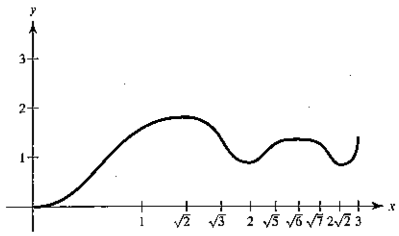
(b)

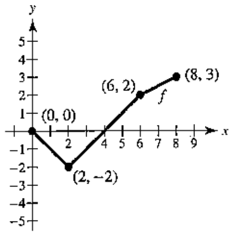



(c) Relative maxima at $x = \sqrt{2}, \sqrt{6}$

Relative minima at $x = 2, 2\sqrt{2}$

(d) Points of inflection at $x = 1, \sqrt{3}, \sqrt{5}, \sqrt{7}$

**5.** (a)




(b)

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| $F(x)$ | 0 | $-\frac{1}{2}$ | $-2$ | $-\frac{7}{2}$ | $-4$ | $-\frac{7}{2}$ | $-2$ | $\frac{1}{4}$ | 3 |

(c) $x = 4, 8$ (d) $x = 2$

**7.** (a) 1.6758; Error of approximation $\approx 0.0071$

(b) $\frac{3}{2}$ (c) Proof

**9.** Proof

**11.** $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^5 \left(\frac{1}{n}\right) = \frac{1}{6}$ **13.** $1 \le \int_0^1 \sqrt{1 + x^4}\, dx \le \sqrt{2}$

**15.** Proof **17.** 100,000 pounds

# Chapter 5

## *Section 5.1 (page 321)*

**1.**

| $x$ | 0.5 | 1.5 | 2 | 2.5 | 3 |
|---|---|---|---|---|---|
| $\int_1^x (1/t)\, dt$ | $-0.6932$ | 0.4055 | 0.6932 | 0.9163 | 1.0987 |

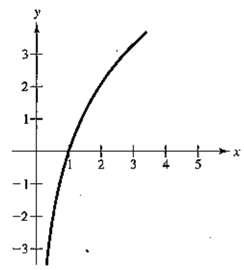
| $x$ | 3.5 | 4 |
|---|---|---|
| $\int_1^x (1/t)\, dt$ | 1.2529 | 1.3865 |

**3.** (a) 3.8067 (b) $\ln 45 = \int_1^{45} \frac{1}{t}\, dt \approx 3.8067$

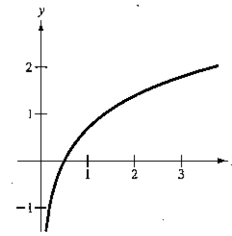
**5.** (a) $-0.2231$ (b) $\ln 0.8 = \int_1^{0.8} \frac{1}{t}\, dt \approx -0.2231$

**7.** b **8.** d **9.** a **10.** c

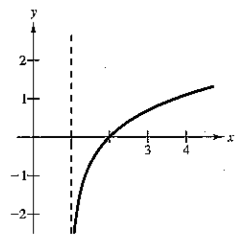
**11.** Domain: $x > 0$



**13.** Domain: $x > 0$



**15.** Domain: $x > 1$



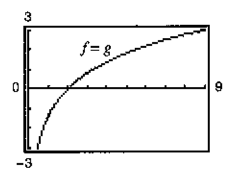
**17.** (a) 1.7917 (b) $-0.4055$ (c) 4.3944 (d) 0.5493

**19.** $\ln 2 - \ln 3$ **21.** $\ln x + \ln y - \ln z$ **23.** $\frac{1}{3}\ln(a^2 + 1)$

**25.** $3[\ln(x + 1) + \ln(x - 1) - 3\ln x]$ **27.** $\ln z + 2\ln(z - 1)$

**29.** $\ln \frac{x - 2}{x + 2}$ **31.** $\ln \sqrt[3]{\frac{x(x + 3)^2}{x^2 - 1}}$ **33.** $\ln \frac{9}{\sqrt{x^2 + 1}}$

**35.**




**37.** $-\infty$

**39.** $\ln 4$ **41.** 3 **43.** 2 **45.** $\frac{2}{x}$ **47.** $\frac{4(\ln x)^3}{x}$

**49.** $\frac{2x^2 - 1}{x(x^2 - 1)}$ **51.** $\frac{1 - x^2}{x(x^2 + 1)}$ **53.** $\frac{1 - 2\ln t}{t^3}$

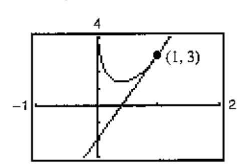
**55.** $\frac{2}{x \ln x^2} = \frac{1}{x \ln x}$ **57.** $\frac{1}{1 - x^2}$ **59.** $\frac{-4}{x(x^2 + 4)}$

**61.** $\frac{\sqrt{x^2 + 1}}{x^2}$ **63.** $\cot x$ **65.** $-\tan x + \frac{\sin x}{\cos x - 1}$

**67.** $\frac{3\cos x}{(\sin x - 1)(\sin x + 2)}$ **69.** $\frac{2}{x}(\sin 2x + x\cos 2x \ln x^2)$

**71.** (a) $5x - y - 2 = 0$

(b)




**73.** $\frac{2xy}{3 - 2y^2}$ **75.** $xy'' + y' = x\left(\frac{-2}{x^2}\right) + \frac{2}{x} = 0$

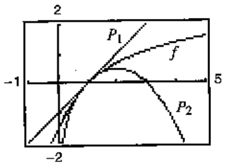
**77.** Relative minimum: $\left(1, \frac{1}{2}\right)$

**79.** Relative minimum: $(e^{-1}, -e^{-1})$

**81.** Relative minimum: $(e, e)$

Point of inflection: $\left(e^2, \frac{e^2}{2}\right)$

**83.** $P_1 = x - 1$; $P_2 = x - 1 - \frac{1}{2}(x - 1)^2$




The values of $f$, $P_1$, and $P_2$ and their first derivatives agree at $x = 1$.

**85.** $x \approx 0.567$ **87.** $\dfrac{2x^2 - 1}{\sqrt{x^2 - 1}}$

**89.** $\dfrac{3x^3 - 15x^2 + 8x}{2(x - 1)^3\sqrt{3x - 2}}$ **91.** $\dfrac{(2x^2 + 2x - 1)\sqrt{x - 1}}{(x + 1)^{3/2}}$

**93.** The domain of the natural logarithmic function is $(0, \infty)$ and the range is $(-\infty, \infty)$. The function is continuous, increasing, and one-to-one and its graph is concave downward. In addition, if $a$ and $b$ are positive numbers and $n$ is rational, then $\ln(1) = 0$, $\ln(a \cdot b) = \ln a + \ln b$, $\ln(a^n) = n \ln a$, and $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$.

**95.** Using properties of logarithms, $\ln e^x$ can be rewritten as $x \ln e$. Then, since $\ln e = 1$ by the definition of $e$, $x \ln e = x(1) = x$.

**97.** (a) Rolle's Theorem does not apply because $f(1) \neq f(3)$.

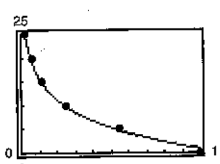
(b) Yes. $f'(2) = 0$ and $2 \in [1, 3]$.

**99.** $\beta = 160 + 10 \log_{10} I$; $\beta = 60$ decibels

**101.** (a) $h = 0$ is not in the domain of the function.

(b) $h = 0.86 - 6.447 \ln p$

(c)




(d) 2.7 kilometers

(e) 0.15 atmosphere

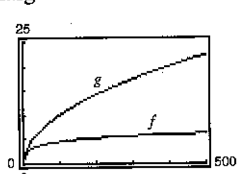
(f) $h = 5$: $\dfrac{dp}{dh} = -0.085$

$h = 20$: $\dfrac{dp}{dh} = -0.009$

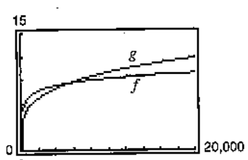
As the altitude increases, the pressure decreases at a slower rate.

**103.** For large values of $x$, $g$ increases at a faster rate than $f$ in both cases. The natural logarithmic function increases very slowly for large values of $x$.

(a)




(b)




**105.** False: $\ln x + \ln 25 = \ln 25x$.

## Section 5.2 (page 330)

**1.** $5 \ln|x| + C$ **3.** $\ln|x + 1| + C$

**5.** $-\frac{1}{2}\ln|3 - 2x| + C$ **7.** $\ln\sqrt{x^2 + 1} + C$

**9.** $\dfrac{x^2}{2} - \ln(x^4) + C$ **11.** $\dfrac{1}{3}\ln|x^3 + 3x^2 + 9x| + C$

**13.** $\dfrac{x^2}{2} - 4x + 6\ln|x + 1| + C$ **15.** $\dfrac{x^3}{3} + 5\ln|x - 3| + C$

**17.** $\dfrac{x^3}{3} - 2x + \ln\sqrt{x^2 + 2} + C$ **19.** $\dfrac{1}{3}(\ln x)^3 + C$

**21.** $2\sqrt{x + 1} + C$ **23.** $2\ln|x - 1| - \dfrac{2}{x - 1} + C$

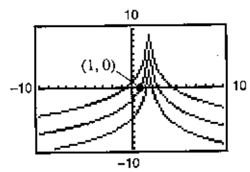
**25.** $\sqrt{2x} - \ln\left|1 + \sqrt{2x}\right| + C$

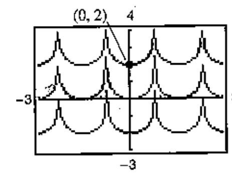
**27.** $x + 6\sqrt{x} + 18\ln\left|\sqrt{x} - 3\right| + C$ **29.** $\ln|\sin\theta| + C$

**31.** $-\frac{1}{2}\ln|\csc 2x + \cot 2x| + C$ **33.** $\ln|1 + \sin t| + C$

**35.** $\ln|\sec x - 1| + C$

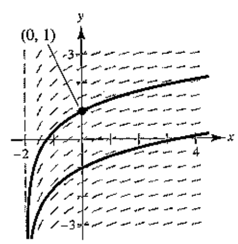
**37.** $y = -3\ln|2 - x| + C$ **39.** $y = -\frac{1}{2}\ln|\cos 2\theta| + C$



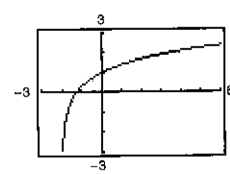




The graph has a hole at $x = 2$.

**41.** (a)




(b) $y = \ln\left|\dfrac{x + 2}{2}\right| + 1$




**43.** $\frac{5}{3}\ln 13 \approx 4.275$ **45.** $\frac{7}{3}$ **47.** $-\ln 3 \approx -1.099$

**49.** $\ln\left|\dfrac{2 - \sin 2}{1 - \sin 1}\right| \approx 1.929$

**51.** $-\ln|\cos x| + C = \ln\left|\dfrac{1}{\cos x}\right| + C = \ln|\sec x| + C$

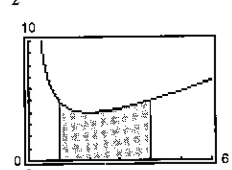
**53.** $\ln|\sec x + \tan x| + C = \ln\left|\dfrac{\sec^2 x - \tan^2 x}{\sec x - \tan x}\right| + C$

$= -\ln|\sec x - \tan x| + C$

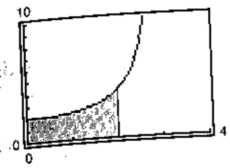
**55.** $2\left[\sqrt{x} - \ln\left(1 + \sqrt{x}\right)\right] + C$ **57.** $-\sin(1 - x) + C$

**59.** $\ln\left(\sqrt{2} + 1\right) - \dfrac{\sqrt{2}}{2} \approx 0.174$ **61.** $\dfrac{1}{x}$ **63.** 0 **65.** d

**67.** $\frac{15}{2} + 8\ln 2 \approx 13.045$



69. $\frac{12}{\pi}\left[2\ln\left(\sqrt{3}+1\right) - \ln 2\right] \approx 5.03$




71. Power Rule  **73.** Log rule

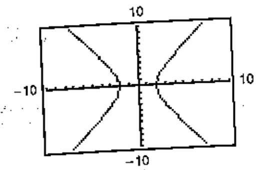
75. Use long division to rewrite the integrand.

77. 1  **79.** $\frac{1}{2(e-1)} \approx 0.291$

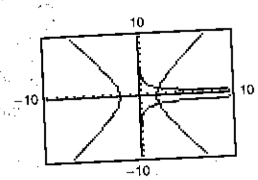
81. $P(t) = 1000(12\ln|1 + 0.25t| + 1)$; $P(3) \approx 7715$

83. \$168.27

85. (a)




(b) Answers will vary. Example: $y^2 = e^{-\ln x + \ln 4} = \frac{4}{x}$



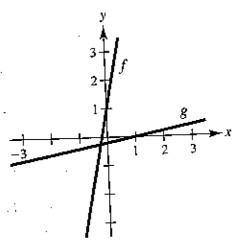

87. False. $\frac{1}{2}(\ln x) = \ln x^{1/2}$  **89.** True

## Section 5.3 (page 338)

1. (a) $f(g(x)) = 5\left(\frac{x-1}{5}\right) + 1 = x$

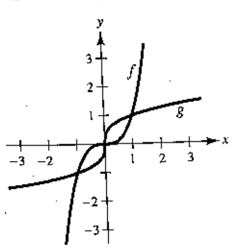
$g(f(x)) = \frac{(5x+1)-1}{5} = x$

(b)




3. (a) $f(g(x)) = \left(\sqrt[3]{x}\right)^3 = x$; $g(f(x)) = \sqrt[3]{x^3} = x$

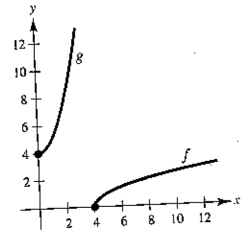
(b)




**5.** (a) $f(g(x)) = \sqrt{x^2 + 4 - 4} = x$;

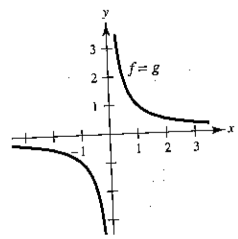
$g(f(x)) = \left(\sqrt{x-4}\right)^2 + 4 = x$

(b)




**7.** (a) $f(g(x)) = \frac{1}{1/x} = x$; $g(f(x)) = \frac{1}{1/x} = x$

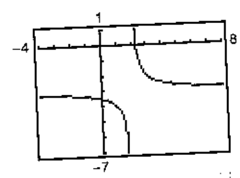
(b)



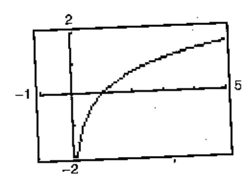

**9.** c  **10.** b  **11.** a  **12.** d

**13.** Inverse exists.  **15.** Inverse does not exist.

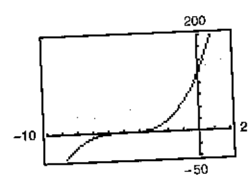
**17.** One-to-one




**19.** One-to-one



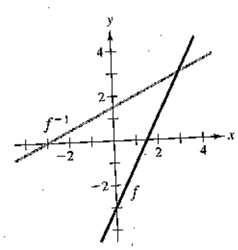

**21.** One-to-one




**23.** Inverse exists.  **25.** Inverse does not exist.

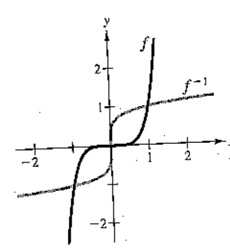
**27.** Inverse exists.

**29.** $f^{-1}(x) = \frac{x+3}{2}$



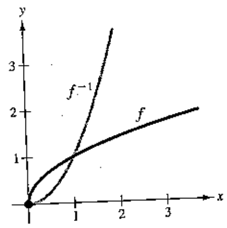

$f$ and $f^{-1}$ are symmetric about $y = x$.

**31.** $f^{-1}(x) = x^{1/5}$



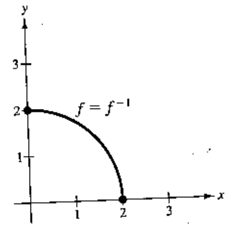

$f$ and $f^{-1}$ are symmetric about $y = x$.

**33.** $f^{-1}(x) = x^2,\ x \ge 0$



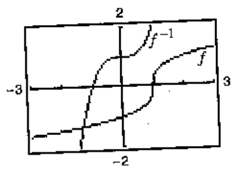

$f$ and $f^{-1}$ are symmetric about $y = x$.

**35.** $f^{-1}(x) = \sqrt{4 - x^2},\ 0 \le x \le 2$



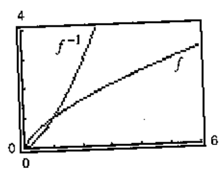

$f$ and $f^{-1}$ are symmetric about $y = x$.

**37.** $f^{-1}(x) = x^3 + 1$



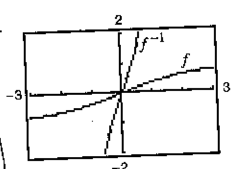

$f$ and $f^{-1}$ are symmetric about $y = x$.

**39.** $f^{-1}(x) = x^{3/2},\ x \ge 0$




$f$ and $f^{-1}$ are symmetric about $y = x$.

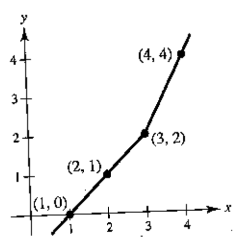
**41.** $f^{-1}(x) = \dfrac{\sqrt{7}x}{\sqrt{1 - x^2}},\ -1 < x < 1$




$f$ and $f^{-1}$ are symmetric about $y = x$.

**43.**

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $f^{-1}(x)$ | 0 | 1 | 2 | 4 |




**45.** (a) Proof

(b) $y = \frac{20}{7}(80 - x)$

$x$: total cost

$y$: number of pounds of the less expensive commodity

(c) $[62.5, 80]$

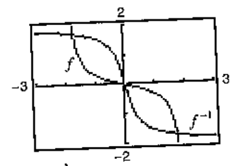
(d) 20 pounds

**47.** $f'(x) = 2(x - 4) > 0$ on $(4, \infty)$

**49.** $f'(x) = -\dfrac{8}{x^3} < 0$ on $(0, \infty)$

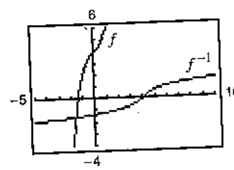
**51.** $f'(x) = -\sin x < 0$ on $(0, \pi)$

**53.** $f^{-1}(x) = \begin{cases} \dfrac{1 - \sqrt{1 + 16x^2}}{2x}, & \text{if } x \ne 0 \\ 0, & \text{if } x = 0 \end{cases}$



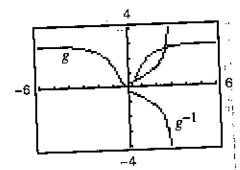

The graph of $f^{-1}$ is a reflection of the graph of $f$ in the line $y = x$.

**55.** (a) and (b)




(c) $f$ is one-to-one and has an inverse function.

**57.** (a) and (b)




(c) $g$ is not one-to-one and does not have an inverse function.

**59.** One-to-one

$f^{-1}(x) = x^2 + 2,\ x \ge 0$

**61.** One-to-one

$f^{-1}(x) = 2 - x,\ x \ge 0$

**63.** $f^{-1}(x) = \sqrt{x} + 3,\ x \ge 0$

(Answer is not unique.)

**65.** $f^{-1}(x) = x - 3,\ x \ge 0$

(Answer is not unique.)

**67.** Inverse exists. Volume is an increasing function, therefore one-to-one. The inverse function gives the time $t$ corresponding to the volume $V$.

**69.** Inverse does not exist. **71.** $\dfrac{1}{5}$ **73.** $\dfrac{2\sqrt{3}}{3}$ **75.** $\dfrac{1}{13}$

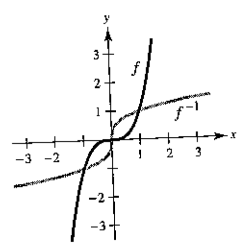
**77.** (a) Domain of $f$: $(-\infty, \infty)$

Domain of $f^{-1}$: $(-\infty, \infty)$

(b) Range of $f$: $(-\infty, \infty)$

Range of $f^{-1}$: $(-\infty, \infty)$

(c)




(d) $f'\left(\frac{1}{2}\right) = \frac{3}{4}$, $(f^{-1})'\left(\frac{1}{8}\right) = \frac{4}{3}$

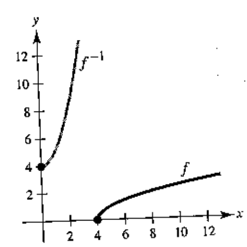
**79.** (a) Domain of $f$: $[4, \infty)$

Domain of $f^{-1}$: $[0, \infty)$

(b) Range of $f$: $[0, \infty)$

Range of $f^{-1}$: $[4, \infty)$

(c)




(d) $f'(5) = \frac{1}{2}$, $(f^{-1})'(1) = 2$

81. $-\frac{1}{11}$ **83.** 32 **85.** 600 **87.** $(g^{-1} \circ f^{-1})(x) = \dfrac{x+1}{2}$

**89.** $(f \circ g)^{-1}(x) = \dfrac{x+1}{2}$

**91.** Let $y = f(x)$ be one-to-one. Solve for $x$ as a function of $y$. Interchange $x$ and $y$ to get $y = f^{-1}(x)$. Let the domain of $f^{-1}$ be the range of $f$. Verify that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

Example: $f(x) = x^3$

$$y = x^3$$
$$x = \sqrt[3]{y}$$
$$y = \sqrt[3]{x}$$
$$f^{-1}(x) = \sqrt[3]{x}$$

**93.** Answers will vary. Example: $y = x^4 - 2x^3$

**95.** Many $x$-values yield the same $y$-value.

For example, $f(\pi) = 0 = f(0)$.

The graph is not continuous at $x = \dfrac{(2n-1)\pi}{2}$, where $n$ is an integer.

**97.** Proof **99.** Proof

**101.** False. Let $f(x) = x^2$. **103.** True

**105.** No. Let $f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 1-x, & 1x \le 2 \end{cases}$. **107.** $\sqrt{17}$

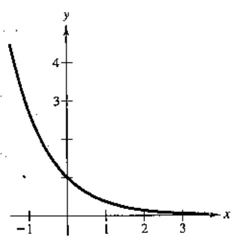
## Section 5.4 (page 347)

**1.** $\ln 1 = 0$ **3.** $e^{0.6931\ldots} = 2$ **5.** $x = 4$

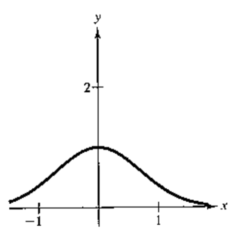
**7.** $x \approx 2.485$ **9.** $x = 0$ **11.** $x \approx 0.511$

**13.** $x \approx 7.389$ **15.** $x \approx 10.389$ **17.** $x \approx 5.389$

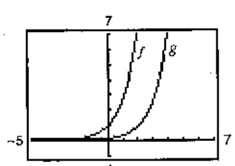
**19.**



**21.**

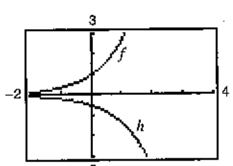


**23.** (a)



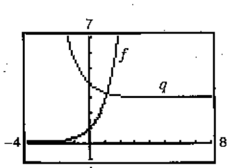

Translation 2 units to the right

(b)




Reflection in the $x$-axis and a vertical shrink

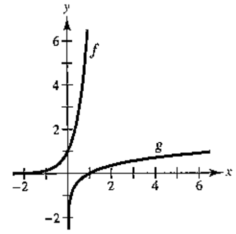
(c)



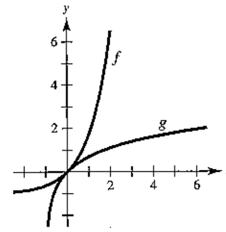

Reflection in the $y$-axis and a translation 3 units upward

**25.** c **26.** d **27.** a **28.** b

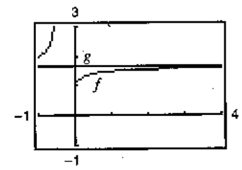
**29.**




**31.**




**33.**




$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} g(x) = e^{0.5}$

**35.** $2.7182805 < e$ **37.** (a) 3 (b) $-3$

**39.** $2e^{2x}$ **41.** $2(x-1)e^{-2x+x^2}$ **43.** $\dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$

**45.** $3(e^{-t} + e^{t})^2(e^{t} - e^{-t})$ **47.** $2x$ **49.** $\dfrac{2e^{2x}}{1+e^{2x}}$

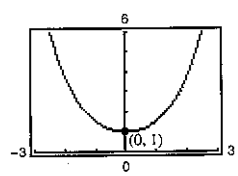
**51.** $\dfrac{-2(e^x - e^{-x})}{(e^x + e^{-x})^2}$ **53.** $x^2e^x$ **55.** $e^{-x}\left(\dfrac{1}{x} - \ln x\right)$

**57.** $2e^x \cos x$ **59.** $\dfrac{10 - e^y}{xe^y + 3}$ **61.** $3(6x+5)e^{-3x}$

**63.** $y'' - 2y' + 3y = 0$

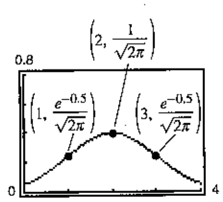
$$e^x\left[-\cos\sqrt{2}x - \sin\sqrt{2}x - 2\sqrt{2}\sin\sqrt{2}x + 2\sqrt{2}\cos\sqrt{2}x\right] -$$
$$2e^x\left[-\sqrt{2}\sin\sqrt{2}x + \sqrt{2}\cos\sqrt{2}x + \cos\sqrt{2}x + \sin\sqrt{2}x\right] +$$
$$3e^x\left[\cos\sqrt{2}x + \sin\sqrt{2}x\right]$$
$$= 0$$
$$0 = 0$$

**65.** Relative minimum: (0, 1)

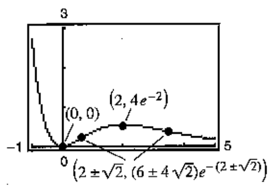



**67.** Relative maximum: $\left(2, \dfrac{1}{\sqrt{2\pi}}\right)$

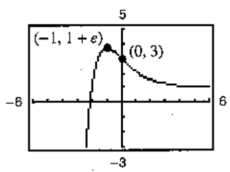
Points of inflection: $\left(1, \dfrac{e^{-0.5}}{\sqrt{2\pi}}\right), \left(3, \dfrac{e^{-0.5}}{\sqrt{2\pi}}\right)$



**69.** Relative minimum: $(0, 0)$
Relative maximum: $(2, 4e^{-2})$
Points of inflection: $\left(2 \pm \sqrt{2}, \left(6 \pm 4\sqrt{2}\right)e^{-(2 \pm \sqrt{2})}\right)$

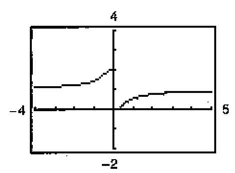



**71.** Relative maximum: $(-1, 1 + e)$
Point of inflection: $(0, 3)$



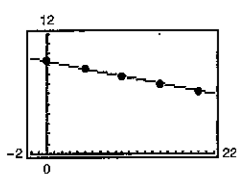

**73.** $A = \sqrt{2}e^{-1/2}$ **75.** Proof **77.** 0.567

**79.** (a)

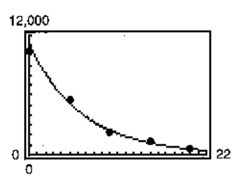



(b) When $x$ increases without bound, $1/x$ approaches zero and $e^{1/x}$ approaches 1. Therefore, $f(x)$ approaches $\frac{2}{1+1} = 1$. Thus, $f(x)$ has a horizontal asymptote at $y = 1$. As $x$ approaches zero from the right, $1/x$ approaches $\infty$, $e^{1/x}$ approaches $\infty$, and $f(x)$ approaches 0. As $x$ approaches zero from the left, $1/x$ approaches $-\infty$, $e^{1/x}$ approaches 0, and $f(x)$ approaches 2. The limit does not exist, because the limit from the left does not equal the limit from the right. Therefore, $x = 0$ is a nonremovable discontinuity.

**81.** (a) $\ln P = -0.1499h + 9.3018$ (b) $P = 10{,}957.7e^{-0.1499h}$

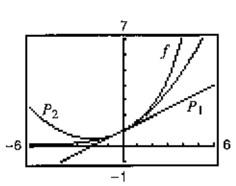



(c)



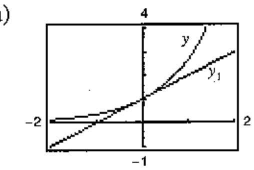

(d) $h = 5$: $-776$
$h = 18$: $-111$

**83.** $P_1 = 1 + \dfrac{x}{2}$; $P_2 = 1 + \dfrac{x}{2} + \dfrac{x^2}{8}$

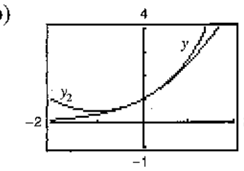



The values of $f$, $P_1$, and $P_2$ and their first derivatives agree at $x = 0$.

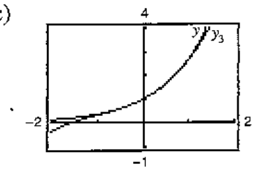
**85.** (a)




(b)




(c)




**87.** $e^{5x} + C$ **89.** $\dfrac{e^2 - 1}{2e^2}$

**91.** $-\frac{1}{2}e^{-x^2} + C$ **93.** $2e^{\sqrt{x}} + C$

**95.** $x - \ln(e^x + 1) + C_1$ or $-\ln(1 + e^{-x}) + C_2$

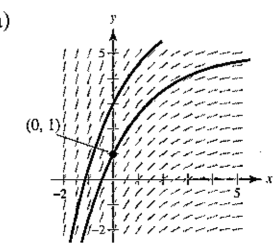
**97.** $\dfrac{e}{3}(e^2 - 1)$ **99.** $-\dfrac{2}{3}(1 - e^x)^{3/2} + C$

**101.** $\ln|e^x - e^{-x}| + C$ **103.** $-\frac{5}{2}e^{-2x} + e^{-x} + C$

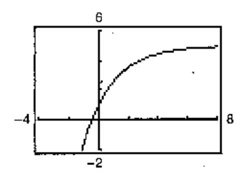
**105.** $\dfrac{1}{\pi}e^{\sin \pi x} + C$ **107.** $\ln|\cos e^{-x}| + C$

**109.** $\dfrac{1}{2a}e^{ax^2} + C$ **111.** $f(x) = \dfrac{1}{2}(e^x + e^{-x})$

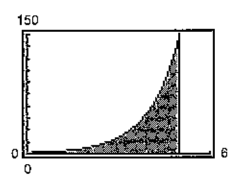
**113.** (a)



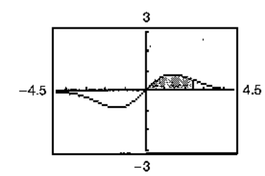

(b) $y = -4e^{-x/2} + 5$




**115.** $e^5 - 1 \approx 147.413$




**117.** $2(1 - e^{-3/2}) \approx 1.554$




**119.** (a)
$$f(x) = e^x$$
$$f(u - v) = e^{u-v}$$
$$= \frac{e^u}{e^v}$$
$$= \frac{f(u)}{f(v)}$$

(b)
$$f(x) = e^x$$
$$f(kx) = e^{xk}$$
$$= [f(x)]^k$$

**121.** The probability that a given battery will last between 48 months and 60 months is approximately 47.72%.

**123.** $\displaystyle\int_0^x e^t\,dt \geq \int_0^x 1\,dt$; $e^x - 1 \geq x$; $e^x > x + 1$ for $x \geq 0$

**125.** $f(x) = e^x$

The domain of $f(x)$ is $(-\infty, \infty)$ and the range of $f(x)$ is $(0, \infty)$. $f(x)$ is continuous, increasing, one-to-one, and concave upward on its entire domain.

$\lim_{x \to -\infty} e^x = 0$ and $\lim_{x \to \infty} e^x = \infty$

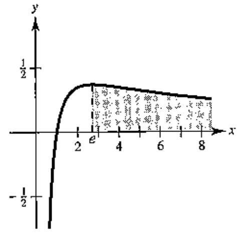
**127.** $f(x) = e^x = f'(x)$

**129.** $e^{-x} > 0$ implies $\int_0^2 e^{-x}\,dx > 0$

**131.** (a) $f'(x) = \dfrac{1 - \ln x}{x^2} = 0$ when $x = e$.

On $(0, e), f'(x) > 0 \Rightarrow f$ is increasing.

On $(e, \infty), f'(x) < 0 \Rightarrow f$ is decreasing.



(b) For $e \le A < B$, we have:

$$\frac{\ln A}{A} > \frac{\ln B}{B}$$
$$B \ln A > A \ln B$$
$$\ln A^B > \ln B^A$$
$$A^B > B^A.$$

(c) Since $e < \pi$, from part (b) we have $e^\pi > \pi^e$.

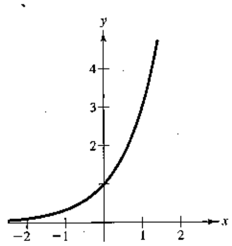
## Section 5.5 (page 357)

**1.** $y(t) = \left(\frac{1}{2}\right)^{t/3}, \frac{1}{4}$ **3.** $y(t) = \left(\frac{1}{2}\right)^{t/7}, \left(\frac{1}{2}\right)^{10/7} \approx 0.371$

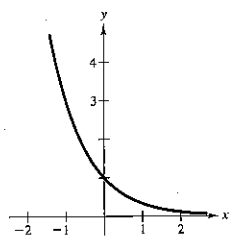
**5.** $-3$ **7.** $0$ **9.** (a) $\log_2 8 = 3$ (b) $\log_3(1/3) = -1$

**11.** (a) $10^{-2} = 0.01$ (b) $\left(\frac{1}{2}\right)^{-3} = 8$

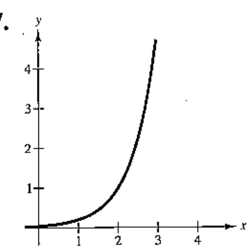
**13.**



**15.**



**17.**



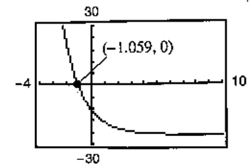
**19.** (a) $x = 3$ (b) $x = -1$

**21.** (a) $x = \frac{1}{3}$ (b) $x = \frac{1}{16}$ **23.** (a) $x = -1, 2$ (b) $x = \frac{1}{3}$

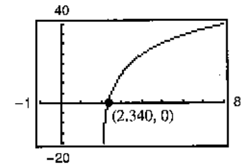
**25.** 1.965 **27.** $-6.288$ **29.** 12.253

**31.** 33.000 **33.** $\pm 11.845$

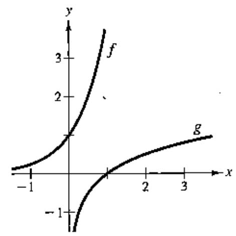
**35.**




**37.**




**39.**



**41.** $(\ln 4)4^x$ **43.** $(\ln 5)5^{x-2}$

**45.** $t2^t(t \ln 2 + 2)$ **47.** $-2^{-\theta}[(\ln 2)\cos \pi\theta + \pi \sin \pi\theta]$

**49.** $\dfrac{1}{x(\ln 3)}$ **51.** $\dfrac{x-2}{(\ln 2)x(x-1)}$ **53.** $\dfrac{x}{(\ln 5)(x^2-1)}$

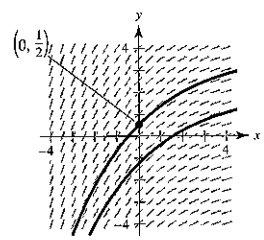
**55.** $\dfrac{5}{(\ln 2)t^2}(1 - \ln t)$ **57.** $2(1 - \ln x)x^{(2/x)-2}$

**59.** $(x-2)^{x+1}\left[\dfrac{x+1}{x-2} + \ln(x-2)\right]$

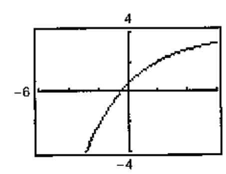
**61.** $\dfrac{3^x}{\ln 3} + C$ **63.** $\dfrac{7}{\ln 4}$ **65.** $-\dfrac{1}{2 \ln 5}(5^{-x^2}) + C$

**67.** $\dfrac{\ln(3^{2x}+1)}{2 \ln 3} + C$

**69.** (a)



(b) $y = \dfrac{3(1 - 0.4^{x/3})}{\ln 2.5} + \dfrac{1}{2}$



**71.** Answers will vary. Example: Growth and decay problems

**73.** (a) False. $y = a^x \Rightarrow 0 = a' \Rightarrow a = 0$, but exponential functions are not defined for $a = 0$.

(b) True: $y = \log_2 x$

(c) True: $2^y = x$

(d) False. (1, 0), (2, 1), and (8, 3) are not collinear.

**75.** $g(x) = x^x, k(x) = 2^x, h(x) = x^2, f(x) = \log_2 x$

**77.** (a) \$40.64 (b) $C'(1) \approx 0.051P, C'(8) \approx 0.072P$

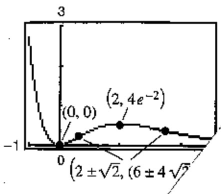
(c) ln 1.05

**79.**

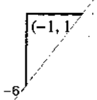
| $n$ | 1 | 2 | 4 | 12 |
|---|---|---|---|---|
| $A$ | \$1410.60 | \$1414.78 | \$1416.91 | \$1418.34 |

| $n$ | 365 | Continuous |
|---|---|---|
| $A$ | \$1419.04 | \$1419.07 |

69. Relative minimum: (0, 0)
Relative maximum: $(2, 4e^{-2})$
Points of inflection: $(2 \pm \sqrt{2}, (6 \pm$

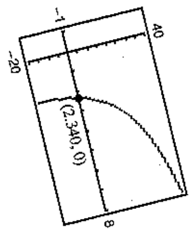



71. Relative maxim
Point of infl






37.






...,582.66

| | |
|---|---|
| $15,589.88 | $8251.24 |

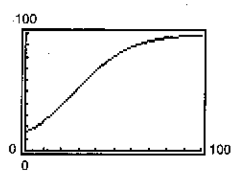
87. c

89. (a) 6.7 million cubic feet per acre

(b) $t = 20: \dfrac{dV}{dt} = 0.073$

$t = 60: \dfrac{dV}{dt} = 0.040$

91. (a)



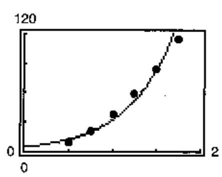

(b) 16.7%

(c) $x \approx 38.8$ or 38,800 egg masses

(d) $x \approx 2.78$ or 27,800 egg masses

93. (a) $B = 4.75(6.774)^d$

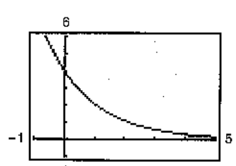
(b)




(c) When $d = 0.8$, the rate of growth is 41.99.
When $d = 1.5$, the rate of growth is 160.21.

95. (a) 5.67

(b)




(c) $f(t) = g(t) = h(t)$. No, because the definite integrals of two functions over a given interval may be equal even though the functions are not equal.

97. $15,039.61  99. $y = 1200(0.6^t)$

101. False: $e$ is an irrational number.

103. True  105. True  107. Proof

## Section 5.6 (page 366)

1. $y = \dfrac{x^2}{2} + 2x + C$  3. $y = Ce^x - 2$

5. $y^2 - 5x^2 = C$  7. $y = Ce^{(2x^{3/2})/3}$  9. $y = C(1 + x^2)$

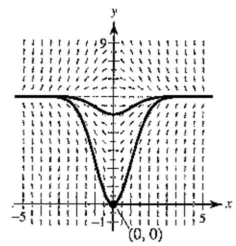
11. $\dfrac{dQ}{dt} = \dfrac{k}{t^2}$

$Q = -\dfrac{k}{t} + C$

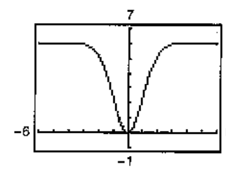
13. $\dfrac{dN}{ds} = k(250 - s)$

$N = -\dfrac{k}{2}(250 - s)^2 + C$

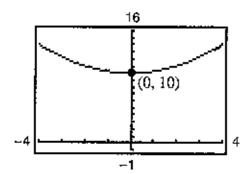
15. (a)



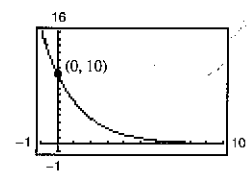

(b) $y = 6 - 6e^{-x^2/2}$




17. $y = \frac{1}{4}t^2 + 10$




19. $y = 10e^{-t/2}$




21. $\dfrac{dy}{dx} = ky$

$y = 4e^{0.3054x}$

$y(6) \approx 25$

23. $\dfrac{dy}{dt} = kV$

$V = 20{,}000e^{-0.1175t}$

$V(6) \approx 9882$

25. $y = \frac{1}{2}e^{0.4605t}$  27. $y = 0.6687e^{0.4024t}$

29. A differential equation in $x$ and $y$ is an equation that involves $x$, $y$, and derivatives of $y$.

Example: $y' = \dfrac{3x}{y}$

31. Quadrants I and III; $dy/dx$ is positive when both $x$ and $y$ are positive (Quadrant I) or when both $x$ and $y$ are negative (Quadrant III).

33. Amount after 1000 years: 6.52 grams
Amount after 10,000 years: 0.14 gram

35. Initial quantity: 36.07 grams
Amount after 1000 years: 23.65 grams

37. Amount after 1000 years: 4.43 grams
Amount after 10,000 years: 1.49 grams

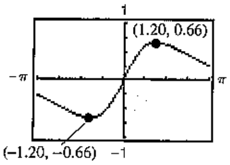
39. Initial quantity: 2.16 grams
Amount after 10,000 years: 1.63 grams

41. 95.81%

43. Time to double: 11.55 years
Amount after 10 years: $1822.12

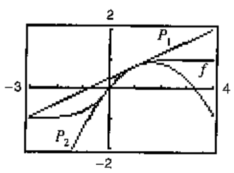
45. Annual rate: 8.94%
Amount after 10 years: $1833.67

**31.** Relative maximum: $(1.20, 0.66)$
Relative minimum: $(-1.20, -0.66)$

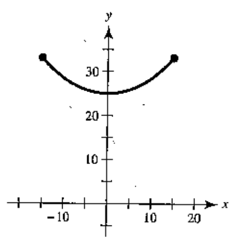



**33.** $y = a \sinh x$
$y' = a \cosh x$
$y'' = a \sinh x$
$y''' = a \cosh x$
Therefore, $y''' - y' = 0$.

**35.** $P_1(x) = 0.76 + 0.42(x - 1)$
$P_2(x) = 0.76 + 0.42(x - 1) - 0.32(x - 1)^2$




**37.** (a)



(b) 33.146 units; 25 units (c) $m = \sinh(1) \approx 1.175$

**39.** $-\frac{1}{2}\cosh(1 - 2x) + C$ **41.** $\frac{1}{3}\cosh^3(x - 1) + C$

**43.** $\ln|\sinh x| + C$ **45.** $-\coth \dfrac{x^2}{2} + C$ **47.** $\operatorname{csch} \dfrac{1}{x} + C$

**49.** $\dfrac{1}{5}\ln 3$ **51.** $\dfrac{\pi}{4}$ **53.** $\dfrac{1}{2}\arctan x^2 + C$ **55.** $\dfrac{3}{\sqrt{9x^2 - 1}}$

**57.** $|\sec x|$ **59.** $2 \sec 2x$ **61.** $2 \sinh^{-1}(2x)$

**63.** See "Definition of the Hyperbolic Functions" on page 395.

**65.** $-\dfrac{\sqrt{a^2 - x^2}}{x}$

**67.** $-\operatorname{csch}^{-1}(e^x) + C = -\ln\left(\dfrac{1 + \sqrt{1 + e^{2x}}}{e^x}\right) + C$

**69.** $2\sinh^{-1}\sqrt{x} + C = 2\ln\left(\sqrt{x} + \sqrt{1 + x}\right) + C$

**71.** $\dfrac{1}{4}\ln\left|\dfrac{x - 4}{x}\right| + C$ **73.** $\dfrac{1}{2\sqrt{6}}\ln\left|\dfrac{\sqrt{2}(x + 1) + \sqrt{3}}{\sqrt{2}(x + 1) - \sqrt{3}}\right| + C$

**75.** $\dfrac{1}{4}\arcsin\left(\dfrac{4x - 1}{9}\right) + C$

**77.** $-\dfrac{x^2}{2} - 4x - \dfrac{10}{3}\ln\left|\dfrac{x - 5}{x + 1}\right| + C$

**79.** $8\arctan(e^2) - 2\pi \approx 5.207$ **81.** $\frac{5}{2}\ln\left(\sqrt{17} + 4\right) \approx 5.237$

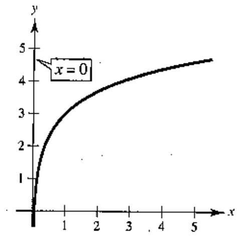
**83.** $\frac{52}{31}$ kilograms

**85.** If $k$ were increased, the time of descent would increase.

**87.** Proof **89.** Proof **91.** Proof

## *Review Exercises for Chapter 5 (page 405)*

**1.** Vertical asymptote: $x = 0$




**3.** $\frac{1}{5}[\ln(2x + 1) + \ln(2x - 1) - \ln(4x^2 + 1)]$

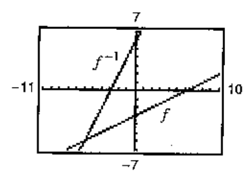
**5.** $\ln\left(\dfrac{3\sqrt[3]{4 - x^2}}{x}\right)$ **7.** $e^4 - 1 \approx 53.598$

**9.** $\dfrac{1}{2x}$ **11.** $\dfrac{1 + 2\ln x}{2\sqrt{\ln x}}$ **13.** $\dfrac{x}{(a + bx)^2}$ **15.** $\dfrac{1}{x(a + bx)}$

**17.** $\frac{1}{7}\ln|7x - 2| + C$ **19.** $-\ln|1 + \cos x| + C$

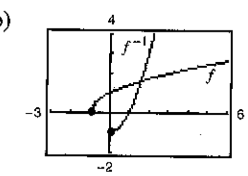
**21.** $3 + \ln 4$ **23.** $\ln\left(2 + \sqrt{3}\right)$

**25.** (a) $f^{-1}(x) = 2x + 6$
(b)




(c) Proof

**27.** (a) $f^{-1}(x) = x^2 - 1, \quad x \geq 0$
(b)



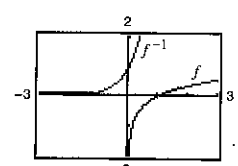

(c) Proof

**29.** (a) $f^{-1}(x) = x^3 - 1$
(b)


(c) Proof

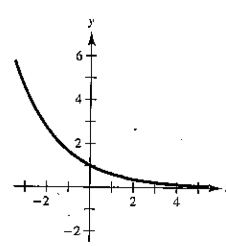
**31.** $\dfrac{1}{3(\sqrt[3]{-3})^2} \approx 0.160$ **33.** $\dfrac{3}{4}$

**35.** (a) $f^{-1}(x) = e^{2x}$
(b)




(c) Proof

**37.**



**41.** $te^t(t + 2)$ **43.** $\dfrac{e^{2x} - e^{-2x}}{\sqrt{e^{2x} + e^{-2x}}}$

**47.** $-\dfrac{y}{x(2y + \ln x)}$

$+ C$ **51.** $\dfrac{e^{4x} - 3e^{2x} - 3}{3e^x} + C$

$+ C$ **55.** $\ln|e^x - 1| + C$

$(a \cos 3x + b \sin 3x)$

$[(-3a + b) \sin 3x + (a + 3b) \cos 3x]$

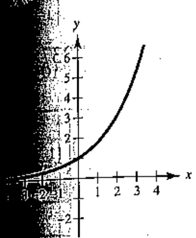
$[(-6a - 8b) \sin 3x + (-8a + 6b) \cos 3x]$

$y' + 10y$

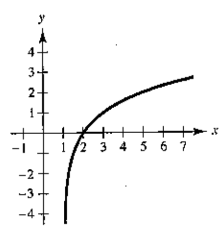
$[(-6a - 8b) - 2(-3a + b) + 10b] \sin 3x +$

$[(-8a + 6b) - 2(a + 3b) + 10a] \cos 3x\} = 0$

$- 1) \approx 0.500$



**63.**



**67.** $x^{2x+1}\left(\dfrac{2x + 1}{x} + 2 \ln x\right)$

$\ln 3(2 - 2x)$ **71.** $\dfrac{5^{(x+1)^2}}{2 \ln 5} + C$

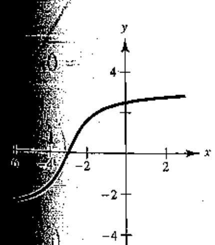
(b) $(\ln a)a^x$ (c) $x^x(1 + \ln x)$ (d) 0

$3499.38 **77.** $\approx 7.79$ inches **79.** About 46.2 years

$+ 3 \ln|x| + C$ **83.** $y = Ce^{x^2}$

$= C$ **87.** Proof; $y = -2x + \frac{1}{2}x^3$

**91.** (a) $\dfrac{1}{2}$ (b) $\dfrac{\sqrt{3}}{2}$



$)^{-3/2}$ **95.** $\dfrac{x}{|x|\sqrt{x^2 - 1}} + \operatorname{arcsec} x$

$)^2$ **99.** $\frac{1}{2} \arctan(e^{2x}) + C$

$x^2 + C$ **103.** $\ln\sqrt{16 + x^2} + C$

$\left(\arctan \dfrac{x}{2}\right)^2 + C$ **107.** $y = A \sin\left(\sqrt{\dfrac{k}{m}}\, t\right)$

$\dfrac{\sinh\sqrt{x}}{2\sqrt{x}}$ **111.** $\dfrac{1}{2}\ln\left(\sqrt{x^4 - 1} + x^2\right) + C$

## *P.S. Problem Solving (page 408)*

**1.** $\theta \approx 1.7263$ or $98.9°$

**3.** (a) $(0, \infty)$

(b) Answers will vary.

Example: $e^{\pi/2} \approx 4.8105$ and $e^{5\pi/2} \approx 2575.9705$

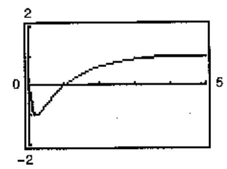
(c) Answers will vary.

Example: $e^{-\pi/2} \approx 0.2079$ and $e^{3\pi/2} \approx 111.3178$

(d) $[-1, 1]$

(e) $f'(x) = \dfrac{\cos(\ln x)}{x}$; Maximum value is 1.

(f)




Limit does not exist.

(g) Limit does not exist.

**5.** (a) Area of sector $= \dfrac{t}{2}$

(b) $A(t) = \frac{1}{2}\text{base} \cdot \text{height} - \displaystyle\int_1^{\cosh t} \sqrt{x^2 - 1}\, dx$

$A(t) = \frac{1}{2}\cosh t \cdot \sinh t - \displaystyle\int_1^{\cosh t} \sqrt{x^2 - 1}\, dx$

$A(t) = \frac{1}{2}t$

**7.** Tangent line: $y = \dfrac{1}{a}x + (b - 1)$

Passes through $(0, c)$, therefore $c = b - 1$.

Distance between $b$ and $c$ is $b - c = 1$.

**9.** $2 \ln\left(\frac{3}{2}\right) \approx 0.8109$

**11.** (a) $y = \dfrac{1}{(1 - 0.01t)^{100}}$; $T = 100$

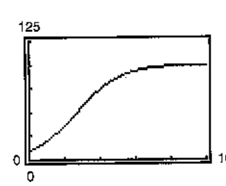
(b) $y = \dfrac{1}{\left[\left(\dfrac{1}{y_0}\right)^e - ket\right]^{1/e}}$; Answers will vary.

**13.** 22.35°F

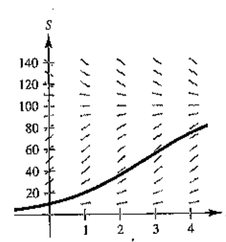
**15.** (a) $\dfrac{dS}{dt} = kS(L - S)$; $S = \dfrac{100}{1 + 9e^{-0.8109t}}$

(b) 2.7 months

(c)




(d)




(e) Sales will decrease toward the line $S = L$.