Area*** Consider the circle $r = 8 \cos \theta$.

(a) Find the area of the circle.
(b) Complete the table giving the areas A of the sectors of the circle between $\theta = 0$ and the values of θ in the table.

θ	0.2	0.4	0.6	0.8	1.0	1.2	1.4
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(c) Use the table in part (b) to approximate the values of θ for which the sector of the circle composes $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ of the total area of the circle.
(d) Use a graphing utility to approximate to two-decimal-place accuracy the angles θ for which the sector of the circle composes $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ of the total area of the circle.
(e) Do the results in part (d) depend on the radius of the circle? Explain.

True or False? **In Exercises 63 and 64, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

63. If $f(\theta) > 0$ for all θ and $g(\theta) < 0$ for all θ, then the graphs of $r = f(\theta)$ and $r = g(\theta)$ do not intersect.
64. If $f(\theta) = g(\theta)$ for $\theta = 0, \pi/2$, and $3\pi/2$, then the graphs of $r = f(\theta)$ and $r = g(\theta)$ have at least four points of intersection.

65. Use the formula for the arc length of a curve in parametric form to derive the formula for the arc length of a polar curve.