

AP[®] CALCULUS AB
2012 SCORING GUIDELINES

Question 1

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W , where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

- (a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.
- (b) Use the data in the table to evaluate $\int_0^{20} W'(t) dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t) dt$ in the context of this problem.
- (c) For $0 \leq t \leq 20$, the average temperature of the water in the tub is $\frac{1}{20} \int_0^{20} W(t) dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20} \int_0^{20} W(t) dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.
- (d) For $20 \leq t \leq 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t} \cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

$$(a) \quad W'(12) \approx \frac{W(15) - W(9)}{15 - 9} = \frac{67.9 - 61.8}{6} \\ = 1.017 \text{ (or 1.016)}$$

The water temperature is increasing at a rate of approximately 1.017 °F per minute at time $t = 12$ minutes.

$$(b) \quad \int_0^{20} W'(t) dt = W(20) - W(0) = 71.0 - 55.0 = 16$$

The water has warmed by 16 °F over the interval from $t = 0$ to $t = 20$ minutes.

$$(c) \quad \frac{1}{20} \int_0^{20} W(t) dt \approx \frac{1}{20} (4 \cdot W(0) + 5 \cdot W(4) + 6 \cdot W(9) + 5 \cdot W(15)) \\ = \frac{1}{20} (4 \cdot 55.0 + 5 \cdot 57.1 + 6 \cdot 61.8 + 5 \cdot 67.9) \\ = \frac{1}{20} \cdot 1215.8 = 60.79$$

This approximation is an underestimate, because a left Riemann sum is used and the function W is strictly increasing.

$$(d) \quad W(25) = 71.0 + \int_{20}^{25} W'(t) dt \\ = 71.0 + 2.043155 = 73.043$$

2 : $\begin{cases} 1 : \text{estimate} \\ 1 : \text{interpretation with units} \end{cases}$

2 : $\begin{cases} 1 : \text{value} \\ 1 : \text{interpretation with units} \end{cases}$

3 : $\begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{approximation} \\ 1 : \text{underestimate with reason} \end{cases}$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$