REVIEW EXERCISES FOR CHAPTER 1

1.1 In Exercises 1 and 2, determine whether the problem can be solved using precalculus or if calculus is required. If the problem can be solved using precalculus, solve it. If the problem seems to require calculus, explain your reasoning. Use a graphical or numerical approach to estimate the solution.

1. Find the distance between the points (1, 1) and (3, 9) along the curve $y = x^2$.
2. Find the distance between the points (1, 1) and (3, 9) along the line $y = 4x - 3$.

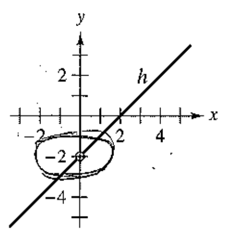
1.2 In Exercises 3 and 4, complete the table and use the result to estimate the limit. Use a graphing utility to graph the function to confirm your result.

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$						

3. $\lim_{x\to 0} \dfrac{[4/(x+2)] - 2}{x}$
4. $\lim_{x\to 0} \dfrac{4(\sqrt{x+2} - \sqrt{2})}{x}$

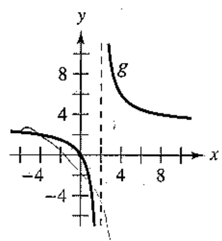
In Exercises 5 and 6, use the graph to determine each limit.

5. $h(x) = \dfrac{x^2 - 2x}{x}$



(a) $\lim_{x\to 0} h(x)$ (b) $\lim_{x\to -1} h(x)$

6. $g(x) = \dfrac{3x}{x - 2}$



(a) $\lim_{x\to 2} g(x)$ (b) $\lim_{x\to 0} g(x)$

In Exercises 7–10, find the limit L. Then use the ε-δ definition to prove that the limit is L.

7. $\lim_{x\to 1} (3 - x)$
8. $\lim_{x\to 9} \sqrt{x}$
9. $\lim_{x\to 2} (x^2 - 3)$
10. $\lim_{x\to 5} 9$

1.3 In Exercises 11–24, find the limit (if it exists).

11. $\lim_{t\to 4} \sqrt{t+2}$
12. $\lim_{y\to 4} 3|y - 1|$
13. $\lim_{t\to -2} \dfrac{t+2}{t^2 - 4}$
14. $\lim_{t\to 3} \dfrac{t^2 - 9}{t - 3}$
15. $\lim_{x\to 4} \dfrac{\sqrt{x} - 2}{x - 4}$
16. $\lim_{x\to 0} \dfrac{\sqrt{4+x} - 2}{x}$
17. $\lim_{x\to 0} \dfrac{[1/(x+1)] - 1}{x}$
18. $\lim_{s\to 0} \dfrac{(1/\sqrt{1+s}) - 1}{s}$
19. $\lim_{x\to -5} \dfrac{x^3 + 125}{x + 5}$
20. $\lim_{x\to -2} \dfrac{x^2 - 4}{x^3 + 8}$
21. $\lim_{x\to 0} \dfrac{1 - \cos x}{\sin x}$
22. $\lim_{x\to \pi/4} \dfrac{4x}{\tan x}$
23. $\lim_{\Delta x\to 0} \dfrac{\sin[(\pi/6) + \Delta x] - (1/2)}{\Delta x}$

    [*Hint:* $\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$]
24. $\lim_{\Delta x\to 0} \dfrac{\cos(\pi + \Delta x) + 1}{\Delta x}$

    [*Hint:* $\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$]

In Exercises 25 and 26, evaluate the limit given $\lim_{x\to c} f(x) = -\frac{3}{4}$ and $\lim_{x\to c} g(x) = \frac{2}{3}$.

25. $\lim_{x\to c} [f(x)g(x)]$
26. $\lim_{x\to c} [f(x) + 2g(x)]$

Numerical, Graphical, and Analytic Analysis **In Exercises 27 and 28, consider**

$$\lim_{x\to 1^+} f(x).$$

(a) Complete the table to estimate the limit.

(b) Use a graphing utility to graph the function and use the graph to estimate the limit.

(c) Rationalize the numerator to find the exact value of the limit analytically.

x	1.1	1.01	1.001	1.0001
$f(x)$				

27. $f(x) = \dfrac{\sqrt{2x+1} - \sqrt{3}}{x - 1}$
28. $f(x) = \dfrac{1 - \sqrt[3]{x}}{x - 1}$

    [*Hint:* $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$]

Free-Falling Object **In Exercises 29 and 30, use the position function**

$$s(t) = -4.9t^2 + 200$$

which gives the height (in meters) of an object that has fallen from a height of 200 meters. The velocity at time $t = a$ seconds is given by

$$\lim_{t\to a} \frac{s(a) - s(t)}{a - t}.$$

29. Find the velocity of the object when $t = 4$.
30. At what velocity will the object impact the ground?

1.4 In Exercises 31–36, find the limit (if it exists). If the limit does not exist, explain why.

31. $\displaystyle\lim_{x\to 3^-} \frac{|x-3|}{x-3}$

32. $\displaystyle\lim_{x\to 4} [\![x-1]\!]$

33. $\displaystyle\lim_{x\to 2} f(x)$, where $f(x) = \begin{cases} (x-2)^2, & x \le 2 \\ 2-x, & x > 2 \end{cases}$

34. $\displaystyle\lim_{x\to 1^+} g(x)$, where $g(x) = \begin{cases} \sqrt{1-x}, & x \le 1 \\ x+1, & x > 1 \end{cases}$

35. $\displaystyle\lim_{t\to 1} h(t)$, where $h(t) = \begin{cases} t^3+1, & t < 1 \\ \frac{1}{2}(t+1), & t \ge 1 \end{cases}$

36. $\displaystyle\lim_{s\to -2} f(s)$, where $f(s) = \begin{cases} -s^2-4s-2, & s \le -2 \\ s^2+4s+6, & s > -2 \end{cases}$

In Exercises 37–46, determine the intervals on which the function is continuous.

37. $f(x) = [\![x+3]\!]$

38. $f(x) = \dfrac{3x^2-x-2}{x-1}$

39. $f(x) = \begin{cases} \dfrac{3x^2-x-2}{x-1}, & x \ne 1 \\ 0, & x = 1 \end{cases}$

40. $f(x) = \begin{cases} 5-x, & x \le 2 \\ 2x-3, & x > 2 \end{cases}$

41. $f(x) = \dfrac{1}{(x-2)^2}$

42. $f(x) = \sqrt{\dfrac{x+1}{x}}$

43. $f(x) = \dfrac{3}{x+1}$

44. $f(x) = \dfrac{x+1}{2x+2}$

45. $f(x) = \csc \dfrac{\pi x}{2}$

46. $f(x) = \tan 2x$

47. Determine the value of c such that the function is continuous on the entire real line.

$$f(x) = \begin{cases} x+3, & x \le 2 \\ cx+6, & x > 2 \end{cases}$$

48. Determine the values of b and c such that the function is continuous on the entire real line.

$$f(x) = \begin{cases} x+1, & 1 < x < 3 \\ x^2+bx+c, & |x-2| \ge 1 \end{cases}$$

49. Use the Intermediate Value Theorem to show that $f(x) = 2x^3 - 3$ has a zero in the interval $[1, 2]$.

50. ***Cost of Overnight Delivery*** The cost of sending an overnight package from New York to Atlanta is $9.80 for the first pound and $2.50 for each additional pound. Use the greatest integer function to create a model for the cost C of overnight delivery of a package weighing x pounds. Use a graphing utility to graph the function and discuss its continuity.

51. Let $f(x) = \dfrac{x^2-4}{|x-2|}$. Find each limit (if possible).

(a) $\displaystyle\lim_{x\to 2^-} f(x)$

(b) $\displaystyle\lim_{x\to 2^+} f(x)$

(c) $\displaystyle\lim_{x\to 2} f(x)$

52. Let $f(x) = \sqrt{x(x-1)}$.

(a) Find the domain of f.

(b) Find $\displaystyle\lim_{x\to 0^-} f(x)$.

(c) Find $\displaystyle\lim_{x\to 1^+} f(x)$.

1.5 In Exercises 53–56, find the vertical asymptotes (if any) of the function.

53. $g(x) = 1 + \dfrac{2}{x}$

54. $h(x) = \dfrac{4x}{4-x^2}$

55. $f(x) = \dfrac{8}{(x-10)^2}$

56. $f(x) = \csc \pi x$

In Exercises 57–68, find the one-sided limit.

57. $\displaystyle\lim_{x\to -2^-} \frac{2x^2+x+1}{x+2}$

58. $\displaystyle\lim_{x\to (1/2)^+} \frac{x}{2x-1}$

59. $\displaystyle\lim_{x\to -1^+} \frac{x+1}{x^3+1}$

60. $\displaystyle\lim_{x\to -1^-} \frac{x+1}{x^4-1}$

61. $\displaystyle\lim_{x\to 1^-} \frac{x^2+2x+1}{x-1}$

62. $\displaystyle\lim_{x\to -1^+} \frac{x^2-2x+1}{x+1}$

63. $\displaystyle\lim_{x\to 0^+} \left(x - \frac{1}{x^3}\right)$

64. $\displaystyle\lim_{x\to 2^-} \frac{1}{\sqrt[3]{x^2-4}}$

65. $\displaystyle\lim_{x\to 0^+} \frac{\sin 4x}{5x}$

66. $\displaystyle\lim_{x\to 0^+} \frac{\sec x}{x}$

67. $\displaystyle\lim_{x\to 0^+} \frac{\csc 2x}{x}$

68. $\displaystyle\lim_{x\to 0^-} \frac{\cos^2 x}{x}$

69. ***Cost of Clean Air*** A utility company burns coal to generate electricity. The cost C in dollars of removing $p\%$ of the air pollutants in the stack emissions is

$$C = \frac{80{,}000p}{100-p}, \qquad 0 \le p < 100.$$

Find the cost of removing (a) 15%, (b) 50%, and (c) 90% of the pollutants. (d) Find the limit of C as $p \to 100^-$.

70. The function f is defined as follows.

$$f(x) = \frac{\tan 2x}{x}, \qquad x \ne 0$$

(a) Find $\displaystyle\lim_{x\to 0} \frac{\tan 2x}{x}$ (if it exists).

(b) Can the function f be defined at $x = 0$ such that it is continuous at $x = 0$?