

Mid Term Exam Review

C

$$\sqrt{x-9}$$

$$y' = \frac{1}{2} (x-9)^{-1/2}$$

$$= \frac{1}{2\sqrt{x-9}}$$

E 5.

B

$$y = \sec \sqrt{t}$$

$$y' = \sec \sqrt{t} \tan \sqrt{t} \left( \frac{1}{2} t^{-1/2} \right)$$

$$= \frac{\sec \sqrt{t} \tan \sqrt{t}}{2\sqrt{t}}$$

B

2. GRAPH

E

3.  $f(x) = \frac{1}{x-1}$

$$f'(x) = \frac{(x-1)(0) - (1)(1)}{(x-1)^2}$$

$$= \frac{-1}{(x-1)^2}$$

$$f'(2) = \frac{-1}{(2-1)^2}$$

$$= \frac{-1}{1}$$

$$y - 1 = -1(x - 2)$$

$$y - 1 = -x + 2$$

$$x + y - 3 = 0$$

D

7.  $5x^2 - 2xy + 7y^2 = 0$

$$10x - \left( 2x \frac{dy}{dx} + y(-2) \right) + 14y \frac{dy}{dx} = 0$$

$$10x - 2x \frac{dy}{dx} - 2y + 14y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} (14y - 2x) = 2y - 10x$$

$$\frac{dy}{dx} = \frac{2y - 10x}{14y - 2x}$$

$$\frac{dy}{dx} = \frac{y - 5x}{7y - x}$$

A

4.  $f(x) = \frac{x^2 - 4x}{x^{1/2}}$

$$f'(x) = \frac{x^{1/2}(2x-4) - (x^2-4x)(\frac{1}{2}x^{-1/2})}{(x^{1/2})^2}$$

$$= \frac{x^{-1/2} \left( x(2x-4) - \frac{1}{2}(x^2-4x) \right)}{x}$$

$$= \frac{2x^2 - 4x - \frac{1}{2}x^2 + 2x}{x^{3/2}}$$

$$= \frac{\frac{3}{2}x^2 - 2x}{x^{3/2}}$$

$$= \frac{1}{2}x \frac{(3x-4)}{x^{3/2}}$$

$$= \frac{x(3x-4)}{2x^{3/2}} = \frac{3x-4}{2x^{1/2}}$$

8.  $\frac{dr}{dt} = -0.05$      $V = 128\pi$   
 $r = 2.5$

$128\pi = \pi(2.5)^2(h)$   
 $128 = 2.5^2 h$   
 $20.48 = h$

$V = \pi r^2 h$

$\pi r^2$

$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt} + h \cdot 2\pi r \frac{dr}{dt}$



$\frac{dV}{dt} = 6.25\pi \frac{dh}{dt} + 20.48(2\pi)(2.5)(-0.05)$

$0 = 6.25\pi \frac{dh}{dt} - 16.085$

$\frac{16.085}{6.25\pi} = \frac{dh}{dt}$

$0.8192 = \frac{dh}{dt}$

10. C

11. B

12. C

9) cont. ✓  
diff. ✓

$\frac{f(-1/2) - f(-3)}{-1/2 - (-3)}$   
 $\frac{2 + \sqrt{3}}{5/2}$



$\frac{7/3}{5/2} = \frac{14}{15}$   
 $\frac{14}{15} = \frac{1}{x^2}$

$\frac{14}{15} = \frac{1}{x^2}$

$14x^2 = 15$

$x^2 = \frac{15}{14}$

$x = \pm \sqrt{\frac{15}{14}}$

D 13.  $f(t) = \frac{t^3 + t}{4t + 1}$

$f'(t) = \frac{(4t+1)(3t^2+1) - (t^3+t)(4)}{(4t+1)^2}$   
 $= \frac{(4(-1)+1)(3(1)+1) - (-1-1)(4)}{(-4+1)^2}$   
 $= \frac{(-3)(4)+8}{9}$   
 $= -\frac{4}{9}$

C 14.  $2 \tan(4x) \sec^2(4x) (4)$   
 $8 \tan 4x \sec^2 4x$

$x = -\sqrt{\frac{15}{14}}$

5.  $V = \frac{1}{3} \pi r^2 h$

$$\frac{dV}{dt} = \frac{1}{3} \pi r^2 \frac{dh}{dt} + h \cdot \frac{2}{3} \pi r \frac{dr}{dt}$$

$r = h$

$$15 = \frac{1}{3} \pi (10)^2 \frac{dh}{dt} + 10 \cdot \frac{2}{3} \pi (10) \frac{dh}{dt}$$

$$45 = 100 \pi \frac{dh}{dt} + 200 \pi \frac{dh}{dt}$$

$$45 = 300 \pi \frac{dh}{dt}$$

$$\frac{45}{300 \pi} = \frac{dh}{dt}$$

$.047$	$=$	$\frac{dh}{dt}$
$\text{ft/min}$		

16. a.  $(1, 3)$  v  $(6, 7)$   $f'$  is mc.

b. possible min's at endpoints  
 $x=1$

A 1.

$$x^2 + xy = 10$$

$$2x + x \frac{dy}{dx} + y = 0$$

$$4 + 2 \frac{dy}{dx} + 3 = 0$$

$$7 + 2 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{7}{2}$$

$$2^2 + 2y = 10$$

$$4 + 2y = 10$$

$$2y = 6$$

$$y = 3$$

MIT  
6/16

12. JEE GRAPH

13.  $y = x^3 + 3x^2 + 7x - 1$

$y' = 3x^2 + 6x + 7$

$m = 3(-1)^2 + 6(-1) + 7$

$= 3 - 6 + 7$

$= 4$

$y + 6 = -\frac{1}{4}(x + 1)$

$y + 6 = -\frac{1}{4}x - \frac{1}{4}$

$4y + 24 = -x - 1$

$x + 4y = -25$

$y = (-1)^3 + 3(-1)^2 + 7(-1) - 1$

$y = -1 + 3 - 7 - 1$

$y = -6$

$\square$  D

$x + \frac{k}{x}$

$1 + \frac{-k}{x^2} = 0$

$1 - \frac{k}{(-2)^2} = 0$

$1 - \frac{k}{4} = 0$

$1 = \frac{k}{4}$

$4 = k$

$\square$  B

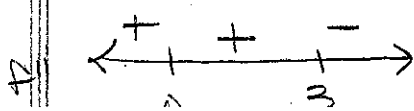
$y = 5x^4 - x^5$

$y' = 20x^3 - 5x^4$

$y'' = 60x^2 - 20x^3$

$0 = 20x^2(3 - x)$

$0 = x \quad x = 3$



at  $x = 3$

$\square$  E

18

19

$\square$  B

$m = \frac{2}{4} = \frac{1}{2}$

$y = \sqrt{x}$

$y' = \frac{1}{2\sqrt{x}}$

$\frac{1}{2} = \frac{1}{2\sqrt{x}}$

$2\sqrt{x} = 2$

$\sqrt{x} = 1$

$x = \pm 1$

$x = 1$

(1,1)

20.  $f(x) = x + 8\sin x$

$f'(x) = 1 + \cos x$

21

$\square$  C

$\frac{-5(3)^2 - (-5(0)^2)}{3 - 0}$

$\frac{-45}{3}$

$-15$

22

$\square$  B

$y' = \frac{1}{x^2} \cdot 2x$

$m = \frac{1}{e^4} \cdot 2e^2 = \frac{2}{e^2}$

21.  $f(x) = \frac{1}{3}x^3 - 4x^2 + 12x - 5$

(23)

$f'(x) = x^2 - 8x + 12$

$0 = x^2 - 8x + 12$

$(x-6)(x-2)$

$x=6, x=2$

E

$f(0) = -5$

$f(2) = \frac{8}{3} - 16 + 24 - 5 = \frac{17}{3}$

$f(6) = -74$

$f(9) = 22 \quad 243 - 324 + 108 - 5$

22.  $S = 4\pi r^2$

(24)  $100\pi = 4\pi r^2$

$25 = r^2$

$5 = r$

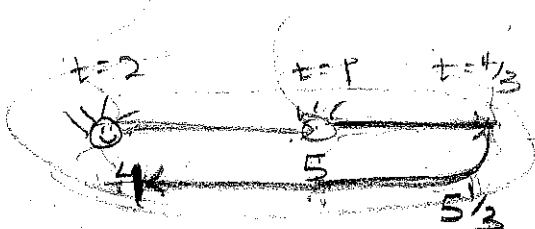
E

$v = \frac{4}{3}\pi r^3$

$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$

$= 4\pi (5)^2 (1/3)$

$= 30\pi$



$t=1 \quad \text{pos. } 5 \quad (+1/3)$

$t=4/3 \quad \text{pos. } 5 1/3$

$t=2 \quad \text{pos. } 4 \quad (20)$

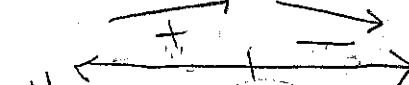
25.  $d = 8t - 3t^2$

(25)  $d' = 8 - 6t$

$0 = 8 - 6t$

$6t = 8$

$t = 4/3$



$t=1 \quad 8(1) - 3(1)^2 = 5 \quad (+1/3)$

$t=4/3 \quad 8(4/3) - 3(4/3)^2 = 10/3$

$t=2 \quad 8(2) - 3(2)^2 = 4 \quad (-4/3)$

total distance =  $1/3 + 4/3 = 5/3$

C

24.  $v = \frac{1}{3}\pi r^2 h$

(26)  $\frac{dv}{dt} = \frac{1}{3}\pi r^2 \frac{dh}{dt} + h \frac{2\pi r}{3} \frac{dr}{dt}$

C  $= \frac{1}{3}\pi (6)^2 (1/2) + 9(2/3)\pi (6) (1/2)$

$\frac{dv}{dt} = 6\pi + 18\pi$

$= 24\pi$

27. 25.  $y = x^2 + 3x^2 + 2$

$y' = 2x^2 + 6x$

B  $y'' = 6x + 6$

$0 = 6x + 6$

$-1 = x$

$y' = 2(-1)^2 + 6(-1)$

$= -3$

$y(-1) = -1 + 3 + 2$

$= 4$

$(-1, 4)$

$y - 4 = -3(x + 1)$

$y - 4 = -3x - 3$

$y = -3x + 1$

26.  $A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

A  $96\pi = 2\pi(8) \frac{dr}{dt}$

$96\pi = 16\pi \frac{dr}{dt}$

$6 = \frac{dr}{dt}$

$4\pi = \pi r^2$

$4 = r^2$

$2 = r$

C  
29

28. E continuous!

30

29.  $x^2 + y^2 = z^2$

$$\frac{dz}{dt} = 3 \frac{dy}{dt}$$

31

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$$

$$2(4) \left( 3 \frac{dy}{dt} \right) + 2(3) \left( \frac{dy}{dt} \right) = 2(5) (0)$$

B

$$24 \frac{dy}{dt} + 6 \frac{dy}{dt} = 10$$

$$30 \frac{dy}{dt} = 10$$

$$\frac{dy}{dt} = \frac{1}{3}$$

$$\frac{dz}{dt} = 1$$

32

30.  $V = \pi r^2 h$

$$10\pi = \pi r^2 h$$

$$A = 2\pi r^2 + 2\pi r h$$

$$16 = r^2 h$$

$$(16 = r^2)$$

$$A = 2\pi \left( \frac{16}{h} \right) + 2\pi \left( \frac{4}{\sqrt{h}} \right) h h$$

$$A = \frac{32\pi}{h} + 8\pi\sqrt{h}$$

$$A' = -\frac{32\pi}{h^2} + 8\pi \left( \frac{1}{2} h^{-1/2} \right)$$

$$A' = -\frac{32\pi}{h^2} + \frac{4\pi}{\sqrt{h}}$$

$$0 = -\frac{32\pi}{h^2} + \frac{4\pi}{\sqrt{h}}$$

$$\frac{32\pi}{h^2} = \frac{4\pi}{\sqrt{h}}$$

$$32\pi\sqrt{h} = 4\pi h^2$$

$$0 = 4\pi h^2 - 32\pi\sqrt{h}$$

$$0 = 4\pi\sqrt{h} (h^{3/2} - 8)$$

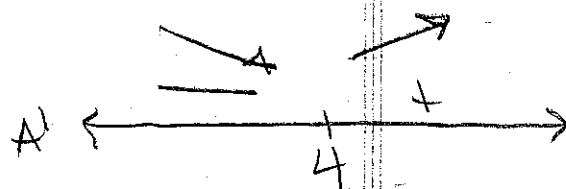
$$0 = h \quad h^{3/2} - 8 = 0$$

$$h^{3/2} = 8$$

$$h^3 = 64$$

$$h = \sqrt[3]{64}$$

$$h = 4$$

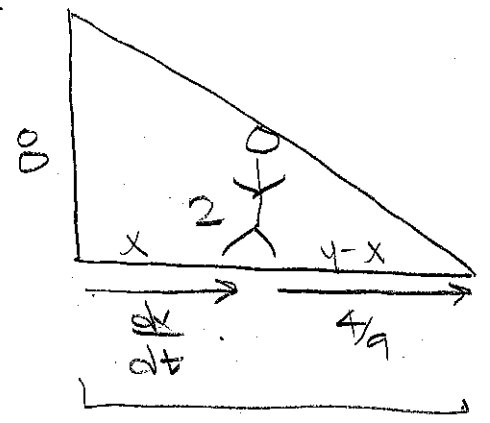


h=4

23

~~21~~

D



$$\frac{dy}{dt} = \frac{dx}{dt} + \frac{4}{9}$$

$$\frac{8}{y} = \frac{2}{4-x}$$

$$8y - 8x = 2y$$

$$6y = 8x$$

$$y = \frac{4}{3}x$$

$$\frac{dy}{dt} = \frac{4}{3} \frac{dx}{dt}$$

$$\frac{dx}{dt} + \frac{4}{9} = \frac{4}{3} \frac{dx}{dt}$$

$$-\frac{1}{3} \frac{dx}{dt} = -\frac{4}{9}$$

$$\frac{1}{3} \frac{dx}{dt} = \frac{4}{9}$$

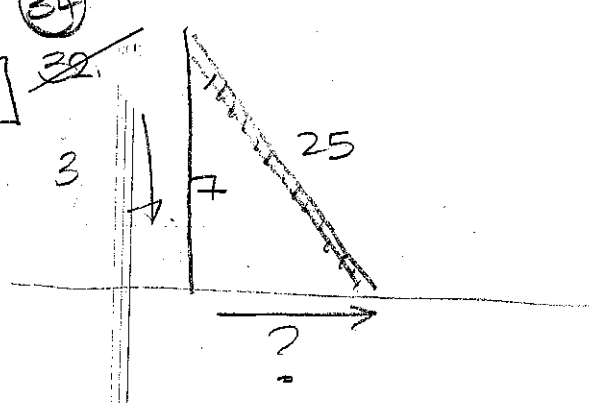
$$\frac{dx}{dt} = \frac{12}{9}$$

$$\frac{dx}{dt} = \frac{4}{3}$$

34

~~32~~

D



$$x^2 + y^2 = z^2$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$$

$$2(24) \frac{dx}{dt} + 2(7)(-3) = 0$$

$$48 \frac{dx}{dt} - 42 = 0$$

$$\frac{dx}{dt} = \frac{42}{48} = \frac{7}{8}$$