

In Exercises 1–4, match the antiderivative with the correct integral. [Integrals are labeled (A), (B), (C), and (D).]

- (a) $\int \ln x \, dx$ (b) $\int x \sin x \, dx$
 (c) $\int x^2 e^x \, dx$ (d) $\int x^2 \cos x \, dx$
1. $y = \sin x - x \cos x$
 2. $y = x^2 \sin x + 2x \cos x - 2 \sin x$
 3. $y = x^3 e^x - 2xe^x + 2e^x$
 4. $y = -x + x \ln x$

In Exercises 5–10, identify u and dv for evaluating the integral using integration by parts. (Do not evaluate the integral.)

5. $\int x e^{2x} \, dx$ 6. $\int x^2 e^{2x} \, dx$
 7. $\int (\ln x)^2 \, dx$ 8. $\int \ln 3x \, dx$
 9. $\int x \sec^2 x \, dx$ 10. $\int x^2 \cos x \, dx$

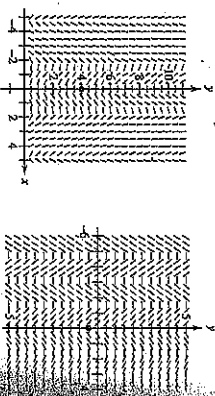
In Exercises 11–36, evaluate the integral. (Note: Solve by the simplest method—not all require integration by parts.)

11. $\int x e^{-x} \, dx$ 12. $\int \frac{2x}{e^{-x}} \, dx$
 13. $\int x^2 e^x \, dx$ 14. $\int \frac{e^{1/x}}{x^2} \, dx$
 15. $\int x^2 e^{x^2} \, dx$ 16. $\int x^4 \ln x \, dx$
 17. $\int \ln(x+1) \, dx$ 18. $\int \frac{1}{x(\ln x)^2} \, dx$
 19. $\int \frac{(\ln x)^2}{x} \, dx$ 20. $\int \frac{x}{(x^2+1)^2} \, dx$
 21. $\int \frac{x e^{2x}}{(2x+1)^2} \, dx$ 22. $\int \frac{x^2}{(x^2+1)^2} \, dx$
 23. $\int (x^2-1)e^x \, dx$ 24. $\int \frac{\ln 2x}{x^2} \, dx$
 25. $\int x\sqrt{x-1} \, dx$ 26. $\int \frac{x}{\sqrt{2+3x}} \, dx$
 27. $\int x \cos x \, dx$ 28. $\int x \sin x \, dx$
 29. $\int x^2 \sin x \, dx$ 30. $\int x^2 \cos x \, dx$
 31. $\int \sec t \cot t \, dt$ 32. $\int 8 \sec \theta \tan \theta \, d\theta$
 33. $\int \arctan x \, dx$ 34. $\int 4 \arccos x \, dx$
 35. $\int e^{2x} \sin x \, dx$ 36. $\int e^x \cos 2x \, dx$

In Exercises 37–42, solve the differential equation.

37. $y' = x e^x$ 38. $y' = \ln x$
 39. $\frac{dy}{dx} = \frac{e}{\sqrt{2+3x}}$ 40. $\frac{dy}{dx} = x^2 \sqrt{x-1}$
 41. $(\cos y)y' = 2x$ 42. $y' = \arctan \frac{x}{2}$

Slope Fields In Exercises 43 and 44, a differential equation and a slope field are given. (a) Sketch two approximate solutions of the differential equation on the slope field, one which passes through the indicated point. (b) Use integration and a graphing utility to graph the solution. Compare the result with the sketches in part (a). To print an enlarged copy of the graph, go to the website www.mathgraphs.com.



43. $\frac{dy}{dx} = x - \sqrt{5} \cos x$, (0, 4) 44. $\frac{dy}{dx} = e^{-y} \sin 2x$, (0, 0)

In Exercises 45 and 46, use a computer algebra system to find the slope field for the differential equation and graph it through the specified initial condition.

45. $\frac{dy}{dx} = \frac{x}{y} e^{y^2}$, $y(0) = 2$
 46. $\frac{dy}{dx} = \frac{x}{y} \sin x$, $y(0) = 4$

In Exercises 47–58, evaluate the definite integral. Use a graphing utility to confirm your result.

47. $\int_0^{\pi/2} x e^{-x/2} \, dx$ 48. $\int_0^1 x^2 e^x \, dx$
 49. $\int_0^{\pi/2} x \cos x \, dx$ 50. $\int_0^{\pi} x \sin 2x \, dx$
 51. $\int_0^{1/2} \arccos x \, dx$ 52. $\int_0^1 x \arcsin x^2 \, dx$
 53. $\int_0^1 e^{x^2} \sin x \, dx$ 54. $\int_0^{\pi} e^{-x} \cos x \, dx$
 55. $\int_0^1 x^2 \ln x \, dx$ 56. $\int_0^1 \ln(1+x^2) \, dx$
 57. $\int_2^4 x \arccos x \, dx$ 58. $\int_0^{\pi/4} x \sec^2 x \, dx$

In Exercises 1–6, give the form of the partial fraction decomposition of the rational expression. Do not solve for the constants.

1. $\frac{5}{x^2 - 10x}$ 2. $\frac{4x^2 + 3}{(x-5)^2}$
 3. $\frac{2x-3}{x^2 + 10x}$ 4. $\frac{x-2}{x^2 + 4x + 3}$
 5. $\frac{16x}{x^2 - 10x^2}$ 6. $\frac{2x-1}{x^2(x+1)^2}$

In Exercises 7–28, use partial fractions to evaluate the integral.

7. $\int \frac{1}{x^2-1} \, dx$ 8. $\int \frac{1}{4x^2-9} \, dx$
 9. $\int \frac{x+3}{x^2+x-3} \, dx$ 10. $\int \frac{x+1}{x^2+4x+3} \, dx$
 11. $\int \frac{-5-x}{2x^2+11x-12} \, dx$ 12. $\int \frac{5x^2-12x-12}{x^3-4x} \, dx$
 13. $\int \frac{x^2+12x+12}{x^2+x-2} \, dx$ 14. $\int \frac{x^2-x+3}{x^2+x-2} \, dx$
 15. $\int \frac{2x^2-4x+15}{x^2-4x+4} \, dx$ 16. $\int \frac{x+2}{x^2-4x} \, dx$
 17. $\int \frac{6x^2+2x-2}{x^2-2x+2} \, dx$ 18. $\int \frac{2x^2-4x}{x^2-4x} \, dx$
 19. $\int \frac{x^2+3x-4}{x^2-4x^2+4x} \, dx$ 20. $\int \frac{4x^2-x-1}{4x^2-x-1} \, dx$
 21. $\int \frac{x^2-1}{x^2+x} \, dx$ 22. $\int \frac{10x^2-18}{x^2-3x} \, dx$
 23. $\int \frac{x^2}{x^4-2x^2-8} \, dx$ 24. $\int \frac{x^2-9}{(x^2+9)^2} \, dx$
 25. $\int \frac{x}{16x^4-1} \, dx$ 26. $\int \frac{x^2-2x+3}{x^2+4x+3} \, dx$
 27. $\int \frac{x^2+5}{x^3-x^2+x+3} \, dx$ 28. $\int \frac{x^2+6x+3}{x^2+6x+3} \, dx$

In Exercises 29–32, evaluate the definite integral. Use a graphing utility to verify your result.

29. $\int_0^1 \frac{3}{2x^2+5x+2} \, dx$ 30. $\int_0^{\pi} \frac{x-1}{x^2(x+1)} \, dx$
 31. $\int_1^2 \frac{x+1}{x(x^2+1)} \, dx$ 32. $\int_0^1 \frac{x^2-x}{x^2+x+1} \, dx$

In Exercises 33–40, use a computer algebra system to determine the antiderivative that passes through the indicated point. Use the system to graph the resulting antiderivative.

33. $\int \frac{3x}{x^2-6x+9} \, dx$, (4, 0) 34. $\int \frac{6x^2+1}{x^2(x-1)^2} \, dx$, (2, 1)
 35. $\int \frac{x^2+x+2}{(x^2+2)^2} \, dx$, (0, 1) 36. $\int \frac{x^2}{(x^2-4)^2} \, dx$, (3, 4)

In Exercises 37–40, use substitution to evaluate the integral.

37. $\int \frac{2x^2-2x+3}{x^3-x^2-x-2} \, dx$, (3, 10)
 38. $\int \frac{x(2x-9)}{x^2-4x^2+12x-8} \, dx$, (3, 2)
 39. $\int \frac{1}{x^2-4} \, dx$, (6, 4) 40. $\int \frac{x^2-x+2}{x^3-x^2+x-1} \, dx$

In Exercises 41–46, use substitution to evaluate the integral.

41. $\int \frac{\sin x}{\cos x(\cos x-1)} \, dx$ 42. $\int \frac{\sin x}{\cos x + \cos^2 x} \, dx$
 43. $\int \frac{3 \cos x}{\sin^2 x + \sin x - 2} \, dx$ 44. $\int \frac{\sec^2 x}{\tan x(\tan x + 1)} \, dx$
 45. $\int \frac{e^x}{(e^x-1)(e^x+4)} \, dx$ 46. $\int \frac{e^x}{(e^x+1)(e^x-1)} \, dx$

In Exercises 47–50, use the method of partial fractions to evaluate the integral.

47. $\int \frac{1}{x(a+bx)} \, dx = \frac{1}{a} \ln \left| \frac{x}{a+bx} \right| + C$
 48. $\int \frac{1}{a^2-x^2} \, dx = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + C$
 49. $\int \frac{x}{(a+bx)^2} \, dx = \frac{1}{b^2} \left(\frac{a}{a+bx} + \ln|a+bx| \right) + C$
 50. $\int \frac{1}{x^2(a+bx)} \, dx = -\frac{1}{x} - \frac{b}{a} \ln \left| \frac{x}{a+bx} \right| + C$

In Exercises 51 and 52, use a computer algebra system to find the slope field for the differential equation, and graph it through the specified initial condition.

51. $\frac{dy}{dx} = \frac{6}{4-x^2}$, $y(0) = 3$ 52. $\frac{dy}{dx} = x^2-2x-3$, $y(0) = 5$

CHALLENGE PROBLEMS

53. What is the first step when integrating $\int \frac{x^2}{x-5} \, dx$?
54. Describe the decomposition of the proper rational function $N(x)/D(x)$ (a) if $D(x) = (ax+b)^n$, and (b) if $(ax^2+bx+c)^n$ where $4a^2+b^2+c^2 < 0$ is irreducible. Do not integrate.
- (a) $\int \frac{x+1}{x^2+2x-8} \, dx$ (b) $\int \frac{7x+4}{x^2+2x-8} \, dx$
 (c) $\int \frac{4}{x^2+2x+5} \, dx$