

Streifenbreite - Parametrisierung / Vektore

[B] 1.

$$xy = 10$$

$$y = \frac{10}{x}$$

$$\frac{dy}{dx} = -\frac{10}{x^2}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$-\frac{10}{x^2} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$-\frac{10}{4} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$-\frac{5}{2} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$-5 \frac{dx}{dt} = 6 \frac{dy}{dt}$$

$$\frac{dx}{dt} = -\frac{6}{5} \frac{dy}{dt}$$

[D] 2.

$$x = 2 \sin t \quad y = \cos^2 t$$

$$\frac{dy}{dx} = \frac{-2 \cos t \sin t}{2 \cos t} = -\sin t$$

$$\frac{d^2y}{dx^2} = \frac{-2 \cos t}{2 \cos t} = -2$$

[C] 3.

$$x = t^3 + t \quad y = t^4 + 2t^2$$

$$\frac{dy}{dx} = \frac{4t^3 + 4t}{3t^2 + 1}$$

at $t=1$ $x=2$

$y=3$

$$\text{at } t=1 \quad \frac{4(1)^3 + 4(1)}{3(1)^2 + 1} = \frac{8}{4} = 2$$

$$y-3 = 2(x-2)$$

$$y-3 = 2x-4$$

$$y = 2x-1$$

[C] 4.

$$x = t^3 + 2t \quad y = t^4 + 2t^2 - 8t$$

$$\frac{dy}{dx} = \frac{4t^3 + 4t - 8}{3t^2 + 2}$$

vertical tangents where $3t^2 + 2 = 0$

$$t(3t+2) = 0$$

$$t = 0 \quad t = -\frac{2}{3}$$

$$\frac{2}{3}$$

C 5. $x = t^2$ $y = 8m(4t)$

position: $\langle t^2, 8m(4t) \rangle$

$v = \langle 2t, 400(4t) \rangle$

speed = $\sqrt{(2t)^2 + (400(4t))^2}$
at $t = 3$

speed = $\sqrt{6^2 + (400(12))^2}$

= $\sqrt{36 + 11.312}$

speed = 6.884

use reactions!

C 6. $x = t^3 - 3t^2$ $y = 2t^3 - 3t^2 - 12t$

position: $\langle t^3 - 3t^2, 2t^3 - 3t^2 - 12t \rangle$

velocity: $\langle 3t^2 - 6t, 6t^2 - 6t - 12 \rangle$

$3t^2 - 6t = 0$

$3t(t-2)$

$3t = 0$ $t-2 = 0$

$t = 0$ $t = 2$

$6t^2 - 6t - 12 = 0$

$6(t^2 - t - 2) = 0$

$(t+1)(t-2) = 0$

$t = -1$ $t = 2$

When $t = 2$ velocity: $\langle 0, 0 \rangle$ so at rest.

D 7. $x = 3\cos t$ $y = 4\sin t$

$\frac{dx}{dt} = \frac{-4\cos t}{-3\sin t} = \frac{-4\cot t}{3}$ at $t = \frac{\pi}{3}$

$\frac{-4}{3} \cot \frac{\pi}{3}$

[A] 8.

$$x = 3 + 2\cos\theta$$

$$y = 1 + \sin\theta$$

$$y - 1 = \sin\theta$$

$$\frac{x-3}{2} = \cos\theta$$

$$y-1 = \sin\theta$$

$$\cos\theta$$

$$\frac{1}{2}$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\left(\frac{x-3}{2}\right)^2 + (y-1)^2 = 1$$

$$\frac{x^2 - 6x + 9}{4} + y^2 - 2y + 1 = 1$$

$$x^2 - 6x + 9 + 4y^2 - 8y + 4 = 4$$

$$x^2 - 6x + 9 + 4y^2 - 8y = 0$$

[E] 9.

$$x = \cos^2 t$$

$$y = 8\sin^2 t$$

$$x'(t) = 2\cos t(-\sin t)$$

$$= -2\sin t \cos t$$

$$\int_0^{\pi/2} \sqrt{9\sin^2 t \cos^4 t + 9\sin^4 t \cos^2 t}$$

$$y'(t) = 16\sin t \cos t$$

[D] 10.

$$x = t^2 - 1$$

$$y = t^4 - 2t^3$$

position: $\langle t^2 - 1, t^4 - 2t^3 \rangle$

velocity: $\langle 2t, 4t^3 - 6t^2 \rangle$

acceleration: $\langle 2, 12t^2 - 12t \rangle$ at $t=1$ $\langle 2, 0 \rangle$

[C] 11.

$$\langle 3-3, -2-7 \rangle$$

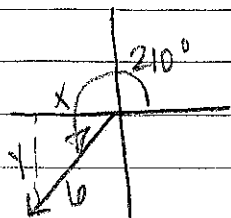
$$\langle 0, -9 \rangle$$

[D] 12.

magnitude: $\frac{\sqrt{25+4}}{\sqrt{29}}$

unit vector: $\frac{\langle -5, 2 \rangle}{\sqrt{29}} = \left\langle \frac{-5}{\sqrt{29}}, \frac{2}{\sqrt{29}} \right\rangle$

13.



$$\cos 210 = \frac{x}{b}$$

$$\sin 210 = \frac{y}{b}$$

$$b \cos 210 = x$$

$$b \sin 210 = y$$

$$b \left(-\frac{\sqrt{3}}{2}\right) = x$$

$$b \left(-\frac{1}{2}\right) = y$$

$$-3\sqrt{3} = x$$

$$-3 = y$$

$$\boxed{\langle -3\sqrt{3}, -3 \rangle}$$

$$14. \text{ CF: } \langle 5, -3, 24, -11 \rangle$$

$$\boxed{\langle 8, 15 \rangle}$$

$$M: \sqrt{8^2 + 15^2}$$

$$D: \tan \theta = \frac{15}{8}$$

$$\sqrt{64 + 225}$$

$$B$$

$$\sqrt{289}$$

$$\boxed{\theta = 61.9^\circ}$$

$$\boxed{17}$$

$$15. \|u\| = \sqrt{2^2 + (-7)^2}$$

$$b. 2u = 3v$$

$$= \sqrt{4 + 49}$$

$$2\langle 2, -7 \rangle = 3\langle -3, 5 \rangle$$

$$= \boxed{\sqrt{53}}$$

$$\langle 4, -14 \rangle = \langle -9, 15 \rangle$$

$$\boxed{\langle 13, 1 \rangle}$$

$$16. \langle 200 \cos 215, 200 \sin 215 \rangle + \langle 150 \cos 162, 150 \sin 162 \rangle$$

$$\langle -163.88, -114.715 \rangle + \langle -142.658, 46.358 \rangle$$

$$\boxed{\langle -306.488, -68.362 \rangle}$$

17. a) A (420, 0) w2 N3.2 $m = \frac{3.2}{-2}$

$$y - 0 = -\frac{3.2}{2}(x - 420)$$

$$y = -1.6(x - 420)$$

$$\boxed{y = -1.6x + 672}$$

b) $x = 420 - 2t$ $y = 3.2t$

c) $x = t$ $y = 300 + 1.5t$

$$\boxed{y = 300 + 1.5x}$$

d) $-1.6x + 672 = 300 + 1.5x$
 $372 = 3.1x$
 $120 = x$

$y = 300 + 1.5(120)$
 $y = 480$

$$\boxed{(120, 480)}$$

e) Plane A: $120 = 420 - 2t$ $t = 150$
 Plane B: $t = 120$

$\boxed{\text{No they don't crash}}$

18. $x^2 + y^2 = (42,000)^2$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\frac{x^2}{(42,000)^2} + \frac{y^2}{(42,000)^2} = 1 \quad \sin^2\left(\frac{2\pi}{8}\right) + \cos^2\left(\frac{2\pi}{8}\right) = 1$$

$$\sqrt{\frac{x^2}{(42,000)^2}} = \sqrt{\cos^2\left(\frac{2\pi}{8}\right)} \quad \sqrt{\frac{y^2}{(42,000)^2}} = \sqrt{\sin^2\left(\frac{2\pi}{8}\right)}$$

$$\frac{x}{42,000} = \cos\left(\frac{2\pi}{8}\right) \quad \frac{y}{42,000} = \sin\left(\frac{2\pi}{8}\right)$$

$$\boxed{x = 42,000 \cos\left(\frac{2\pi}{8} + 2\right)} \quad \boxed{y = 42,000 \sin\left(\frac{2\pi}{8} + 2\right)}$$

$$\boxed{y = 42,000 \sin\left(\frac{2\pi}{8} + 2\right)}$$

$$19. a) \frac{dx}{dt} = -3t^2 \quad \frac{dy}{dt} = 2t$$

$$\langle -3t^2, 2t \rangle$$

$$b) \langle -3(-1)^2, 2(-1) \rangle = \langle -3, -2 \rangle$$

left b/c
negative

down b/c
negative

$$c) \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t}{-3t^2} = -\frac{2}{3t} \text{ at } t = .5$$

$$= -\frac{2}{3(.5)}$$

$$= \boxed{\frac{-4}{3}}$$

$$d) \langle -3t^2, 2t \rangle = \langle 0, 0 \rangle \text{ when } t=0$$

$3t^2=0 \quad 2t=0$
 $t=0 \quad t=0$

$$\text{acceleration} = \langle -6t, 2 \rangle$$

$$\boxed{\text{When } t=0 \text{ acceleration} = \langle 0, 2 \rangle}$$

$$20. \mathbf{v} = \langle -\tan(e^{-t}), \sec(e^{-t}) \rangle$$

$$a) \langle -e^{-t} \sec^2(e^{-t}), -e^{-t} \sec(e^{-t}) \tan(e^{-t}) \rangle$$

$$\langle -e^{-1} \sec^2(e^{-1}), -e^{-1} \sec(e^{-1}) \tan(e^{-1}) \rangle$$

$$\left\langle \frac{-\sec^2\left(\frac{1}{e}\right)}{e}, \frac{-\sec\left(\frac{1}{e}\right) \tan\left(\frac{1}{e}\right)}{e} \right\rangle$$

$$b) \langle \tan\left(\frac{1}{e}\right), \sec\left(\frac{1}{e}\right) \rangle$$

$$s = \sqrt{\tan^2\left(\frac{1}{e}\right) + \sec^2\left(\frac{1}{e}\right)}$$

$$s = \sqrt{\tan^2\left(\frac{1}{e}\right) + \sec^2\left(\frac{1}{e}\right)}$$

$$s = \sqrt{1.149 + 1.149}$$

$$s = \boxed{1.139}$$

$$c) \int_1^2 \tan(e^{-t}) dt \quad \int_1^2 \sec(e^{-t}) dt$$

$$\langle 0.238, 1.030 \rangle \quad \text{displacement}$$

at $t=1$ position is $(2, -3)$

$$at \ t=2 \text{ position is } (2+0.238, -3+1.030) = \boxed{(2.238, -1.97)}$$

$$d) \int_1^2 \sqrt{\tan^2(e^{-t}) + \sec^2(e^{-t})} dt$$

$$= \boxed{1.059}$$