EXERCISES FOR SECTION 8.6

In Exercises 1–4, verify the formula.

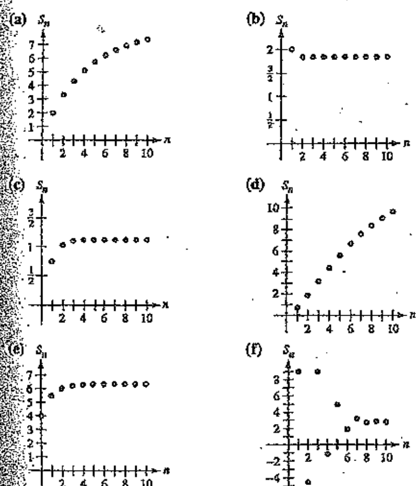
1. $\frac{(n+1)!}{(n-2)!} = (n+1)(n)(n-1)$

2. $\frac{(2k-2)!}{(2k)!} = \frac{1}{(2k)(2k-1)}$

3. $1 \cdot 3 \cdot 5 \cdots (2k-1) = \frac{(2k)!}{2^k k!}$

4. $\frac{1}{1 \cdot 3 \cdot 5 \cdots (2k-5)} = \frac{2^k k!(2k-3)(2k-1)}{(2k)!}, \quad k \ge 3$

In Exercises 5–10, match the series with the graph of its sequence of partial sums. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]



5. $\sum_{n=1}^{\infty} n\left(\frac{3}{4}\right)^n$

6. $\sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n \left(\frac{1}{n!}\right)$

7. $\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{n!}$

8. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}4}{(2n)!}$

9. $\sum_{n=1}^{\infty} \left(\frac{4n}{5n-3}\right)^n$

10. $\sum_{n=0}^{\infty} 4e^{-n}$

Numerical, Graphical, and Analytic Analysis In Exercises 11 and 12, (a) verify that the series converges. (b) Use a graphing utility to find the indicated partial sum S_n and complete the table. (c) Use a graphing utility to graph the first ten terms of the sequence of partial sums. (d) Use the table to estimate the sum of the series. (e) Explain the relationship between the magnitude of the terms of the series and the rate at which the sequence of partial sums approaches the sum of the series.

n	5	10	15	20	25
S_n					

11. $\sum_{n=1}^{\infty} n^2\left(\frac{5}{8}\right)^n$

12. $\sum_{n=1}^{\infty} \frac{n^2+1}{n!}$

In Exercises 13–32, use the Ratio Test to determine the convergence or divergence of the series.

13. $\sum_{n=0}^{\infty} \frac{n!}{3^n}$

14. $\sum_{n=0}^{\infty} \frac{3^n}{n!}$

15. $\sum_{n=1}^{\infty} n\left(\frac{3}{4}\right)^n$

16. $\sum_{n=1}^{\infty} n\left(\frac{3}{2}\right)^n$

17. $\sum_{n=1}^{\infty} \frac{n}{2^n}$

18. $\sum_{n=1}^{\infty} \frac{n^3}{2^n}$

19. $\sum_{n=1}^{\infty} \frac{2^n}{n^2}$

20. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+2)}{n(n+1)}$

21. $\sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{n!}$

22. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(3/2)^n}{n^2}$

23. $\sum_{n=1}^{\infty} \frac{n!}{n3^n}$

24. $\sum_{n=1}^{\infty} \frac{(2n)!}{n^5}$

25. $\sum_{n=0}^{\infty} \frac{4^n}{n!}$

26. $\sum_{n=1}^{\infty} \frac{n^n}{n!}$

27. $\sum_{n=0}^{\infty} \frac{3^n}{(n+1)^n}$

28. $\sum_{n=0}^{\infty} \frac{(n!)^2}{(3n)!}$

29. $\sum_{n=0}^{\infty} \frac{4^n}{3^n+1}$

30. $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{4n}}{(2n+1)!}$

31. $\sum_{n=0}^{\infty} \frac{(-1)^{n+1} n!}{1 \cdot 3 \cdot 5 \cdots (2n+1)}$

32. $\sum_{n=1}^{\infty} \frac{(-1)^n[2 \cdot 4 \cdot 6 \cdots (2n)]}{2 \cdot 5 \cdot 8 \cdots (3n-1)}$

In Exercises 33 and 34, verify that the Ratio Test is inconclusive for the p-series.

33. (a) $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ (b) $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$

34. (a) $\sum_{n=1}^{\infty} \frac{1}{n^4}$ (b) $\sum_{n=1}^{\infty} \frac{1}{n^p}$

In Exercises 35–42, use the Root Test to determine the convergence or divergence of the series.

35. $\sum_{n=1}^{\infty} \left(\frac{n}{2n+1}\right)^n$

36. $\sum_{n=1}^{\infty} \left(\frac{2n}{n+1}\right)^n$

37. $\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^n}$

38. $\sum_{n=1}^{\infty} \left(\frac{-3n}{2n+1}\right)^{3n}$

39. $\sum_{n=1}^{\infty} \left(2\sqrt[n]{n}+1\right)^n$

40. $\sum_{n=0}^{\infty} e^{-n}$

41. $\frac{1}{(\ln 3)^3} + \frac{1}{(\ln 4)^4} + \frac{1}{(\ln 5)^5} + \frac{1}{(\ln 6)^6} + \cdots$

42. $1 + \frac{2}{3} + \frac{3}{3^2} + \frac{4}{3^3} + \frac{5}{3^4} + \frac{6}{3^5} + \cdots$

In Exercises 43–60, determine the convergence or divergence of the series using any appropriate test from this chapter. Identify the test used.

43. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}5}{n}$

44. $\sum_{n=1}^{\infty} \frac{5}{n}$

45. $\sum_{n=1}^{\infty} \frac{3}{n\sqrt{n}}$

46. $\sum_{n=1}^{\infty} \left(\frac{\pi}{4}\right)^n$

47. $\sum_{n=1}^{\infty} \frac{2n}{n+1}$

48. $\sum_{n=1}^{\infty} \frac{n}{2n^2+1}$

49. $\sum_{n=1}^{\infty} \frac{(-1)^n 3^{n-2}}{2^n}$

50. $\sum_{n=1}^{\infty} \frac{10}{3\sqrt{n^3}}$

51. $\sum_{n=1}^{\infty} \frac{10n+3}{n2^n}$

52. $\sum_{n=1}^{\infty} \frac{2^n}{4n^2-1}$

53. $\sum_{n=1}^{\infty} \frac{\cos n}{2^n}$

54. $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

55. $\sum_{n=1}^{\infty} \frac{n7^n}{n!}$

56. $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

57. $\sum_{n=1}^{\infty} \frac{(-1)^n 3^{n-1}}{n!}$

58. $\sum_{n=1}^{\infty} \frac{(-1)^n 3^n}{n2^n}$

59. $\sum_{n=1}^{\infty} \frac{(-3)^n}{3 \cdot 5 \cdot 7 \cdots (2n+1)}$

60. $\sum_{n=1}^{\infty} \frac{3 \cdot 5 \cdot 7 \cdots (2n+1)}{18^n(2n-1)n!}$

In Exercises 61–64, identify the two series that are the same.

61. (a) $\sum_{n=1}^{\infty} \frac{n5^n}{n!}$
(b) $\sum_{n=0}^{\infty} \frac{n5^n}{n!}$
(c) $\sum_{n=0}^{\infty} \frac{(n+1)5^{n+1}}{(n+1)!}$

62. (a) $\sum_{n=4}^{\infty} n\left(\frac{3}{4}\right)^n$
(b) $\sum_{n=0}^{\infty} (n+1)\left(\frac{3}{4}\right)^n$
(c) $\sum_{n=1}^{\infty} n\left(\frac{3}{4}\right)^{n-1}$

63. (a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}$
(b) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)!}$
(c) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n+1)!}$

64. (a) $\sum_{n=2}^{\infty} \frac{(-1)^n}{(n-1)2^{n-1}}$
(b) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n2^n}$
(c) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{(n+1)2^n}$

In Exercises 65 and 66, write an equivalent series with the index of summation beginning at $n = 0$.

65. $\sum_{n=1}^{\infty} \frac{n}{4^n}$

66. $\sum_{n=2}^{\infty} \frac{2^n}{(n-2)!}$

In Exercises 67 and 68, (a) determine the number of terms required to approximate the sum of the series with an error less than 0.0001, and (b) use a graphing utility to approximate the sum of the series with an error less than 0.0001.

67. $\sum_{k=1}^{\infty} \frac{(-3)^k}{2^k k!}$

68. $\sum_{k=0}^{\infty} \frac{(-3)^k}{1 \cdot 3 \cdot 5 \cdots (2k+1)}$

Getting at the Concept

69. State the Ratio Test.

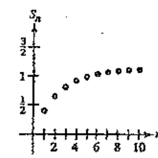
70. State the Root Test.

71. You are told that the terms of a positive series appear to approach zero rapidly as n approaches infinity. In fact, $a_7 \le 0.0001$. Given no other information, does this imply that the series converges? Support your conclusion with examples.

72. The graph shows the first ten terms of the sequence of partial sums of the convergent series

$$\sum_{n=1}^{\infty} \left(\frac{2n}{3n+2}\right)^n.$$

Find a series such that the terms of its sequence of partial sums are less than the corresponding terms of the sequence in the figure, but such that the series diverges.



73. Using the Ratio Test, it is determined that an alternating series converges. Does the series converge conditionally or absolutely?

74. Prove Property 2 of Theorem 8.17.

75. Prove Theorem 8.18. (*Hint for Property 1:* If the limit equals $r < 1$, choose a real number R such that $r < R < 1$. By the definition of the limit, there exists some $N > 0$ such that

$\sqrt[n]{|a_n|} < R$ for $n > N$.)

76. ***Writing*** Read the article "A Differentiation Test for Absolute Convergence" by Yaser S. Abu-Mostafa in *Mathematics Magazine*. (To view this article, go to the website *www.matharticles.com*.) Then write a paragraph that describes the test. Include examples of series that converge and examples of series that diverge.