EXERCISES FOR SECTION 8.4

1. ***Graphical Analysis*** The figures show the graphs of the first ten terms, and the graphs of the first ten terms of the sequence of partial sums, of each series.

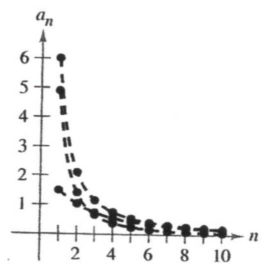
$$\sum_{n=1}^{\infty} \frac{6}{n^{3/2}}, \quad \sum_{n=1}^{\infty} \frac{6}{n^{3/2}+3}, \text{ and } \sum_{n=1}^{\infty} \frac{6}{n\sqrt{n^2+0.5}}$$

(a) Identify the series in each figure.

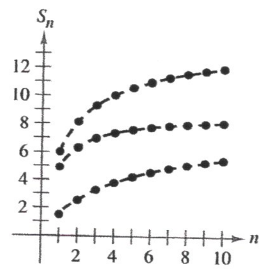
(b) Which series is a p-series? Does it converge or diverge?

(c) For the series that are not p-series, how do the magnitudes of the terms compare with the magnitudes of the terms of the p-series? What conclusion can you draw about the convergence or divergence of the series?

(d) Explain the relationship between the magnitudes of the terms of the series and the magnitudes of the terms of the partial sums.



Graphs of terms



Graphs of partial sums

2. ***Graphical Analysis*** The figures show the graphs of the first ten terms, and the graphs of the first ten terms of the sequence of partial sums, of each series.

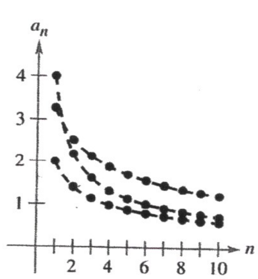
$$\sum_{n=1}^{\infty} \frac{2}{\sqrt{n}}, \quad \sum_{n=1}^{\infty} \frac{2}{\sqrt{n}-0.5}, \text{ and } \sum_{n=1}^{\infty} \frac{4}{\sqrt{n}+0.5}$$

(a) Identify the series in each figure.

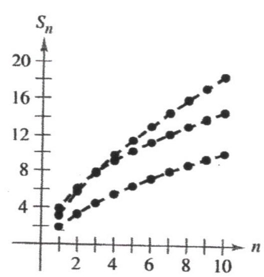
(b) Which series is a p-series? Does it converge or diverge?

(c) For the series that are not p-series, how do the magnitudes of the terms compare with the magnitudes of the terms of the p-series? What conclusion can you draw about the convergence or divergence of the series?

(d) Explain the relationship between the magnitudes of the terms of the series and the magnitudes of the terms of the partial sums.



Graphs of terms



Graphs of partial sums

In Exercises 3–14, use the Direct Comparison Test to determine the convergence or divergence of the series.

3. $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

4. $\sum_{n=1}^{\infty} \frac{1}{3n^2+2}$

5. $\sum_{n=2}^{\infty} \frac{1}{n-1}$

6. $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$

7. $\sum_{n=0}^{\infty} \frac{1}{3^n+1}$

8. $\sum_{n=0}^{\infty} \frac{3^n}{4^n+5}$

9. $\sum_{n=2}^{\infty} \frac{\ln n}{n+1}$

10. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}}$

11. $\sum_{n=0}^{\infty} \frac{1}{n!}$

12. $\sum_{n=1}^{\infty} \frac{1}{4\sqrt[3]{n}-1}$

13. $\sum_{n=0}^{\infty} e^{-n^2}$

14. $\sum_{n=1}^{\infty} \frac{4^n}{3^n-1}$

In Exercises 15–28, use the Limit Comparison Test to determine the convergence or divergence of the series.

15. $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

16. $\sum_{n=1}^{\infty} \frac{2}{3^n-5}$

17. $\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+1}}$

18. $\sum_{n=3}^{\infty} \frac{3}{\sqrt{n^2-4}}$

19. $\sum_{n=1}^{\infty} \frac{2n^2-1}{3n^5+2n+1}$

20. $\sum_{n=1}^{\infty} \frac{5n-3}{n^2-2n+5}$

21. $\sum_{n=1}^{\infty} \frac{n+3}{n(n+2)}$

22. $\sum_{n=1}^{\infty} \frac{1}{n(n^2+1)}$

23. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2+1}}$

24. $\sum_{n=1}^{\infty} \frac{n}{(n+1)2^{n-1}}$

25. $\sum_{n=1}^{\infty} \frac{n^{k-1}}{n^k+1}, \quad k > 2$

26. $\sum_{n=1}^{\infty} \frac{5}{n+\sqrt{n^2+4}}$

27. $\sum_{n=1}^{\infty} \sin\frac{1}{n}$

28. $\sum_{n=1}^{\infty} \tan\frac{1}{n}$

In Exercises 29–36, test for convergence or divergence, using each test at least once. Identify the test used.

(a) nth-Term Test **(b) Geometric Series Test**
(c) p-Series Test **(d) Telescoping Series Test**
(e) Integral Test **(f) Direct Comparison Test**
(g) Limit Comparison Test

29. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n}$

30. $\sum_{n=0}^{\infty} 5\left(-\frac{1}{5}\right)^n$

31. $\sum_{n=1}^{\infty} \frac{1}{3^n+2}$

32. $\sum_{n=4}^{\infty} \frac{1}{3n^2-2n-15}$

33. $\sum_{n=1}^{\infty} \frac{n}{2n+3}$

34. $\sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+2}\right)$

35. $\sum_{n=1}^{\infty} \frac{n}{(n^2+1)^2}$

36. $\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$

37. Use the Limit Comparison Test with the harmonic series to show that the series Σa_n (where $0 < a_n < a_{n-1}$) diverges if

$$\lim_{n\to\infty} na_n$$

is finite and nonzero.

38. Prove that, if $P(n)$ and $Q(n)$ are polynomials of degree j and k, respectively, then the series

$$\sum_{n=1}^{\infty} \frac{P(n)}{Q(n)}$$

converges if $j < k - 1$ and diverges if $j \geq k - 1$.

In Exercises 39–42, use the polynomial test given in Exercise 38 to determine whether the series converges or diverges.

39. $\frac{1}{2} + \frac{2}{5} + \frac{3}{10} + \frac{4}{17} + \frac{5}{26} + \cdots$

40. $\frac{1}{3} + \frac{1}{8} + \frac{1}{15} + \frac{1}{24} + \frac{1}{35} + \cdots$

41. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3 + 1}$

42. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^3 + 1}$

In Exercises 43 and 44, use the divergence test given in Exercise 37 to show that the series diverges.

43. $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{5n^4 + 3}$

44. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\ln n}$

Getting at the Concept

45. State the Direct Comparison Test and give an example of its use.

46. State the Limit Comparison Test and give an example of its use.

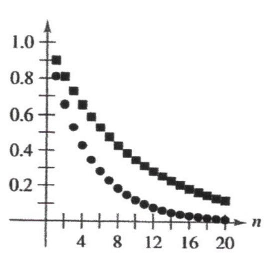
47. The figure shows the first 20 terms of the convergent series

$$\sum_{n=1}^{\infty} a_n$$

and the first 20 terms of the series

$$\sum_{n=1}^{\infty} a_n^2.$$

Identify the two series and explain your reasoning in making the selection.



Getting at the Concept *(continued)*

48. It appears that the terms of the series

$$\frac{1}{1000} + \frac{1}{1001} + \frac{1}{1002} + \frac{1}{1003} + \cdots$$

are less than the corresponding terms of the convergent series

$$1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots.$$

If the statement above is correct, the first series converges. Is this correct? Why or why not? Make a statement about how the divergence or convergence of a series is affected by inclusion or exclusion of the first finite number of terms.

In Exercises 49–52, determine the convergence or divergence of the series.

49. $\frac{1}{200} + \frac{1}{400} + \frac{1}{600} + \frac{1}{800} + \cdots$

50. $\frac{1}{200} + \frac{1}{210} + \frac{1}{220} + \frac{1}{230} + \cdots$

51. $\frac{1}{201} + \frac{1}{204} + \frac{1}{209} + \frac{1}{216} + \cdots$

52. $\frac{1}{201} + \frac{1}{208} + \frac{1}{227} + \frac{1}{264} + \cdots$

53. ***Think About It*** Review the results of Exercises 49–52. Explain why careful analysis is required to determine the convergence or divergence of a series and why only considering the magnitudes of the terms of a series could be misleading.

54. Consider the following series and its sum.

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$$

(a) Verify that the series converges.

(b) Use a graphing utility to complete the table.

n	5	10	20	50	100
S_n					

(c) Find the sum of the series

$$\sum_{n=3}^{\infty} \frac{1}{(2n-1)^2}$$

by hand. Describe how you found the sum.

(d) Use a graphing utility to find the sum of the series

$$\sum_{n=10}^{\infty} \frac{1}{(2n-1)^2}.$$

True or False? **In Exercises 55–58, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

55. If $0 < a_n \leq b_n$ and $\displaystyle\sum_{n=1}^{\infty} a_n$ converges, then $\displaystyle\sum_{n=1}^{\infty} b_n$ diverges.

56. If $0 < a_{n+10} \leq b_n$ and $\displaystyle\sum_{n=1}^{\infty} b_n$ converges, then $\displaystyle\sum_{n=1}^{\infty} a_n$ converges.