AP Calculus Review - Integration Part I

1. a) $1\left[f(0)+f(1)+f(2)\right]$

$1\left[1+2+5\right]$

$\boxed{8}$

b) $1\left[f(0)+f(0)+f(1)\right]$

$1\left[1+1+2\right]$

$\boxed{4}$

c) $1\left[f(-\frac{1}{2})+f(\frac{1}{2})+f(\frac{3}{2})\right]$

$1\left[\frac{5}{4}+\frac{5}{4}+\frac{13}{4}\right]$

$\boxed{\frac{23}{4}}$

d) $\frac{3}{6}\left[f(-1)+2f(0)+2f(1)+2f(2)\right]$

$\frac{1}{2}\left[2+2(1)+2(2)+5\right]$

$\frac{1}{2}\left[2+2+4+5\right]$

$\boxed{\frac{13}{2}}$

e) $\int_{-1}^{2} 1+x^2\,dx$

$\left[x+\frac{x^3}{3}\right]_{-1}^{2}$

$2+\frac{8}{3}-\left(-1-\frac{1}{3}\right)$

$3+\frac{9}{3}$

$\boxed{6}$

1 5 3 x 1

2. $\frac{1}{2}(1)(5+11) + \frac{1}{2}(3)(11+8) + \frac{1}{2}(2)(18+14)$

$8 + \frac{87}{2} + 32$

$\frac{40+87}{2}$

$\boxed{\frac{167}{2}}$

3. $f(x) = \frac{1}{x+1}$

$f'(x) = \frac{-1}{(x+1)^2}$

$f''(x) = \frac{(x+1)^2(0) - (-1)2(x+1)}{(x+1)^4}$

$= \frac{2(x+1)}{(x+1)^4}$

$= \frac{2}{(x+1)^3}$

on $[0,1]$

$E \leq \frac{(1-0)^3}{12(4)^2}$

$E \leq \frac{1}{192} \max \left| \frac{2}{(x+1)^3} \right|$

$E \leq \frac{1}{192}(2)$

$E \leq \frac{1}{96}$

$\left| \frac{2}{(x+1)^3} \right|$ is decreasing from $x=0$ to $x=1$ so max is at $x=0$

max value = 2

4. $f(x) = (x+1)^{2/3}$

$f'(x) = \frac{2}{3}(x+1)^{-1/3}$

$f''(x) = -\frac{2}{9}(x+1)^{-4/3}$

$\left|-\frac{2}{9}(x+1)^{-4/3}\right|$ ✓

decreasing from $x=0$ to $x=2$

do not D at $x=0$

max value $= .222$

$E \le \frac{(2-0)^3}{12n^2} \max\left|-\frac{2}{9}(x+1)^{-4/3}\right|$

$E < \frac{8}{12n^2}(.222)$

$E \le \left(\frac{4}{3n^2}\right)(.222) < \frac{1}{100{,}000}$

$\frac{.888}{3n^2} < \frac{1}{100{,}000}$

$88800 < 3n^2$

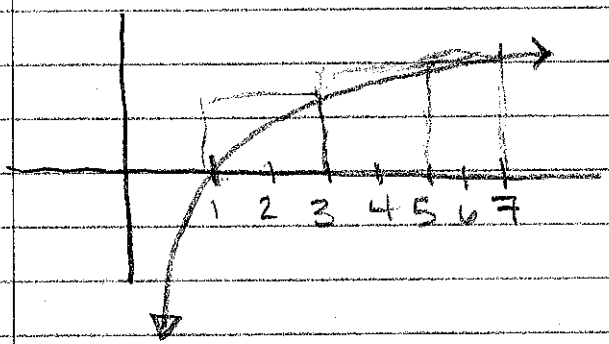
$29600 < n^2$

$172.047 < n$

173 trapezoids are needed

Multiple Choice

[C] 1.



$2(\ln 3 + \ln 5 + \ln 7)$

[B] 2. $\frac{1}{2}(1)(2) + (1)(2) + \frac{1}{2}(1)(2) - \frac{1}{2}(1)(1) - (1)(1)$

$1 + 2 + 1 - \frac{1}{2} - 1$

$\frac{5}{2}$

$\frac{x}{5} \quad -\frac{\pi}{3} \quad -\frac{1}{x} + \cdots$

[A] 3. $f''(x) = 0$

Area under $0 \geq 0$

[C] 4. $\int_0^c (6x^2 - 3)\,dx = 10$

$$\frac{6x^3}{3} - 3x \Big]_0^c = 10$$

$$2c^3 - 3c - 0 = 10$$

$$2c^3 - 3c = 10$$

$$2c^3 - 3c - 10 = 0$$

$$c = 2$$

[C] 5. Definition of definite integral

L. Integration Rules

1. $\int_2^5 f(x)\,dx = \int_2^1 f(x) - \int_5^1 f(x)$

$= -5 - -6$

$= \boxed{1}$

2. D

Basic Integration

1. $\int \frac{1}{x^4}\,dx$

$= \int x^{-4}\,dx$

$= \frac{x^{-3}}{-3} + C$

$= \boxed{-\frac{1}{3x^3} + C}$

2. $\int 5x^{-1/2}$

$\frac{5x^{1/2}}{1/2}$

$\boxed{10x^{1/2} + C}$

3. $\int x^3 + 7x^{-2}$

$\frac{x^4}{4} + \frac{7x^{-1}}{-1}$

$\boxed{\frac{x^4}{4} - \frac{7}{x} + C}$

4. $\int (5x^4 - 3x^2 + 2x + 6)\,dx$

$\frac{5x^5}{5} - \frac{3x^3}{3} + \frac{2x^2}{2} + 6x$

$\boxed{x^5 - x^3 + x^2 + 6x + C}$

5. $\int (3x^{-3} - 2x^2 + x^4 + 16x^7)\,dx$

$\frac{3x^{-2}}{-2} - \frac{2x^3}{3} + \frac{x^5}{5} + \frac{16x^8}{8}$

$\boxed{-\frac{3}{2x^2} - \frac{2x^3}{3} + \frac{x^5}{5} + 2x^8 + C}$

6. $\int (x - 2 + x^3 - 2x^2)\,dx$

$\boxed{\frac{x^2}{2} - 2x + \frac{x^4}{4} - \frac{2x^3}{3} + C}$

7. $\int (2x^{1/3} + x^{4/3})\,dx$

$\frac{2x^{4/3}}{4/3} + \frac{x^{7/3}}{7/3}$

$\frac{6x^{4/3}}{4} + \frac{3x^{7/3}}{7}$

$\boxed{\frac{3x^{4/3}}{2} + \frac{3x^{7/3}}{7} + C}$

8. $\int (x^6 + 2x^4 + x^2)\,dx$

$\boxed{\frac{x^7}{7} + \frac{2x^5}{5} + \frac{x^3}{3} + C}$

9. $\int (x^4 - 2x^2 + x^{-2})\,dx$

$\frac{x^5}{5} - \frac{2x^3}{3} + \frac{x^{-1}}{-1}$

$\boxed{\frac{x^5}{5} - \frac{2x^3}{3} - \frac{1}{x} + C}$

10. $\int x(x-1)^3\,dx$

$\int x(x^3+3x-3x^2-1)\,dx$

$\int(x^4-3x^2+3x^3-x)\,dx$

$\frac{x^5}{5}+\frac{3x^3}{3}+\frac{3x^4}{4}-\frac{x^2}{2}$

$\boxed{\frac{x^5}{5}+\frac{3}{4}x^4+x^3-\frac{x^2}{2}+C}$

11. $\int(\cos x-5\sin x)\,dx$

$\sin x-5(-\cos x)+C$

$\boxed{\sin x+5\cos x+C}$

12. $\int(\sec^2 x+\sec x\tan x)\,dx$

$\boxed{\tan x+\sec x+C}$

13. $\int(\sec^2 x+x)\,dx$

$\boxed{\tan x+\frac{x^2}{2}+C}$

14. $\int\frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}\,dx$

$\int\tan x\sec x\,dx$

$\boxed{\sec x+C}$

15. $\int_0^{\pi}(4\sin\theta-3\cos\theta)\,d\theta$

$-4\cos\theta-3\sin\theta\Big]_0^{\pi}$

$(-4\cos\pi+3\sin\pi)-(-4\cos 0-3\sin 0)$

$(4-0)-(-4-0)$

$4+4$

$\boxed{8}$

16. $\int_0^{\pi}\sec^2 x$

$\tan x\Big]_0^{\pi}$

$\tan\pi-\tan 0$

$0-0$

$\boxed{0}$

17. $\int_0^1\left(x^{4/3}+x^{5/4}\right)dx$

$\frac{3x^{7/3}}{7}+\frac{4x^{9/4}}{9}\Big]_0^1$

$\left(\frac{3}{7}+\frac{4}{9}\right)-0$

$\boxed{\frac{55}{63}}$

18. $\int_{\pi/6}^{\pi/2} \csc x \cot x \, dx$

$-\csc x \Big]_{\pi/6}^{\pi/2}$

$-\csc \pi/2 - (-\csc \pi/6)$

$-1 - -2$

$-1 + 2$

$\boxed{1}$

19. $\int_0^2 6x^2 - 4x + 5 \, dx$

$\frac{6x^3}{3} - \frac{4x^2}{2} + 5x$

$2x^3 - 2x^2 + 5x \Big]_0^2$

$2(8) - 2(4) + 5(2) - 0$

$16 - 8 + 10$

$\boxed{18}$

20. $\int_1^2 (x^{-2} + 5x^4) \, dx$

$\frac{x^{-1}}{-1} + \frac{5x^5}{5} \Big]_1^2$

$\frac{-1}{x} + x^5 \Big]_1^2$

$-\frac{1}{2} + 32 - (-1 + 1)$

$\boxed{\frac{63}{2}}$