BC Calculus Integration
Review

Name ANSWER KEY
Date ____________

1. $\int_0^8 \frac{dx}{\sqrt{1+x}}$ 1969 AB-4

$\boxed{4}$

2. $\int_0^1 \sqrt{x^2-2x+1}\,dx$ 1969-AB-26

$\boxed{-1/2}$

3. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x}\,dx$ 1969 AB-29

$\boxed{\ln\frac{\sqrt{2}}{2}}$ $\boxed{\ln\sqrt{2}}$

4. $\int_0^1 \frac{x^2}{x^2+1}\,dx$ 1969 BC-10

$\boxed{1-\frac{\pi}{4}}$

5. $\int_0^1 (x+1)e^{x^2+2x}\,dx$ 1973 AB-21

$\boxed{\frac{e^3-1}{2}}$

6. $\int_0^{\frac{\pi}{4}} tan^2x\ dx$ 1973 AB-25

$\boxed{\frac{4-\pi}{4}}$

7. $\int_0^{\frac{1}{2}} \frac{2x}{\sqrt{1-x^2}}\,dx$ 1973 AB-27

$\boxed{-\sqrt{3}+2}$

8. $\int_1^2 \frac{x-4}{x^2}\,dx$ 1973 AB-30

$\boxed{\ln 2-2}$

9. $\int_0^1 xe^{-x}\,dx$ 1985 AB-17

$\boxed{-2e^{-1}+1}$

10. $\int_1^2 \frac{x^2-1}{x+1}\,dx$ 1985 AB-22

$\boxed{1/2}$

11. $\int_0^{\frac{\pi}{3}} \sin(3x)\,dx$ 1985 AB-32

$\boxed{2/3}$

12. $\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sqrt{1+\sin\theta}}\,d\theta$ 1988 AB-14

$\boxed{2(\sqrt{2}-1)}$

13. $\int_2^3 \frac{x}{x^2+1}\,dx$ 1988 AB-19

$\boxed{\frac{1}{2}\ln 2}$

14. $\int_0^{\frac{\pi}{2}} x\cos x\,dx$ 1988 AB-26

$\boxed{\frac{\pi-2}{2}}$

15. $\int_0^1 x\,(x^2+2)^2\,dx$ 1988 BC-2

$\boxed{19/6}$

16. $\int_2^3 \frac{3}{(x-1)(x+2)}\,dx$ 1988 BC-17

$\boxed{\ln 8/5}$

17. $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}}$ 1993 AB-32

$\boxed{\pi/3}$

18. $\int_0^1 x^3e^{x^4}\,dx$ 1993 BC-7

$\boxed{\frac{1}{4}(e-1)}$

19. $\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{cos^2x}\,dx$ 1997-AB-18

$\boxed{e-1}$

20. $\int_0^1 \sqrt{x}(x+1)\,dx$ 1997 BC-1

$\boxed{16/15}$

21. $\int x^2\sin x\,dx$ 1997 BC-84

$\boxed{-x^2\cos x+2x\sin x+2\cos x+C}$

22. $\int \frac{dx}{(x-1)(x+3)}$ 1997 BC-86

$\boxed{\frac{1}{4}\ln\left|\frac{x-1}{x+3}\right|+C}$

23. $\int_1^2 \frac{1}{x^2}\,dx$ 1998 AB-3

$\boxed{1/2}$

24. $\int_1^e \frac{x^2-1}{x}\,dx$ 1998 AB-7

e $\boxed{\frac{e^2}{2}-3/2} = \boxed{\frac{e^2-3}{2}}$

25. $\int \frac{1}{x^2-6x+8}\,dx$ 1998 BC-4

$\boxed{\frac{1}{2}\ln\left|\frac{x-4}{x-2}\right|+C}$

BC CALCULUS INTEGRATION REVIEW

(1) $\int_0^8 \frac{dx}{\sqrt{1+x}} = \int_0^8 u^{-1/2}\,du = 2u^{1/2} = 2\sqrt{1+x}\Big]_0^8 = 2\sqrt{1+8} - 2\sqrt{1+0} = 2\sqrt{9} - 2\sqrt{1} = 2(3) - 2(1)$

$6 - 2 = \boxed{4}$

$u = 1+x$

$du = dx$

(2) $\int_0^1 \sqrt{x^2-2x+1}\,dx = \int_0^1 \sqrt{(x-1)(x-1)}\,dx = \int_0^1 \sqrt{(x-1)^2}\,dx = \int_0^1 x-1\,dx$

$= \int_0^1 x\,dx - \int_0^1 dx = \frac{x^2}{2} - x\Big]_0^1 = \frac{1^2}{2} - 1 - \frac{0^2}{2} + 0 = \frac{1}{2} - 1 = \boxed{-\frac{1}{2}}$

(3) $\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x}\,dx = \int_{\pi/4}^{\pi/2} \frac{1}{u}\,du = \ln|\sin x|\Big]_{\pi/4}^{\pi/2} = \ln|\sin(\pi/2)| - \ln|\sin(\pi/4)| = \ln(1) - \ln(\sqrt{2}/2) = \boxed{-\ln\left(\frac{\sqrt{2}}{2}\right)}$

$u = \sin x$

$du = \cos x\,dx$

$\ln\left(\frac{\sqrt{2}}{2}\right)^{-1} = \ln\frac{2}{\sqrt{2}} = \ln\frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \ln\frac{2\sqrt{2}}{2} = \boxed{\ln\sqrt{2}}$

(4) $\int_0^1 \frac{x^2}{x^2+1}\,dx = \int_0^1 \frac{x^2+1-1}{x^2+1}\,dx = \int_0^1 \frac{x^2+1}{x^2+1} - \int_0^1 \frac{1}{x^2+1}\,dx = x - \arctan x\Big]_0^1$

$1 - \arctan(1) - 0 + \arctan(0) = 1 - \frac{\pi}{4} + 0 = \boxed{1-\frac{\pi}{4}} = \boxed{\frac{4-\pi}{4}}$

(5) $\int_0^1 (x+1)e^{x^2+2x}\,dx = \frac{1}{2}\int_0^1 e^u\,du = \frac{1}{2}e^{x^2+2x}\Big]_0^1 = \frac{1}{2}e^{1^2+2(1)} - \frac{1}{2}e^{0^2+2(0)} = \frac{1}{2}e^3 - \frac{1}{2} = \boxed{\frac{e^3-1}{2}}$

$u = x^2+2x$

$du = 2x+2\,dx \rightarrow \frac{1}{2}du = x+1\,dx$

(6) $\int_0^{\pi/4} \tan^2 x\,dx = \int_0^{\pi/4} \sec^2 x - 1\,dx = \int_0^{\pi/4} \sec^2 x\,dx - \int_0^{\pi/4} 1\,dx = \tan x - x\Big]_0^{\pi/4}$

$\tan\frac{\pi}{4} - \frac{\pi}{4} - \tan(0) + 0 = 1 - \frac{\pi}{4} - 0 + 0 = \frac{4}{4} - \frac{\pi}{4} = \boxed{1-\frac{\pi}{4}} = \boxed{\frac{4-\pi}{4}}$

⑦ $\int_0^{1/2} \frac{2x}{\sqrt{1-x^2}}\,dx = -\int_0^{1/2} u^{-1/2}\,du = -2u^{1/2} = -2\sqrt{1-x^2}\Big]_0^{1/2}$

$u = 1-x^2$

$du = -2x\,dx$

$-2\sqrt{1-\tfrac{1}{2}^2} + 2\sqrt{1-0^2} = -2\sqrt{3/4} + 2 = -2\,\sqrt{3}/\sqrt{4} + 2$

$-\cancel{2}\sqrt{3}\cdot\tfrac{1}{\cancel{2}} + 2 = \boxed{-\sqrt{3}+2}$

⑧ $\int_1^2 \frac{x-4}{x^2}\,dx = \int_1^2 (x-4)(x^{-2})\,dx = \int_1^2 x^{-1} - 4x^{-2}\,dx = \ln x + 4(\tfrac{1}{x})\Big]_1^2$

$\ln(2) + 4/2 - \ln(1) - 4/1 = \ln(2) + 2 - 0 - 4 = \boxed{\ln 2 - 2}$

⑨ $\int_0^1 xe^{-x}\,dx = -xe^{-x} - \int -e^{-x}\,dx = -xe^{-x} + \int e^{-x}\,dx = -xe^{-x} - e^{-x}\Big]_0^1$

$u = x \quad dv = e^{-x}\,dx$

$du = dx \quad v = -e^{-x}$

$-1e^{-1} - e^{-1} + 0e^{0} + e^{-0} = -1e^{-1} - e^{-1} + 1 = \boxed{-2e^{-1}+1}$

$\int e^{-x}\,dx$

$u = -x$

$du = -dx$

$-du = dx$

$-\int e^{u}$

$-e^{-x}$

⑩ $\int_1^2 \frac{x^2-1}{x+1}\,dx = \int_1^2 \frac{(x-1)\cancel{(x+1)}}{\cancel{(x+1)}}\,dx = \int_1^2 x-1\,dx = \frac{x^2}{2} - x\Big]_1^2$

$\left(\frac{2^2}{2} - 2\right) - \left(\frac{1^2}{2} - 1\right) = 0 - (-\tfrac{1}{2}) = \boxed{\tfrac{1}{2}}$

⑪ $\int_0^{\pi/3} \sin(3x)\,dx = \tfrac{1}{3}\int_0^{\pi/3} \sin(u)\,du = -\tfrac{1}{3}\cos 3x\Big]_0^{\pi/3}$

$u = 3x$

$du = 3dx \rightarrow \tfrac{1}{3}du = dx$

$-\tfrac{1}{3}\cos(3\cdot\tfrac{\pi}{3}) + \tfrac{1}{3}\cos(3\cdot 0) = -\tfrac{1}{3}\cos(\pi) + \tfrac{1}{3}\cos(0) = -\tfrac{1}{3}(-1) + \tfrac{1}{3}(1)$

$\tfrac{1}{3} + \tfrac{1}{3} = \boxed{\tfrac{2}{3}}$

⑫ $\int_0^{\pi/2} \frac{\cos\theta}{\sqrt{1+\sin\theta}}\,d\theta = \int_0^{\pi/2} u^{-1/2}\,du = 2u^{1/2} = 2\sqrt{1+\sin\theta}\Big]_0^{\pi/2}$

$u = 1+\sin\theta$

$du = \cos\theta\,d\theta$

$2\sqrt{1+\sin(\pi/2)} - 2\sqrt{1+\sin(0)} = 2\sqrt{1+1} - 2\sqrt{1+0} = \boxed{2\sqrt{2}-2}$

$\boxed{2(\sqrt{2}-1)}$

13) $\int_2^3 \frac{x}{x^2+1}\,dx = \frac{1}{2}\int_2^3 \frac{1}{u}\,du = \frac{1}{2}\ln|x^2+1|\Big]_2^3 = \frac{1}{2}\ln(3^2+1) - \frac{1}{2}\ln(2^2+1)$

$\frac{1}{2}\ln(10) - \frac{1}{2}\ln(5) =$

$u = x^2+1$ $\qquad \frac{1}{2}\ln(\frac{10}{5}) = \boxed{\frac{1}{2}\ln 2}$

$du = 2x\,dx \to \frac{1}{2}du = x\,dx$

14) $\int_0^{\pi/2} x\cos x\,dx = x\sin x - \int_0^{\pi/2} \sin x\,dx = x\sin x + \cos x\Big]_0^{\pi/2}$

$u = x \quad dv = \cos x\,dx$

$du = dx \quad v = \sin x$ $\qquad \frac{\pi}{2}\sin(\frac{\pi}{2}) + \cos(\frac{\pi}{2}) - 0\sin(0) - \cos(0)$

$\frac{\pi}{2}(1) + 0 - 0 - 1 = \boxed{\frac{\pi}{2} - 1} = \boxed{\frac{\pi-2}{2}}$

15) $\int_0^1 x(x^2+2)^2\,dx = \frac{1}{2}\int_0^1 u^2\,du = \frac{1}{2}\frac{u^3}{3} = \frac{(x^2+2)^3}{6}\Big]_0^1$

$u = x^2+2$

$du = 2x\,dx \to \frac{1}{2}du = x\,dx$ $\qquad \frac{(1^2+2)^3}{6} - \frac{(0^2+2)^3}{6} = \frac{3^3}{6} - \frac{2^3}{6} = \frac{27}{6} - \frac{8}{6} = \boxed{\frac{19}{6}}$

16) $\int_2^3 \frac{3}{(x-1)(x+2)}\,dx = \int_2^3 \frac{1}{x-1}\,dx + \int_2^3 \frac{-1}{x+2}\,dx = \ln|x-1| - \ln|x+2|\Big]_2^3$

$\frac{3}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$ $\qquad \ln|3-1| - \ln|3+2| - \ln|2-1| + \ln|2+2|$

$\ln|2| - \ln|5| - \ln|1| + \ln|4|$

$3 = A(x+2) + B(x-1)$ $\qquad \ln\frac{2}{5}\cdot 4 = \boxed{\ln\frac{8}{5}}$

$x = -2$ $\qquad x = 1$

$3 = A(-2+2) + B(-2-1)$ $\qquad 3 = A(1+2) + B(1-1)$

$3 = -3B$ $\qquad 3 = 3A$

$-1 = B$ $\qquad 1 = A$

17) $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}} = \arcsin\frac{x}{2}\Big]_0^{\sqrt{3}} = \arcsin\frac{\sqrt{3}}{2} - \arcsin\frac{0}{2} = \arcsin\frac{\sqrt{3}}{2} - \arcsin 0$

$= \frac{\pi}{3} - 0 = \boxed{\frac{\pi}{3}}$

$a = 2, u = x$

(18) $\int_0^1 x^3 e^{x^4}\,dx = \frac{1}{4}\int_0^1 e^u\,du = \frac{1}{4}e^{x^4}\Big]_0^1 = \frac{1}{4}e^{(1)^4} - \frac{1}{4}e^{0^4} = \frac{1}{4}e - \frac{1}{4} = \boxed{\frac{1}{4}(e-1)}$

$u = x^4$

$du = 4x^3\,dx \rightarrow \frac{1}{4}du = x^3\,dx$

(19) $\int_0^{\pi/4} \frac{e^{\tan x}}{\cos^2 x}\,dx = \int_0^{\pi/4} \sec^2 x \cdot e^{\tan x}\,dx = \int_0^{\pi/4} e^u\,du = e^{\tan x}\Big]_0^{\pi/4}$

$u = \tan x$

$du = \sec^2 x\,dx$

$= e^{\tan \pi/4} - e^{\tan 0} = e^1 - e^0 = \boxed{e-1}$

(20) $\int_0^1 \sqrt{x}\,(x+1)\,dx = \int_0^1 (x^{1/2})(x+1)\,dx = \int_0^1 x^{3/2} + x^{1/2}\,dx = \frac{2x^{5/2}}{5} + \frac{2x^{3/2}}{3}\Big]_0^1$

$\frac{2(1)^{5/2}}{5} + \frac{2(1)^{3/2}}{3} - \frac{2(0)^{5/2}}{5} - \frac{2(0)^{3/2}}{3} = \frac{2}{5} + \frac{2}{3} = \frac{6}{15} + \frac{10}{15} = \boxed{\frac{16}{15}}$

(21) $\int x^2 \sin x\,dx = \boxed{-x^2\cos x + 2x\sin x + 2\cos x + C}$

u	dv	± 1
x^2	$\sin x$	$+1$
$2x$	$-\cos x$	-1
2	$-\sin x$	$+1$
0	$\cos x$	-1
		$+1$

(22) $\int \frac{dx}{(x-1)(x+3)} = \int \frac{1/4}{x-1} + \frac{-1/4}{x+3}\,dx = \frac{1}{4}\int \frac{1}{x-1}\,dx - \frac{1}{4}\int \frac{1}{x+3}$

$\frac{1}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$

$= \boxed{\frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x+3| + C}$

$1 = A(x+3) + B(x-1)$

$= \boxed{\frac{1}{4}\ln\left|\frac{x-1}{x+3}\right| + C}$

$x = -3$

$1 = A(-3+3) + B(-3-1)$

$1 = -4B$

$-\frac{1}{4} = B$

$x = 1$

$1 = A(1+3) + B(1-1)$

$1 = A(4)$

$\frac{1}{4} = A$

$-2+1=-1 \quad \frac{4^{-1}}{-1} = -4^{-1}$

(23) $\int_1^2 \frac{1}{x^2}\,dx = \int_1^2 x^{-2}\,dx = \frac{-1}{x}\Big]_1^2 = \frac{-1}{2} + \frac{1}{1} = -\frac{1}{2}+1 = \boxed{\frac{1}{2}}$

(24) $\int_1^e \frac{x^2-1}{x}\,dx = \int_1^e (x^2-1)(x^{-1})\,dx = \int_1^e x - x^{-1}\,dx = \frac{x^2}{2} - \ln x\Big]_1^e$

$= \frac{e^2}{2} - \ln e - \frac{1^2}{2} + \ln(1) = \frac{e^2}{2} - 1 - \frac{1}{2} = \boxed{\frac{e^2}{2} - \frac{3}{2}}$

(25) $\int \frac{1}{x^2-6x+8}\,dx = \int \frac{\frac{1}{2}}{x-4} + \frac{-\frac{1}{2}}{x-2}\,dx = \boxed{\frac{1}{2}\ln|x-4| - \frac{1}{2}\ln|x-2| + C}$

$\boxed{\frac{1}{2}\ln\frac{x-4}{x-2} + C}$

$\frac{1}{(x-4)(x-2)} = \frac{A}{x-4} + \frac{B}{x-2}$

$1 = A(x-2) + B(x-4)$

$x=2$ | $x=4$
$1 = A(2-2) + B(2-4)$ | $1 = A(4-2) + B(4-4)$
$1 = -2B$ | $1 = 2A$
$-\frac{1}{2} = B$ | $\frac{1}{2} = A$