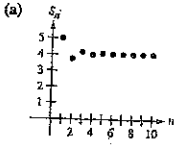
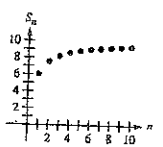
EXERCISES FOR SECTION 8.5

In Exercises 1–4, match the series with the graph of its sequence of partial sums. [The graphs are labeled (a), (b), (c), and (d).]

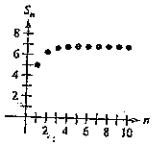
(a)



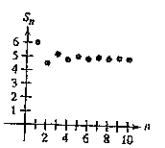
(b)



(c)



(d)



1. $\sum_{n=1}^{\infty} \frac{6}{n^2}$
2. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} 6}{n^2}$
3. $\sum_{n=1}^{\infty} \frac{10}{n2^n}$
4. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} 10}{n2^n}$

Numerical and Graphical Analysis **In Exercises 5–8, explore the Alternating Series Remainder.**

(a) Use a graphing utility to find the indicated partial sum S_n and complete the table.

(b) Use a graphing utility to graph the first ten terms of the sequence of partial sums and a horizontal line representing the sum.

(c) What pattern exists between the plot of the successive points in part (b) relative to the horizontal line representing the sum of the series? Do the distances between the successive points and the horizontal line increase or decrease?

(d) Discuss the relationship between the answers in part (c) and the Alternating Series Remainder as given in Theorem 8.15.

n	1	2	3	4	5	6	7	8	9	10
S_n										

5. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} = \frac{\pi}{4}$
6. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(n-1)!} = \frac{1}{e}$
7. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} = \frac{\pi^2}{12}$
8. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)!} = \sin 1$

In Exercises 9–28, determine the convergence or divergence of the series.

9. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$
10. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{2n-1}$
11. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1}$
12. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$
13. $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^2+1}$
14. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{n^2+1}$
15. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$
16. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^2}{n^2+5}$
17. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+1)}{\ln(n+1)}$
18. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \ln(n+1)}{n+1}$
19. $\sum_{n=1}^{\infty} \sin \frac{(2n-1)\pi}{2}$
20. $\sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{(2n-1)\pi}{2}$
21. $\sum_{n=1}^{\infty} \cos n\pi$
22. $\sum_{n=1}^{\infty} \frac{1}{n} \cos n\pi$
23. $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$
24. $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)}$
25. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{n+2}$
26. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{\sqrt{n}}$
27. $\sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{e^n - e^{-n}} = \sum_{n=1}^{\infty} (-1)^{n+1} \operatorname{csch} n$
28. $\sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{e^n + e^{-n}} = \sum_{n=1}^{\infty} (-1)^{n+1} \operatorname{sech} n$

In Exercises 29–32, approximate the sum of the series by using the first six terms. (See Example 4.)

29. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 3}{n^2}$
30. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 4}{\ln(n+1)}$
31. $\sum_{n=0}^{\infty} \frac{(-1)^n 2}{n!}$
32. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{2^n}$

In Exercises 33–38, (a) use Theorem 8.15 to determine the number of terms required to approximate the sum of the convergent series with an error of less than 0.001, and (b) use a graphing utility to approximate the sum of the series with an error of less than 0.001.

33. $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = \frac{1}{e}$
34. $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n n!} = \frac{1}{\sqrt{e}}$
35. $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} = \sin 1$
36. $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} = \cos 1$
37. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = \ln 2$
38. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n4^n} = \ln \frac{5}{4}$

EXERCISES FOR SECTION 8.4

1. ***Graphical Analysis*** The figures show the graphs of the first ten terms, and the graphs of the first ten terms of the sequence of partial sums, of each series.

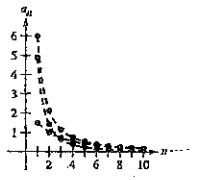
$\sum_{n=1}^{\infty} \frac{6}{n^{3/2}}$, $\sum_{n=1}^{\infty} \frac{6}{n^{3/2}+3}$, and $\sum_{n=1}^{\infty} \frac{6}{n\sqrt{n^2+0.5}}$

(a) Identify the series in each figure.

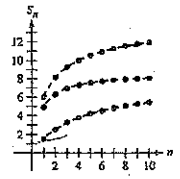
(b) Which series is a p-series? Does it converge or diverge?

(c) For the series that are not p-series, how do the magnitudes of the terms compare with the magnitudes of the terms of the p-series? What conclusion can you draw about the convergence or divergence of the series?

(d) Explain the relationship between the magnitudes of the terms of the series and the magnitudes of the terms of the partial sums.



Graphs of terms



Graphs of partial sums

2. ***Graphical Analysis*** The figures show the graphs of the first ten terms, and the graphs of the first ten terms of the sequence of partial sums, of each series.

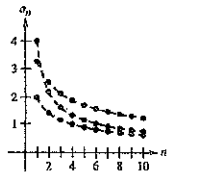
$\sum_{n=1}^{\infty} \frac{2}{\sqrt{n}}$, $\sum_{n=1}^{\infty} \frac{2}{\sqrt{n}-0.5}$, and $\sum_{n=1}^{\infty} \frac{4}{\sqrt{n}+0.5}$

(a) Identify the series in each figure.

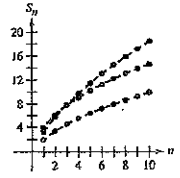
(b) Which series is a p-series? Does it converge or diverge?

(c) For the series that are not p-series, how do the magnitudes of the terms compare with the magnitudes of the terms of the p-series? What conclusion can you draw about the convergence or divergence of the series?

(d) Explain the relationship between the magnitudes of the terms of the series and the magnitudes of the terms of the partial sums.



Graphs of terms



Graphs of partial sums

In Exercises 3–14, use the Direct Comparison Test to determine the convergence or divergence of the series.

3. $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$
4. $\sum_{n=1}^{\infty} \frac{1}{3n^2+2}$
5. $\sum_{n=2}^{\infty} \frac{1}{n-1}$
6. $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$
7. $\sum_{n=0}^{\infty} \frac{1}{3^n+1}$
8. $\sum_{n=0}^{\infty} \frac{3^n}{4^n+5}$
9. $\sum_{n=2}^{\infty} \frac{\ln n}{n+1}$
10. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}}$
11. $\sum_{n=0}^{\infty} \frac{1}{n!}$
12. $\sum_{n=1}^{\infty} \frac{1}{4\sqrt[3]{n}-1}$
13. $\sum_{n=0}^{\infty} e^{-n^2}$
14. $\sum_{n=1}^{\infty} \frac{4^n}{3^n-1}$

In Exercises 15–28, use the Limit Comparison Test to determine the convergence or divergence of the series.

15. $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$
16. $\sum_{n=1}^{\infty} \frac{2}{3^n-5}$
17. $\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+1}}$
18. $\sum_{n=3}^{\infty} \frac{3}{\sqrt{n^2-4}}$
19. $\sum_{n=1}^{\infty} \frac{2n^2-1}{3n^3+2n+1}$
20. $\sum_{n=1}^{\infty} \frac{5n-3}{n^2-2n+5}$
21. $\sum_{n=1}^{\infty} \frac{n+3}{n(n+2)}$
22. $\sum_{n=1}^{\infty} \frac{1}{n(n^2+1)}$
23. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2+1}}$
24. $\sum_{n=1}^{\infty} \frac{n}{(n+1)2^{n-1}}$
25. $\sum_{n=1}^{\infty} \frac{n^{k-1}}{n^k+1}$, $k > 2$
26. $\sum_{n=1}^{\infty} \frac{5}{n+\sqrt{n^2+4}}$
27. $\sum_{n=1}^{\infty} \sin \frac{1}{n}$
28. $\sum_{n=1}^{\infty} \tan \frac{1}{n}$

In Exercises 29–36, test for convergence or divergence, using each test at least once. Identify the test used.

(a) nth-Term Test **(b) Geometric Series Test**
(c) p-Series Test **(d) Telescoping Series Test**
(e) Integral Test **(f) Direct Comparison Test**
(g) Limit Comparison Test

29. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n}$
30. $\sum_{n=0}^{\infty} 5\left(-\frac{1}{5}\right)^n$
31. $\sum_{n=1}^{\infty} \frac{1}{3^n+2}$
32. $\sum_{n=4}^{\infty} \frac{1}{3n^2-2n-15}$
33. $\sum_{n=1}^{\infty} \frac{n}{2n+3}$
34. $\sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+2}\right)$
35. $\sum_{n=1}^{\infty} \frac{n}{(n^2+1)^2}$
36. $\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$