$\int_0^{\frac{\pi}{2}} \frac{\cos x\,dx}{1+\sin^2 x}$ $\quad u=\sin x$ $\quad du=\cos x\,dx$

$\int \frac{du}{1+u^2}$ $\quad a=1$

$\frac{1}{a}\arctan\frac{u}{a} = \arctan(\sin x)\Big|_0^{\frac{\pi}{2}}$

$\arctan(\sin\frac{\pi}{2}) - \arctan(\sin 0)$

$\arctan(1) - \arctan(0)$

$\frac{\pi}{4} - 0$

$\boxed{\frac{\pi}{4}}$

⑥ $\int_0^1 x10^{x^2}\,dx$ $\quad u=x^2$ $\quad du=2x\,dx$ $\quad \frac{1}{2}du=x\,dx$

$\int 10^u\,du$

$= \frac{10^u}{\ln 10}$

$\frac{1}{2} = \frac{10^{x^2}}{\ln 10}\Big|_0^1$

$= \frac{10}{2\ln 10} - \frac{10^0}{2\ln 10} = \frac{10}{2\ln 10} - \frac{1}{2\ln 10}$

$\boxed{= \frac{9}{2\ln 10}} \rightarrow \frac{9}{\ln 100}$

⑦ $\int_0^4 \frac{t}{\sqrt{9+t^2}}\,dt$ $\quad u=9+t^2$ $\quad du=2t\,dt$ $\quad \frac{1}{2}du=t\,dt$

$\frac{1}{2}\int \frac{du}{\sqrt{u}}$

$\frac{1}{2}\int u^{-\frac{1}{2}}\,du$

$\frac{1}{2}\cdot\frac{2}{1}u^{\frac{1}{2}} = \sqrt{9+t^2}\Big|_0^4$

$\sqrt{9+16} - \sqrt{9+0}$

$\sqrt{25} - \sqrt{9}$

$5-3 = \boxed{2}$

⑧ $\int_0^{\frac{\pi}{2}} e^{\cos x}\sin x\,dx$ $\quad u=\cos x$ $\quad du=-\sin x\,dx$ $\quad -du=\sin x\,dx$

$-\int e^u\,du$

$-e^{\cos x}\Big|_0^{\frac{\pi}{2}}$

$-e^{\cos\frac{\pi}{2}} + e^{\cos 0}$

$-e^0 + e^1$

$\boxed{-1+e}$