Study Guide - Parametric/Vectors

$\boxed{B}$ 1. $xy = 10$

$y = \frac{10}{x}$

$\frac{dy}{dx} = -\frac{10}{x^2}$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

$\frac{-10}{x^2} = \frac{3}{\frac{dx}{dt}}$

$\frac{-10}{4} = \frac{3}{\frac{dx}{dt}}$

$\frac{-5}{2} = \frac{3}{\frac{dx}{dt}}$

$-5\frac{dx}{dt} = 6$

$\frac{dx}{dt} = -\frac{6}{5}$

$\boxed{D}$ 2. $x = \sin t \quad y = \cos^2 t$

$\frac{dy}{dx} = \frac{-2\cancel{\cos t}\sin t}{\cancel{\cos t}} = -2\sin t$

$\frac{d^2y}{dx^2} = \frac{-2\cancel{\cos t}}{\cancel{\cos t}} = -2$

$\boxed{C}$ 3. $x = t^3 + t \quad y = t^4 + 2t^2$

$\frac{dy}{dx} = \frac{4t^3 + 4t}{3t^2 + 1}$

at $t = 1$ $x = 2$

$y = 3$

at $t = 1$ $\frac{4(1)^3 + 4(1)}{3(1)^2 + 1} = \frac{8}{4} = 2$

$y - 3 = 2(x - 2)$

$y - 3 = 2x - 4$

$y = 2x - 1$

$\boxed{C}$ 4. $x = t^3 - t^2 - 1 \quad y = t^4 + 2t^2 - 8t$

$\frac{dy}{dx} = \frac{4t^3 + 4t - 8}{3t^2 - 2t}$

vertical tangent where $3t^2 - 2t = 0$

$t(3t - 2) = 0$

$t = 0 \quad t = \frac{2}{3}$

$\boxed{C}$ 5. $x = t^2 \quad y = \sin(4t)$

speed $= \sqrt{(2t)^2 + (4\cos(4t))^2}$

at $t = 3$

position: $\langle t^2, \sin 4t \rangle$

speed $= \sqrt{6^2 + (4\cos(12))^2}$

use radians!

$v = \langle 2t, 4\cos(4t) \rangle$

$= \sqrt{36 + 11.3?3}$

speed $= 6.884$

$\boxed{C}$ 6. $x = t^3 - 3t^2 \quad y = 2t^3 - 3t^2 - 12t$

position: $\langle t^3 - 3t^2, 2t^3 - 3t^2 - 12t \rangle$

velocity: $\langle 3t^2 - 6t, 6t^2 - 6t - 12 \rangle$

$3t^2 - 6t = 0$ $\quad$ $6t^2 - 6t - 12 = 0$

$3t(t-2)$ $\quad$ $6(t^2 - t - 2) = 0$

$3t = 0 \quad t - 2 = 0$ $\quad$ $(t+1)(t-2) = 0$

$t = 0 \quad \underline{t = 2}$ $\quad$ $t = -1 \quad \underline{t = 2}$

when $t = 2$ velocity: $\langle 0, 0 \rangle$ so at rest.

$\boxed{D}$ 7. $x = 3\cos t \quad y = 4\sin t$

$\frac{dx}{dy} = \frac{4\cos t}{-3\sin t} = -\frac{4}{3}\cot t$ at $t = 13$ $\quad \boxed{\frac{-4}{3\tan 13}}$

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A 8. $x = 3 + 2\cos\theta \quad y = 1 + \sin\theta \qquad \frac{y-1}{\cos\theta} = \frac{\sin\theta}{\cos\theta}$

$\frac{x-3}{2} = \cos\theta \quad y - 1 = \sin\theta$

$\cos^2\theta + \sin^2\theta = 1$

$\left(\frac{x-3}{2}\right)^2 + (y-1)^2 = 1$

$\frac{x^2 - 6x + 9}{4} + y^2 - 2y + 1 = 1$

$x^2 - 6x + 9 + 4y^2 - 8y + \cancel{4} = \cancel{4}$

$x^2 - 6x + 9 + 4y^2 - 8y = 0$

E 9. $x = \cos^3 t \quad y = \sin^3 t \qquad x'(t) = 3\cos^2 t(-\sin t)$

$= -3\sin t\cos^2 t$

$\int_0^{\pi/2} \sqrt{9\sin^2\cos^4 t + 9\sin^4 t\cos^2 t} \qquad y'(t) = 3\sin^2 t\cos t$

D 10. $x = t^2 - 1 \quad y = t^4 - 2t^3$

position: $\langle t^2 - 1, t^4 - 2t^3 \rangle$

velocity: $\langle 2t, 4t^3 - 6t^2 \rangle$

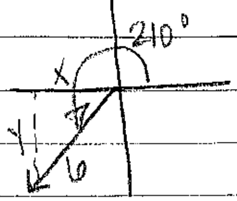
acceleration: $\langle 2, 12t^2 - 12t \rangle$ at $t = 1$ $\langle 2, 0 \rangle$

C 11. $\langle 3 - 3, -2 - 7 \rangle$

$\langle 0, -9 \rangle$

D 12. magnitude: $\sqrt{25 + 4}$ unit vector: $\frac{\langle -5, 2 \rangle}{\sqrt{29}} = \left\langle \frac{-5}{\sqrt{29}}, \frac{2}{\sqrt{29}} \right\rangle$

$\sqrt{29}$

13. 

$\cos 210 = \frac{x}{6}$ $\sin 210 = \frac{y}{6}$

$6\cos 210 = x$ $6\sin 210 = y$

$6\left(-\frac{\sqrt{3}}{2}\right) = x$ $6\left(-\frac{1}{2}\right) = y$

$-3\sqrt{3} = x$ $-3 = y$

$\boxed{\langle -3\sqrt{3}, -3\rangle}$

14. CF: $\langle 5 - -3, 26 - 11\rangle$

$\boxed{\langle 8, 15\rangle}$

M: $\sqrt{8^2 + 15^2}$

$\sqrt{64 + 225}$

$\sqrt{289}$

$\boxed{17}$

D: $\tan\theta = \frac{15}{8}$

$\boxed{\theta = 61.9^\circ}$

15. $||u|| = \sqrt{2^2 + (-7)^2}$

$= \sqrt{4 + 49}$

$= \boxed{\sqrt{53}}$

b. $2u - 3v$

$2\langle 2, -7\rangle - 3\langle -3, -5\rangle$

$\langle 4, -14\rangle - \langle -9, -15\rangle$

$\boxed{\langle 13, 1\rangle}$

16. $\langle 200\cos 215, 200\sin 215\rangle + \langle 150\cos 162, 150\sin 162\rangle$

$\langle -163.83, -114.715\rangle + \langle -142.658, 46.353\rangle$

$\boxed{\langle -306.488, -68.362\rangle}$

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17. a) A $(420,0)$ W2 N3.2 $m = \frac{3.2}{-2}$

$$y - 0 = -\frac{3.2}{2}(x - 420)$$

$$y = -1.6(x - 420)$$

$$\boxed{y = -1.6x + 672}$$

b) $x = 420 - 2t$ $y = 3.2t$

c) $x = t$ $y = 300 + 1.5t$ $\boxed{y = 300 + 1.5x}$

d) $-1.6x + 672 = 300 + 1.5x$ $y = 300 + 1.5(120)$

$372 = 3.1x$ $y = 480$

$120 = x$

$\boxed{(120, 480)}$

e) Plane A: $120 = 420 - 2t$ $t = 150$

Plane B: $t = 120$

$\boxed{\text{No they don't crash}}$

18. $x^2 + y^2 = (42{,}000)^2$ $\sin^2 t + \cos^2 \theta = 1$

$$\frac{x^2}{(42{,}000)^2} + \frac{y^2}{(42{,}000)^2} = 1 \qquad \sin^2(2\pi/8) + \cos^2(2\pi/8) = 1$$

$$\sqrt{\frac{x^2}{(42{,}000)^2}} = \sqrt{\cos^2(2\pi/8)} \qquad \sqrt{\frac{y^2}{(42{,}000)^2}} = \sqrt{\sin^2\left(\frac{2\pi}{8}\right)}$$

$$\frac{x}{42{,}000} = \cos(2\pi/8) \qquad \frac{y}{42{,}000} = \sin\left(\frac{2\pi}{8}\right)$$

$\boxed{x = 42{,}000\cos(\tfrac{2\pi}{8}t + 2)}$

$\boxed{y = 42{,}000\sin(\tfrac{2\pi}{8}t + 2)}$

19. a) $\frac{dx}{dt} = -3t^2$ $\quad \frac{dy}{dt} = 2t$ $\quad \boxed{\langle -3t^2, 2t \rangle}$

b) $\langle -3(-1)^2, 2(-1) \rangle = \langle -3, -2 \rangle$

left b/c negative

down b/c negative

c) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \quad \frac{2t}{-3t^2} = \frac{-2}{3t}$ at $t = .5$

$= \frac{-2}{3(.5)}$

$= \boxed{\frac{-4}{3}}$

d) $\langle -3t^2, 2t \rangle = \langle 0, 0 \rangle$ when $t=0$

$-3t^2=0$ $\quad 2t=0$

$t=0$ $\quad t=0$

acceleration $= \langle -6t, 2 \rangle$

$\boxed{\text{when } t=0 \text{ acceleration} = \langle 0, 2 \rangle}$

20. $v = \langle \tan(e^{-t}), \sec(e^{-t}) \rangle$

a) $\langle -e^{-t}\sec^2(e^{-t}), -e^{-t}\sec(e^{-t})\tan(e^{-t}) \rangle$

$\langle -e^{-1}\sec^2(e^{-1}), -e^{-1}\sec e^{-1}\tan e^{-1} \rangle$

$\boxed{\left\langle -\frac{\sec^2(\frac{1}{e})}{e}, \frac{-\sec\frac{1}{e}\tan\frac{1}{e}}{e} \right\rangle}$

b) $\langle \tan(1/e), \sec(1/e) \rangle$

$S = \sqrt{(\tan(1/e))^2}$

$S = \sqrt{\tan^2(1/e) + \sec^2(1/e)}$

$S = \sqrt{.149 + 1.149}$

$S = \boxed{1.139}$

c) $\int_1^2 \tan(e^{-t})\,dt \qquad \int_1^2 \sec(e^{-t})\,dt$

$\langle .238, 1.030 \rangle$ displacement

at $t=1$ position is $(2,-3)$
at $t=2$ position is $(2+.238, -3+1.030) = \boxed{(2.238, -1.97)}$

d) $\int_1^2 \sqrt{\tan^2(e^{-t}) + \sec^2(e^{-t})}\,dt$

$= \boxed{1.059}$