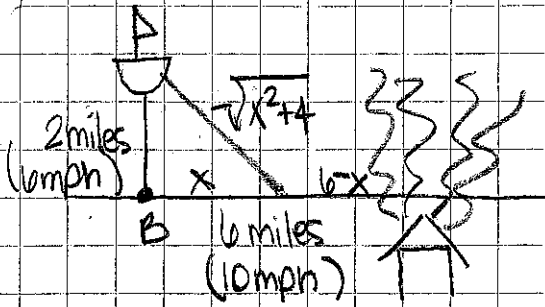


in cumulative practice



rate \* time = distance  
 time =  $\frac{\text{distance}}{\text{rate}}$

Time to house = Time rowing ( $T_1$ ) + Time running ( $T_2$ )

primary  $T_H = T_1 + T_2$   
 secondary  $T_1 = \frac{D_1}{R_1}$       $T_2 = \frac{D_2}{R_2}$

$T_H = \frac{D_1}{R_1} + \frac{D_2}{R_2}$

$T_H = \frac{\sqrt{x^2+4}}{6} + \frac{6-x}{10}$

$T_H = \frac{1}{6}\sqrt{x^2+4} + \frac{1}{10}(6-x)$

$x \geq 0$       $x \leq 6$

$T_H' = \frac{1}{6} \left( \frac{1}{2}(x^2+4) \right)^{-1/2} (2x) + \frac{1}{10}(-1)$   
 $T_H' = \frac{x}{6\sqrt{x^2+4}} - \frac{1}{10}$

$0 = \frac{x}{6\sqrt{x^2+4}} - \frac{1}{10}$

$\frac{1}{10} = \frac{x}{6\sqrt{x^2+4}}$

$(6\sqrt{x^2+4})^2 = (10x)^2$

$36(x^2+4) = 100x^2$   
 $36x^2 + 144 = 100x^2$   
 $-64x^2 + 144 = 0$

$-64x^2 = -144$   
 $x^2 = \frac{144}{64}$

$x = \frac{12}{8}$       $x = \frac{3}{2}$

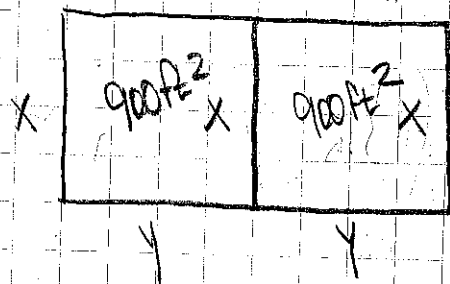
$T_H(0) = \frac{1}{3} + \frac{3}{5} = \frac{14}{15}$

$T_H(1.5) = .502 + .375 = .877$

$T_H(6) = 1.054 + 0 = 1.054$

Time is at a minimum when  $x=1.5$ . Henry should row to a point 1.5 miles from point B and run the rest of the way.

②



\* b)  $Cost = (2x + 4y)3 + 2x$

$A = xy$

$900 = xy$

$\frac{900}{x} = y$

a)  $P = 3x + 4y$  (amount of fence needed)

$A = xy \Rightarrow 900 = xy$   
 $\frac{900}{y} = x$

$P = 3\left(\frac{900}{y}\right) + 4y$

$y > 0$

$P = \frac{2700}{y} + 4y$

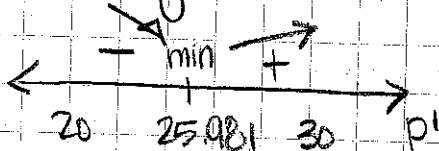
$P' = -\frac{2700}{y^2} + 4$

$0 = -\frac{2700}{y^2} + 4$

$-\frac{4y^2}{-4} = \frac{-2700}{-4}$

$y^2 = 675$

$y = 25.981$



$900 = xy$

$900 = x(25.981)$

$x = 34.641$

$Cost = 3\left(2x + 4\left(\frac{900}{x}\right)\right) + 2x$

$Cost = 6x + \frac{10800}{x} + 2x$

$Cost = 8x + \frac{10800}{x}$

$x > 0$

$C' = 8 - \frac{10800}{x^2}$

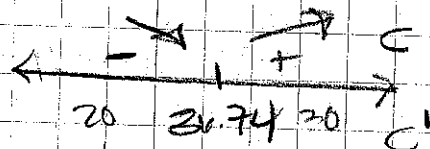
$0 = 8 - \frac{10800}{x^2}$

$-8 = -\frac{10800}{x^2}$

$-8x^2 = -10800$

$x^2 = 1350$

$x = 36.74$



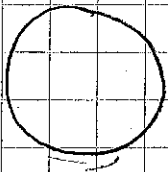
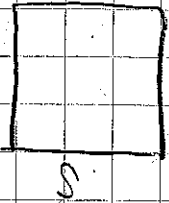
$900 = xy$

$900 = (36.74)(y)$

$y = 24.495$

The lowest cost fence is produced when  $x = 36.74$  &  $y = 24.495$ .

The least amount of fence will be used when the pens are 25.981 ft x 34.641 ft.



$$A = s^2 + \pi r^2$$

$$W = 4s + 2\pi r$$

$$16 = 4s + 2\pi r$$

$$\frac{16 - 2\pi r}{4} = s$$

$$\frac{8 - \pi r}{2} = s$$

$$A = \left(\frac{8 - \pi r}{2}\right)^2 + \pi r^2$$

$$r > 0 \quad r < 16$$

$$A = \frac{64 - 16\pi r + \pi^2 r^2}{4} + \pi r^2$$

$$A' = \frac{1}{4}(-16\pi + 2\pi^2 r) + 2\pi r$$

$$A' = -4\pi + \frac{1}{2}\pi^2 r + 2\pi r$$

$$A' = -12.566 + 4.9348r + 6.283r$$

$$A' = -12.566 + 11.218r$$

$$0 = -12.566 + 11.218r$$

$$12.566 = 11.218r$$

$$1.120 = r$$

Circumference =  $2\pi r$   
 $= 2\pi(1.120)$   
 $= 7.038$

The cut should be made 7.038 in from the end of the wire.

or 8.902 (from other end)

④



$$V = w^2 h$$

$$SA = w^2 + 4wh$$

$$1000 = w^2 + 4wh$$

$$\frac{1000 - w^2}{4w} = h$$

$$V = w^2 \left(\frac{1000 - w^2}{4w}\right)$$

$$w > 0, \quad w < \sqrt{1000}$$

$$V = \frac{w(1000 - w^2)}{4}$$

$$V = \frac{1000w - w^3}{4}$$

$$V' = \frac{1}{4}(1000 - 3w^2)$$

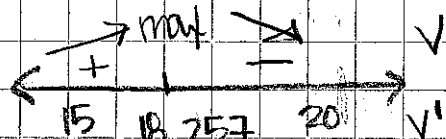
$$0 = \frac{1}{4}(1000 - 3w^2)$$

$$0 = 1000 - 3w^2$$

$$\frac{-1000}{-3} = \frac{-3w^2}{-3}$$

$$333.333 = w^2$$

$$18.257 = w$$



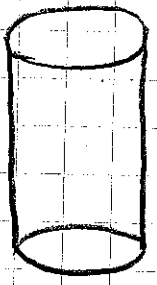
$$w = 18.257$$

$$h = \frac{1000 - w^2}{4w}$$

$$h = 9.129$$

The dimensions for maximum volume are 18.257 x 18.257 x 9.129

5



$$SA = 2\pi r h + 2\pi r^2$$

$$V = \pi r^2 h$$

$$16\pi = \pi r^2 h$$

$$\frac{16\pi}{\pi r^2} = h$$

$$\frac{16}{r^2} = h$$

$$SA = 2\pi r \cdot \frac{16}{r^2} + 2\pi r^2$$

$$r > 0$$

$$SA = \frac{32\pi}{r} + 2\pi r^2$$

$$SA' = -\frac{32\pi}{r^2} + 4\pi r$$

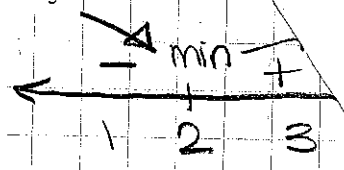
$$0 = -\frac{32\pi}{r^2} + 4\pi r$$

$$\frac{32\pi}{r^2} = 4\pi r$$

$$32\pi = 4\pi r^3$$

$$8 = r^3$$

$$2 = r$$

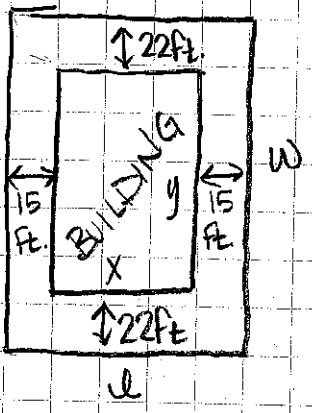


$$r = 2$$

$$h = \frac{16}{4} = 4$$

The dimensions that minimize the amount of material needed are  $r=2$  in and  $h=4$  in.

6



$$A_{lot} = w \cdot l$$

$$w = x + 30 \quad l = y + 44$$

$$xy = 13200$$

$$y = \frac{13200}{x}$$

$$A_{lot} = (x+30)(y+44)$$

$$A_{lot} = (x+30)\left(\frac{13200}{x} + 44\right)$$

$$A_{lot} = 13200 + 44x + \frac{396000}{x} + 1320$$

$$x > 0 \quad x \leq \sqrt{13200}$$

$$A_{lot} = 14520 + 44x + \frac{396000}{x}$$

$$A' = 44 - \frac{396000}{x^2}$$

continued

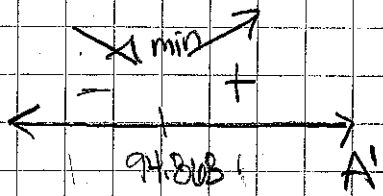
$$0 = 44 - \frac{391000}{x^2}$$

$$\frac{391000}{x^2} = 44$$

$$\frac{391000}{44} = \frac{44x^2}{44}$$

$$9000 = x^2$$

$$94.868 = x$$



$$x = 94.868 + 30 \Rightarrow w = 124.868$$

$$xy = 13200$$

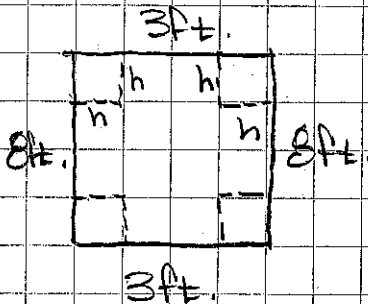
$$y = \frac{13200}{x}$$

$$y = \frac{13200}{124.868}$$

$$y = 139.141 + 44 = 183.141$$

The smallest lot on which the building can be located is  $124.868 \times 183.141$

7.



$$V = lwh = h$$

$$l = 8 - 2h \quad w = 3 - 2h$$

$$V = (8 - 2h)(3 - 2h)h$$

$$V = (24 - 16h - 6h + 4h^2)h$$

$$V = 24h - 22h^2 + 4h^3$$

$$h > 0 \quad h < 3$$

$$V' = 24 - 44h + 12h^2$$

$$0 = 12h^2 - 44h + 24$$

$$h = 3, \frac{2}{3}$$

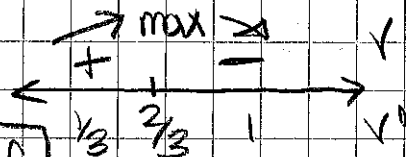
QUAD FORM PROGRAM - CALCULATOR

$$h = \frac{2}{3}$$

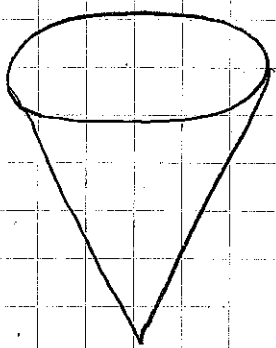
$$l = 8 - 2\left(\frac{2}{3}\right) = 6.667$$

$$w = 3 - 2\left(\frac{2}{3}\right) = 1.667$$

The dimensions that yield the maximum volume are  $6.667 \text{ ft} \times 1.667 \text{ ft} \times \frac{2}{3} \text{ ft}$



8



$$V = 10 \text{ cm}^3$$

$$SA = \pi r \sqrt{r^2 + h^2} + \pi r^2$$

$$V = \frac{1}{3} \pi r^2 h$$

$$10 = \frac{1}{3} \pi r^2 h$$

$$30 = \pi r^2 h$$

$$\frac{30}{\pi r^2} = h$$

$$SA = \pi r \sqrt{r^2 + \left(\frac{30}{\pi r^2}\right)^2} + \pi r^2$$

$$SA = \pi r \sqrt{r^2 + \frac{900}{\pi^2 r^4}} + \pi r^2$$

$$r > 0$$

$$SA = \pi r \left( r^2 + \frac{900}{\pi^2 r^4} \right)^{1/2} + \pi r^2$$

$$SA' = \pi r^2 \cdot \frac{1}{2} \left( r^2 + \frac{900}{\pi^2 r^4} \right)^{-1/2} \cdot \left( 2r - \frac{2700}{\pi^2 r^5} \right) + \left( r^2 + \frac{900}{\pi^2 r^4} \right)^{1/2} \cdot \pi + 2\pi r$$

$$SA' = \frac{\pi r^2}{2} \left( \frac{2r - \frac{2700}{\pi^2 r^5}}{\sqrt{r^2 + \frac{900}{\pi^2 r^4}}} \right) + \pi \sqrt{r^2 + \frac{900}{\pi^2 r^4}} + 2\pi r$$

$$= \frac{\pi r^3 - \frac{1350}{\pi r^3}}{\sqrt{r^2 + \frac{900}{\pi^2 r^4}}} + \frac{\pi \sqrt{r^2 + \frac{900}{\pi^2 r^4}}}{\pi r^2} + 2\pi r$$

=