Lab Series 11

EXERCISES FOR SECTION 8.2

In Exercises 1–6, find the first five terms of the sequence of partial sums.

1. $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots$

2. $\frac{1}{2\cdot 3} + \frac{2}{3\cdot 4} + \frac{3}{4\cdot 5} + \frac{4}{5\cdot 6} + \frac{5}{6\cdot 7} + \cdots$

3. $3 - \frac{9}{2} + \frac{27}{4} - \frac{81}{8} + \frac{243}{16} - \cdots$

4. $\frac{1}{1} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \frac{1}{11} + \cdots$

5. $\sum_{n=1}^{\infty} \frac{3}{2^{n-1}}$

6. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!}$

In Exercises 7–16, verify that the infinite series diverges.

7. $\sum_{n=0}^{\infty} 3\left(\frac{3}{2}\right)^n$

8. $\sum_{n=0}^{\infty} \left(\frac{4}{3}\right)^n$

9. $\sum_{n=0}^{\infty} 1000(1.055)^n$

10. $\sum_{n=0}^{\infty} 2(-1.03)^n$

11. $\sum_{n=1}^{\infty} \frac{n}{n+1}$

12. $\sum_{n=1}^{\infty} \frac{n}{2n+3}$

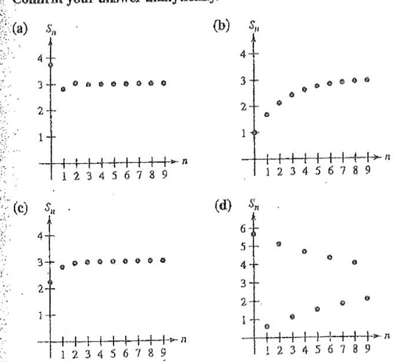
13. $\sum_{n=1}^{\infty} \frac{n^2}{n^2+1}$

14. $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$

15. $\sum_{n=1}^{\infty} \frac{2^n+1}{2^{n+1}}$

16. $\sum_{n=1}^{\infty} \frac{n!}{2^n}$

In Exercises 17–20, match the series with the graph of its sequence of partial sums. [The graphs are labeled (a), (b), (c), and (d).] Use the graph to estimate the sum of the series. Confirm your answer analytically.



17. $\sum_{n=0}^{\infty} \frac{9}{4}\left(\frac{1}{4}\right)^n$

18. $\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n$

19. $\sum_{n=0}^{\infty} \frac{15}{4}\left(-\frac{1}{4}\right)^n$

20. $\sum_{n=0}^{\infty} \frac{17}{3}\left(-\frac{8}{9}\right)^n$

In Exercises 21–26, verify that the infinite series converges.

21. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ (Use partial fractions.)

22. $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$ (Use partial fractions.)

23. $\sum_{n=0}^{\infty} 2\left(\frac{3}{4}\right)^n$

24. $\sum_{n=1}^{\infty} 2\left(-\frac{1}{2}\right)^n$

25. $\sum_{n=0}^{\infty} (0.9)^n = 1 + 0.9 + 0.81 + 0.729 + \cdots$

26. $\sum_{n=0}^{\infty} (-0.6)^n = 1 - 0.6 + 0.36 - 0.216 + \cdots$

Numerical, Graphical, and Analytic Analysis **In Exercises 27–32, (a) find the sum of the series, (b) use a graphing utility to find the indicated partial sum S_n and complete the table, (c) use a graphing utility to graph the first ten terms of the sequence of partial sums and a horizontal line representing the sum, and (d) explain the relationship between the magnitude of the terms of the series and the rate at which the sequence of partial sums approaches the sum of the series.**

n	5	10	20	50	100
S_n					

27. $\sum_{n=1}^{\infty} \frac{6}{n(n+3)}$

28. $\sum_{n=1}^{\infty} \frac{4}{n(n+4)}$

29. $\sum_{n=1}^{\infty} 2(0.9)^{n-1}$

30. $\sum_{n=1}^{\infty} 3(0.85)^{n-1}$

31. $\sum_{n=1}^{\infty} 10(0.25)^{n-1}$

32. $\sum_{n=1}^{\infty} 5\left(-\frac{1}{3}\right)^{n-1}$

In Exercises 33–46, find the sum of the convergent series.

33. $\sum_{n=2}^{\infty} \frac{1}{n^2-1}$

34. $\sum_{n=1}^{\infty} \frac{4}{n(n+2)}$

35. $\sum_{n=1}^{\infty} \frac{8}{(n+1)(n+2)}$

36. $\sum_{n=1}^{\infty} \frac{1}{(2n+1)(2n+3)}$

37. $\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$

38. $\sum_{n=0}^{\infty} 6\left(\frac{4}{5}\right)^n$

39. $\sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n$

40. $\sum_{n=0}^{\infty} 2\left(-\frac{2}{3}\right)^n$

41. $1 + 0.1 + 0.01 + 0.001 + \cdots$

42. $8 + 6 + \frac{9}{2} + \frac{27}{8} + \cdots$

43. $3 - 1 + \frac{1}{3} - \frac{1}{9} + \cdots$

44. $4 - 2 + 1 - \frac{1}{2} + \cdots$

45. $\sum_{n=0}^{\infty} \left(\frac{1}{2^n} - \frac{1}{3^n}\right)$

46. $\sum_{n=1}^{\infty} [(0.7)^n + (0.9)^n]$

In Exercises 1–10, use the Integral Test to determine the convergence or divergence of the series.

1. $\sum_{n=1}^{\infty} \frac{1}{n+1}$

2. $\sum_{n=1}^{\infty} \frac{2}{3n+5}$

3. $\sum_{n=1}^{\infty} e^{-n}$

4. $\sum_{n=1}^{\infty} ne^{-n/2}$

5. $\frac{1}{2} + \frac{1}{5} + \frac{1}{10} + \frac{1}{17} + \frac{1}{26} + \cdots$

6. $\frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \frac{1}{11} + \cdots$

7. $\frac{\ln 2}{2} + \frac{\ln 3}{3} + \frac{\ln 4}{4} + \frac{\ln 5}{5} + \frac{\ln 6}{6} + \cdots$

8. $\frac{1}{4} + \frac{2}{7} + \frac{3}{12} + \cdots + \frac{n}{n^2+3} + \cdots$

9. $\sum_{n=1}^{\infty} \frac{n^{k-1}}{n^k+c}$, k is a positive integer

10. $\sum_{n=1}^{\infty} n^k e^{-n}$, k is a positive integer

In Exercises 11 and 12, use the Integral Test to determine the convergence or divergence of the p-series.

11. $\sum_{n=1}^{\infty} \frac{1}{n^3}$

12. $\sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$

In Exercises 13–20, use Theorem 8.11 to determine the convergence or divergence of the p-series.

13. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$

14. $\sum_{n=1}^{\infty} \frac{3}{n^{5/3}}$

15. $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \cdots$

16. $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots$

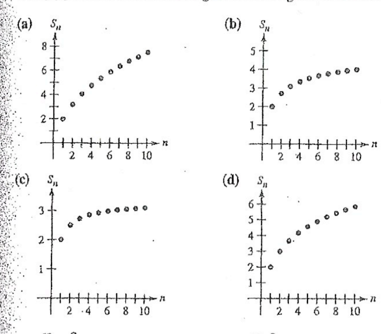
17. $1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \frac{1}{5\sqrt{5}} + \cdots$

18. $1 + \frac{1}{\sqrt[3]{4}} + \frac{1}{\sqrt[3]{9}} + \frac{1}{\sqrt[3]{16}} + \frac{1}{\sqrt[3]{25}} + \cdots$

19. $\sum_{n=1}^{\infty} \frac{1}{n^{1.04}}$

20. $\sum_{n=1}^{\infty} \frac{1}{n^{\pi}}$

In Exercises 21–24, match the series with the graph of its sequence of partial sums. [The graphs are labeled (a), (b), (c), and (d).] Determine the convergence or divergence of the series.



21. $\sum_{n=1}^{\infty} \frac{2}{\sqrt[4]{n^3}}$

22. $\sum_{n=1}^{\infty} \frac{2}{n}$

23. $\sum_{n=1}^{\infty} \frac{2}{n\sqrt{n}}$

24. $\sum_{n=1}^{\infty} \frac{2}{n^2}$

25. ***Writing*** In Exercises 21–24, $\lim_{n\to\infty} a_n = 0$ for each series but they do not all converge. Is this a contradiction of Theorem 8.9? Why do you think some converge and others diverge?

26. ***Numerical and Graphical Analysis*** (a) Use a graphing utility to find the indicated partial sum S_n and complete the table. (b) Use a graphing utility to graph the first ten terms of the sequence of partial sums. (c) Compare the rate at which the sequence of partial sums approaches the sum of the series for each series.

n	5	10	20	50	100
S_n					

28. The **Riemann zeta function** for real numbers is defined for all x for which the series

$$\zeta(x) = \sum_{n=1}^{\infty} n^{-x}$$

converges. Find the domain of the function.

In Exercises 29 and 30, find the positive values of p for which the series converges.

29. $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$

30. $\sum_{n=2}^{\infty} \frac{\ln n}{n^p}$

Getting at the Concept

31. State the Integral Test and give an example of its use.

32. Define a p-series and state the requirements for its convergence.

33. A friend in your calculus class tells you that the following series converges because the terms are very small and approach 0 rapidly. Is your friend correct? Explain.

$$\frac{1}{10{,}000} + \frac{1}{10{,}001} + \frac{1}{10{,}002} + \cdots$$

34. Find a series such that the nth term goes to 0, but the series diverges.

35. Let f be a positive, continuous, and decreasing function for $x \ge 1$, such that $a_n = f(n)$. Prove that if the series

$$\sum_{n=1}^{\infty} a_n$$

converges to S, then the remainder $R_N = S - S_N$ is bounded by

$$0 \le R_N \le \int_N^{\infty} f(x)\,dx.$$

36. Show that the result of Exercise 35 can be written as