## EXERCISES FOR SECTION 5.7

In Exercises 1–6, verify the solution of the differential equation.

| Solution | Differential Equation |
|---|---|
| 1. $y = Ce^{4x}$ | $y' = 4y$ |
| 2. $x^2 + y^2 = Cy$ | $y' = 2xy/(x^2 - y^2)$ |
| 3. $y = C_1 \cos x + C_2 \sin x$ | $y'' + y = 0$ |
| 4. $y = C_1 e^{-x} \cos x + C_2 e^{-x} \sin x$ | $y'' + 2y' + 2y = 0$ |
| 5. $y = -\cos x \ln\lvert\sec x + \tan x\rvert$ | $y'' + y = \tan x$ |
| 6. $y = \frac{2}{3}(e^{-2x} + e^x)$ | $y'' + 2y' = 2e^x$ |

In Exercises 7–12, determine whether the function is a solution of the differential equation $y^{(4)} - 16y = 0$.

7. $y = 3 \cos x$
8. $y = 3 \cos 2x$
9. $y = e^{-2x}$
10. $y = 5 \ln x$
11. $y = C_1 e^{2x} + C_2 e^{-2x} + C_3 \sin 2x + C_4 \cos 2x$
12. $y = 3e^{2x} - 4 \sin 2x$

In Exercises 13–18, determine whether the function is a solution of the differential equation $xy' - 2y = x^3 e^x$.

13. $y = x^2$
14. $y = x^2 e^x$
15. $y = x^2(2 + e^x)$
16. $y = \sin x$
17. $y = \ln x$
18. $y = x^2 e^x - 5x^2$

19. *Think About It* It is known that $y = Ce^{kx}$ is a solution of the differential equation $y' = 0.07y$. Is it possible to determine $C$ or $k$ from the information given? If so, find its value.

20. *Think About It* It is known that $y = A \sin \omega t$ is a solution of the differential equation $y'' + 16y = 0$. Find the value of $\omega$.

In Exercises 21 and 22, some of the curves corresponding to different values of $C$ in the general solution of the differential equation are given. Find the particular solution that passes through the point indicated on the graph.

| Solution | Differential Equation |
|---|---|
| 21. $y^2 = Cx^3$ | $2xy' - 3y = 0$ |
| 22. $2x^2 - y^2 = C$ | $yy' - 2x = 0$ |

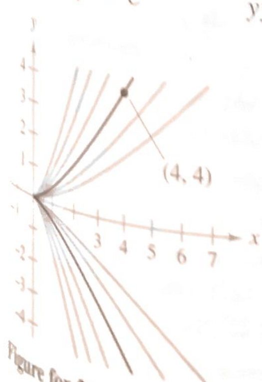


Figure for 21

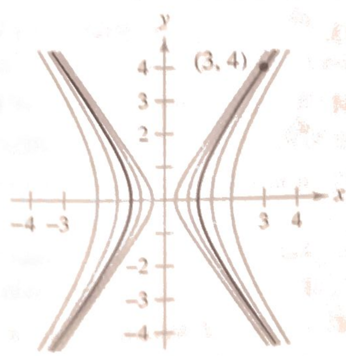


Figure for 22

In Exercises 23 and 24, the general solution of the differential equation is given. Use a graphing utility to graph the particular solutions for the given values of $C$.

23. $4yy' - x = 0$
$4y^2 - x^2 = C$
$C = 0, C = \pm 1, C = \pm 4$

24. $yy' + x = 0$
$x^2 + y^2 = C$
$C = 0, C = 1, C = 4$

In Exercises 25–30, verify that the general solutions satisfy the differential equation. Then find the particular solution that satisfies the initial condition.

25. $y = Ce^{-2x}$
$y' + 2y = 0$
$y = 3$ when $x = 0$

26. $3x^2 + 2y^2 = C$
$3x + 2yy' = 0$
$y = 3$ when $x = 1$

27. $y = C_1 \sin 3x + C_2 \cos 3x$
$y'' + 9y = 0$
$y = 2$ when $x = \pi/6$
$y' = 1$ when $x = \pi/6$

28. $y = C_1 + C_2 \ln x$
$xy'' + y' = 0$
$y = 0$ when $x = 2$
$y' = \frac{1}{2}$ when $x = 2$

29. $y = C_1 x + C_2 x^3$
$x^2 y'' - 3xy' + 3y = 0$
$y = 0$ when $x = 2$
$y' = 4$ when $x = 2$

30. $y = e^{2x/3}(C_1 + C_2 x)$
$9y'' - 12y' + 4y = 0$
$y = 4$ when $x = 0$
$y = 0$ when $x = 3$

In Exercises 31–42, use integration to find a general solution of the differential equation.

31. $\dfrac{dy}{dx} = 3x^2$
32. $\dfrac{dy}{dx} = x^3 - 4x$
33. $\dfrac{dy}{dx} = \dfrac{x}{1 + x^2}$
34. $\dfrac{dy}{dx} = \dfrac{e^x}{1 + e^x}$
35. $\dfrac{dy}{dx} = \dfrac{x - 2}{x}$
36. $\dfrac{dy}{dx} = x \cos x^2$
37. $\dfrac{dy}{dx} = \sin 2x$
38. $\dfrac{dy}{dx} = \tan^2 x$
39. $\dfrac{dy}{dx} = x\sqrt{x - 3}$
40. $\dfrac{dy}{dx} = x\sqrt{5 - x}$
41. $\dfrac{dy}{dx} = xe^{x^2}$
42. $\dfrac{dy}{dx} = 5e^{-x/2}$

In Exercises 43–54, find the general solution of the differential equation.

43. $\dfrac{dy}{dx} = \dfrac{x}{y}$
44. $\dfrac{dy}{dx} = \dfrac{x^2 + 2}{3y^2}$
45. $\dfrac{dr}{ds} = 0.05r$
46. $\dfrac{dr}{ds} = 0.05s$
47. $(2 + x)y' = 3y$
48. $xy' = y$
49. $yy' = \sin x$
50. $yy' = 6 \cos(\pi x)$
51. $\sqrt{1 - 4x^2}\, y' = x$
52. $\sqrt{x^2 - 9}\, y' = 5x$
53. $y \ln x - xy' = 0$
54. $4yy' - 3e^x = 0$

In Exercises 55–64, find the particular solution that satisfies the initial condition.

| *Differential Equation* | *Initial Condition* |
|---|---|
| 55. $yy' - e^x = 0$ | $y(0) = 4$ |
| 56. $\sqrt{x} + \sqrt{y}\,y' = 0$ | $y(1) = 4$ |
| 57. $y(x + 1) + y' = 0$ | $y(-2) = 1$ |
| 58. $2xy' - \ln x^2 = 0$ | $y(1) = 2$ |
| 59. $y(1 + x^2)y' - x(1 + y^2) = 0$ | $y(0) = \sqrt{3}$ |
| 60. $y\sqrt{1 - x^2}\,y' - x\sqrt{1 - y^2} = 0$ | $y(0) = 1$ |
| 61. $\dfrac{du}{dv} = uv \sin v^2$ | $u(0) = 1$ |
| 62. $\dfrac{dr}{ds} = e^{r-2s}$ | $r(0) = 0$ |
| 63. $dP - kP\,dt = 0$ | $P(0) = P_0$ |
| 64. $dT + k(T - 70)\,dt = 0$ | $T(0) = 140$ |

In Exercises 65 and 66, find an equation of the graph that passes through the point and has the indicated slope.

| *Point* | *Slope* |
|---|---|
| 65. (1, 1) | $y' = -\dfrac{9x}{16y}$ |
| 66. (8, 2) | $y' = \dfrac{2y}{3x}$ |

In Exercises 67 and 68, find all functions *f* having the indicated property.

67. The tangent to the graph of $f$ at the point $(x, y)$ intersects the $x$-axis at $(x + 2, 0)$.

68. All tangents to the graph of $f$ pass through the origin.

In Exercises 69–76, determine whether the function is homogeneous, and if it is, determine its degree.

69. $f(x, y) = x^3 - 4xy^2 + y^3$

70. $f(x, y) = x^3 + 3x^2y^2 - 2y^2$

71. $f(x, y) = \dfrac{x^2y^2}{\sqrt{x^2 + y^2}}$

72. $f(x, y) = \dfrac{xy}{\sqrt{x^2 + y^2}}$

73. $f(x, y) = 2 \ln xy$

74. $f(x, y) = \tan(x + y)$

75. $f(x, y) = 2 \ln \dfrac{x}{y}$

76. $f(x, y) = \tan \dfrac{y}{x}$

In Exercises 77–82, solve the homogeneous differential equation.

77. $y' = \dfrac{x + y}{2x}$

78. $y' = \dfrac{x^3 + y^3}{xy^2}$

79. $y' = \dfrac{x - y}{x + y}$

80. $y' = \dfrac{x^2 + y^2}{2xy}$

81. $y' = \dfrac{xy}{x^2 - y^2}$

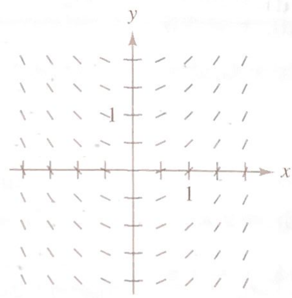
82. $y' = \dfrac{2x + 3y}{x}$

In Exercises 83–86, find the particular solution that satisfies the initial condition.

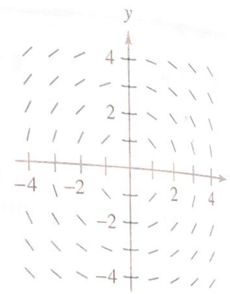
| *Differential Equation* | *Initial Condition* |
|---|---|
| 83. $x\,dy - (2xe^{-y/x} + y)\,dx = 0$ | $y(1) = 0$ |
| 84. $-y^2\,dx + x(x + y)\,dy = 0$ | $y(1) = 1$ |
| 85. $\left(x \sec \dfrac{y}{x} + y\right)dx - x\,dy = 0$ | $y(1) = 0$ |
| 86. $(2x^2 + y^2)\,dx + xy\,dy = 0$ | $y(1) = 0$ |

*Slope Fields* In Exercises 87–90, sketch a few solutions of the differential equation on the slope field and then find the general solution analytically. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

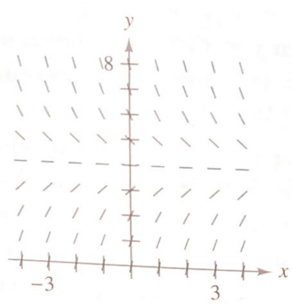
87. $\dfrac{dy}{dx} = x$



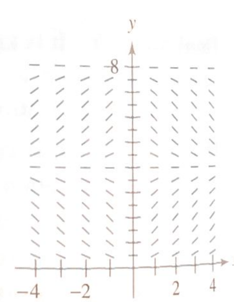
88. $\dfrac{dy}{dx} = -\dfrac{x}{y}$



89. $\dfrac{dy}{dx} = 4 - y$



90. $\dfrac{dy}{dx} = 0.25x(4 - y)$



In Exercises 91–94, use a computer algebra system to sketch the slope field for the differential equation, and graph the solution satisfying the specified initial condition.

91. $\dfrac{dy}{dx} = 0.5y, \; y(0) = 6$

92. $\dfrac{dy}{dx} = 2 - y, \; y(0) = 4$

93. $\dfrac{dy}{dx} = 0.02y(10 - y), \; y(0) = 2$

94. $\dfrac{dy}{dx} = 0.2x(2 - y), \; y(0) = 9$

95. ***Radioactive Decay*** The rate of decomposition of radioactive radium is proportional to the amount present at any time. The half-life of radioactive radium is 1620 years. What percent of a present amount will remain after 25 years?

96. ***Chemical Reaction*** In a chemical reaction, a certain compound changes into another compound at a rate proportional to the unchanged amount. If initially there is 20 grams of the original compound, and there is 16 grams after 1 hour, when will 75 percent of the compound be changed?