

$$\textcircled{1} \int_1^{5e} \frac{1}{x} dx$$

$$\ln x \Big|_1^{5e}$$

$$\ln 5e - \ln 1$$

$$\boxed{\ln 5e}$$

$$\textcircled{2} \int_2^3 \frac{dx}{(x-1)^2}$$

$$u = x-1$$

$$du = dx$$

$$\int u^{-2} du$$

$$-\frac{1}{u} \Big|_2^3$$

$$-\frac{1}{3} + \frac{1}{2}$$

$$\boxed{\frac{1}{2}}$$

$$\textcircled{3} \int_1^2 \frac{dx}{x\sqrt{x^2-1}}$$

$$\operatorname{arccsc} x \Big|_1^2$$

$$\operatorname{arccsc} 2 - \operatorname{arccsc} 1$$

$$\frac{\pi}{3} - 0$$

$$\frac{\pi}{3}$$

~~$$\textcircled{4} \int_3^4 \frac{x^2-1}{x-2} dx$$~~

~~$$\int_3^4 \frac{x^2}{x-2} - \int_3^4 \frac{1}{x-2} dx$$~~

~~$$u = x-2$$~~

~~$$du = dx$$~~

~~$$x = u+2$$~~

~~$$\int \frac{(u+2)^2}{u} du = \ln|x-2| \Big|_3^4$$~~

~~$$\frac{u^2+4u+4}{u}$$~~

~~$$\int u+4+\frac{4}{u} = \ln|x-2| \Big|_3^4$$~~