1976 AB7/BC6
Solution

(a) For $f(x) = x^2 + x$

$$f^*(x) = \lim_{h\to 0} \frac{\left((x+h)^2 + (x+h)\right) - \left((x-h)^2 + (x-h)\right)}{h}$$

$$= \lim_{h\to 0} \frac{(x^2 + 2xh + h^2 + x + h) - (x^2 - 2xh + h^2 + x - h)}{h}$$

$$= \lim_{h\to 0} \frac{4xh + 2h}{h}$$

$$= 4x + 2 = 2(2x+1)$$

(b) For $f(x) = \cos x$

$$f^*(x) = \lim_{h\to 0} \frac{\cos(x+h) - \cos(x-h)}{h}$$

ANGLE ANGLE SUM IDENTITIES

$$= \lim_{h\to 0} \frac{(\cos x \cos h - \sin x \sin h) - (\cos x \cos h + \sin x \sin h)}{h}$$

$$= \lim_{h\to 0} \frac{-2\sin x \sin h}{h}$$

$$= -2\sin x \cdot \lim_{h\to 0} \frac{\sin h}{h}$$

$$= -2\sin x \cdot 1 = -2\sin x$$

(c) $f^*(x) = 2f'(x)$

① $f(x) = \frac{1}{\sqrt{x}}$

$$\lim_{h\to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} = \lim_{h\to 0} \frac{\frac{\sqrt{x}-\sqrt{x+h}}{(\sqrt{x})(\sqrt{x+h})}}{h} = \lim_{h\to 0} \frac{\sqrt{x}-\sqrt{x+h}}{(\sqrt{x})(\sqrt{x+h})h}\left(\frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}\right)$$

$$= \lim_{h\to 0} \frac{\cancel{x}-\cancel{x}-\cancel{h}}{\cancel{h}(\sqrt{x})(\sqrt{x+h})(\sqrt{x}+\sqrt{x+h})} = \frac{-1}{(\sqrt{x})(\sqrt{x+0})(\sqrt{x}+\sqrt{x+0})} = \frac{-1}{(\sqrt{x})(\sqrt{x})(\sqrt{x}+\sqrt{x})} = \frac{-1}{x(2\sqrt{x})} = \boxed{\frac{-1}{2x\sqrt{x}}}$$

② $g(2)=3;\ g'(2)=-2;\ h(2)=-1;\ h'(2)=4$

a) $\frac{d}{dx}[2g(2)-3h(2)] = 2g'(2)-3h'(2) = 2(-2)-3(4) = -2-12 = \boxed{-14}$

b) $\frac{d}{dx}[g(2)h(2)] = g(2)h'(2)+g'(2)h(2) = 3(4)+(-2)(-1) = 12+2 = \boxed{14}$

c) $\frac{d}{dx}\left[\frac{h(2)}{g(2)}\right] = \frac{g(2)h'(2)-h(2)g'(2)}{(g(2))^2} = \frac{(3)(-1)-(4)(-2)}{(3)^2} = \frac{-3+8}{9} = \boxed{\frac{5}{9}}$

1 ③ a) $f(x) = x^2 - \frac{3}{x^3} = x^2 - 3x^{-3}$

$\cancel{f'(x) = 2x+9x^{-4}}$

$\boxed{f'(x) = 2x + \frac{9}{x^4}}$

b) $h(x) = x^{4/5} - x^{2/3}$

$h'(x) = 4/5\, x^{-1/5} - 2/3\, x^{-1/3}$

$\boxed{h'(x) = \frac{4}{5x^{1/5}} - \frac{2}{3x^{1/3}}}$

④ $f(x) = (3x^2-5)(2x^4-x)$

$f'(x) = (3x^2-5)(8x^3-1) + (6x)(2x^4-x)$

$f'(x) = 24x^5 - 3x^2 - 40x^3 + 5 + 12x^5 - 6x^2$

$\boxed{f'(x) = 36x^5 - 40x^3 - 9x^2 + 5}$

⑤ $f(x) = \frac{3x-5}{x^2+7}$

$f'(x) = \frac{(x^2+7)(3)-(3x-5)(2x)}{(x^2+7)^2}$

$f'(x) = \frac{3x^2+21-6x^2+10x}{(x^2+7)^2}$

$\boxed{f'(x) = \frac{-3x^2+10x+21}{(x^2+7)^2}}$

⑥ $f(x) = (2x-1)(x^2+3)$ @ $x=-1$

$f'(x) = (2x-1)(2x) + (2)(x^2+3)$

$f'(x) = 4x^2 - 2x + 2x^2 + 6$

$f'(x) = 6x^2 - 2x + 6$

$f'(-1) = 6(-1)^2 - 2(-1) + 6$

$= 6 + 2 + 6 = 14$

$f(-1) = (2(-1)-1)((-1)^2+3)$

$= (-3)(4) = -12$

$\boxed{y + 12 = 14(x+1)}$

⑦ $f(x) = \frac{x-1}{x^2-1}$

$\boxed{f(x) \text{ HAS A HORIZONTAL TANGENT @ } x=1}$

$f'(x) = \frac{(x^2-1)(1) - (x-1)(2x)}{(x^2-1)^2}$

$f'(x) = \frac{x^2 - 1 - 2x^2 + 2x}{(x^2-1)^2}$

$f'(x) = \frac{-x^2 + 2x - 1}{(x^2-1)^2} = \frac{-1(x^2-2x+1)}{(x^2-1)^2}$

$x^2 - 2x + 1 = 0$

$(x-1)(x-1) = 0$

$x = 1$

⑧ $h(t) = -16t^2 + 1362$

$h'(t) = -32t$

a) AVG. VELOCITY FROM $t=1$ to $t=3$

$\frac{h(3) - h(1)}{3-1} = \frac{1218 - 1346}{2} = \boxed{-64 \text{ ft/sec}}$

OR

$\frac{h'(1) + h'(3)}{2} = \boxed{-64 \text{ ft/sec}}$

b) $-16t^2 + 1362 = 0$

$-16t^2 = -1362$

$t^2 = 85.125$

$t = \pm 9.226$

$\boxed{\text{HITS THE GROUND @ } 9.226 \text{ sec}}$

c) $h'(9.226) = -32(9.226)$

$= \boxed{-295.232 \text{ ft/sec}}$

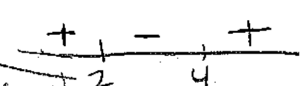
⑨ $S = t^3 - 9t^2 + 24t$

a) $S' = 3t^2 - 18t + 24$

$0 = 3t^2 - 18t + 24$

$3(t^2 - 6t + 8)$

$3(t-4)(t-2) \Rightarrow t=4, t=2$



MOVING TO THE LEFT $(2,4)$ & MOVING TO THE RIGHT $(-\infty,2) \cup (4,\infty)$

b) $S(0) = 0$ > 20
$S(2) = 20$ > 4
$S(4) = 16$ > 20
$S(6) = 36$

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pg 153 # 1,3,5,6, 11 & 12 (part a), 17-31 odd, 39, 41-57 odd, 58, 59, 61, 89, 91

REWRITE AS $2x^2 + \sin x$

① $f(x) = x^2 - 2x + 3$

$\lim_{h\to 0} \frac{(x+h)^2 - 2(x+h) + 3 - (x^2 - 2x + 3)}{h} = \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{2x} - 2h + \cancel{3} - \cancel{x^2} + \cancel{2x} - \cancel{3}}{h}$

$\lim_{h\to 0} \frac{\cancel{h}(2x + h - 2)}{\cancel{h}} = 2x + 0 - 2 = \boxed{2x - 2}$

③ $f(x) = \sqrt{x} + 1$

$\lim_{h\to 0} \frac{\sqrt{x+h} + 1 - (\sqrt{x} + 1)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h} \cancel{+1} - \sqrt{x} \cancel{-1}}{h} = \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}\left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}\right)$

$\lim_{h\to 0} \frac{\cancel{x} + \cancel{h} - \cancel{x}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})} = \frac{1}{\sqrt{x+0} + \sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}}}$

⑤ $f(x) = (x+1)^{2/3}$

$f(x)$ IS DIFFERENTIABLE

$(-\infty, 1) \cup (1, \infty)$

⑥ $f(x) = \frac{4x}{x+3}$

$f(x)$ IS DIFFERENTIABLE

$(-\infty, 3) \cup (-3, \infty)$

⑪ $f(x) = x^3 - 1$ @ $(-1, -2)$

$f'(x) = 3x^2$

$f'(-1) = 3(-1)^2 = 3$

$\boxed{y + 2 = 3(x+1)}$

⑫ $f(x) = \frac{2}{x+1}$ @ $(0, 2)$

$f'(x) = \frac{(x+1)(0) - (2)(1)}{(x+1)^2}$

$f'(x) = -2/(x+1)^2$

$f'(0) = -2/(0+1)^2 = -2$

$y - 2 = -2(x - 0)$

$\boxed{y = -2x + 2}$

⑰ $y = 25$

$\boxed{y' = 0}$

⑲ $f(x) = x^8$

$\boxed{f'(x) = 8x^7}$

㉑ $h(t) = 3t^4$

$\boxed{h'(t) = 12t^3}$

㉓ $f(x) = x^3 - 3x^2$

$\boxed{f'(x) = 3x^2 - 6x}$

㉕ $h(x) = 6\sqrt{x} + 3\sqrt[3]{x}$

$h(x) = 6x^{1/2} + 3x^{1/3}$

$h'(x) = 3x^{-1/2} + x^{-2/3}$

$\boxed{h'(x) = \frac{3}{\sqrt{x}} + \frac{1}{\sqrt[3]{x^2}}}$

(27) $g(t) = \frac{2}{3t^2}$

$g(t) = \frac{2}{3}t^{-2}$

$g'(t) = -\frac{4}{3}t^{-3}$

$\boxed{g'(t) = \frac{-4}{3t^3}}$

(29) $f(\theta) = 2\theta - 3\sin\theta$

$\boxed{f'(\theta) = 2 - 3\cos\theta}$

(31) $f(\theta) = 3\cos\theta - \frac{\sin\theta}{4}$

$f(\theta) = 3\cos\theta - \frac{1}{4}\sin\theta$

$\boxed{f'(\theta) = -3\sin\theta - \frac{1}{4}\cos\theta}$

(39) $x(t) = t^2 - 3t + 2$

a) $v(t) = 2t - 3$

b) $2t - 3 = 0$

$t = 3/2$

$+ \mid -$ at $3/2$

$(3/2, \infty) \leftarrow$ MOVING LEFT

c) $x(3/2) = (3/2)^2 - 3(3/2) + 2$

$9/4 - 9/2 + 2$

$9/4 - 18/4 + 8/4 = \boxed{-1/4}$

d) $0 = t^2 - 3t + 2$

$(t-2)(t-1)$

$t = 2,\ t = 1$

$|v(2)| = |2(2) - 3| = \boxed{1}$

$|v(1)| = |2(1) - 3| = \boxed{1}$

(41) $f(x) = (3x^2+7)(x^2-2x+3)$

$\boxed{f'(x) = (6x)(x^2-2x+3) + (3x^2+7)(2x-2)}$

$6x^3 - 12x^2 + 18x + 6x^3 - 6x^2 + 14x - 14$

$\boxed{12x^3 - 18x^2 + 32x - 14}$

(43) $h(x) = \sqrt{x}\sin x$

$h(x) = x^{1/2}\sin x$

$h'(x) = \frac{1}{2}x^{-1/2}\sin x + x^{1/2}\cos x$

$\boxed{h'(x) = \frac{1}{2\sqrt{x}}\sin x + \sqrt{x}\cos x}$

(45) $f(x) = \frac{2x^3-1}{x^2}$

$f'(x) = \frac{(x^2)(6x^2) - (2x-1)(2x)}{x^4}$

$\boxed{f'(x) = \frac{6x^4 - 2x^2 + 2x}{x^4}}$

(47) $f(x) = \frac{x^2+x-1}{x^2-1}$

$f'(x) = \frac{(x^2-1)(2x+1) - (x^2+x-1)(2x)}{(x^2-1)^2}$

$f'(x) = \frac{\cancel{2x^3} + x^2 - \cancel{2x} - 1 - \cancel{2x^3} - 2x^2 + \cancel{2x}}{(x^2-1)^2}$

$\boxed{f'(x) = \frac{-x^2-1}{(x^2-1)^2}}$

(49) $f(x) = \frac{1}{4-3x^2}$

$f'(x) = \frac{(4-3x^2)(0) - (1)(-6x)}{(4-3x^2)^2}$

$\boxed{f'(x) = \frac{6x}{(4-3x^2)^2}}$

(51) $y = \frac{x^2}{\cos x}$

$y' = \frac{\cos x(2x) - (x^2)(-\sin x)}{(\cos x)^2}$

$\boxed{y' = \frac{2x\cos x + x^2\sin x}{\cos^2 x}}$

(53) $y = 3x^2 \sec x$

$\boxed{y' = 3x^2 \sec x \tan x + 6x \sec x}$

$\boxed{y' = 3x \sec x\,(x \tan x + 3)}$

(55) $y = -x \tan x$

$y' = -x(\sec^2 x) + (-1)(\tan x)$

$\boxed{y' = -x \sec^2 x - \tan x}$

(57) $y = x \cos x - \sin x$

$y' = x(-\sin x) + (1)(\cos x) - \cos x$

$y' = -x \sin x + \cos x - \cos x$

$\boxed{y' = -x \sin x}$

(58) $v(t) = 36 - t^2$

$v(4) = 36 - (4)^2 = 36 - 16 = \boxed{20}$

$a(t) = -2t$

$a(4) = -2(4) = \boxed{-8}$

(59) $g(t) = t^3 - 3t + 2$

$g'(t) = 3t^2 - 3$

$\boxed{g''(t) = 6t}$

(61) $f(\theta) = 3 \tan \theta$

$f'(\theta) = 3 \sec^2 \theta$

$f'(\theta) = 3 \sec \theta \sec \theta$

$f''(\theta) = 3 \sec \theta (\sec \theta \tan \theta) + 3(\sec \theta \tan \theta)(\sec \theta)$

$f''(\theta) = 3 \sec^2 \theta \tan \theta + 3 \sec^2 \theta \tan \theta$

$\boxed{f''(\theta) = 6 \sec^2 \theta \tan \theta}$

(89) $y = 2x^2 + \sin x$

$y' = 4x + \cos x$

$\boxed{y'' = 4 - \sin x}$

(91) $f(x) = \cot x$

$f'(x) = \sec^2 x$

$f'(x) = \sec x \sec x$

$f''(x) = \sec x(\sec x \tan x) + (\sec x \tan x)(\sec x)$

$f''(x) = \sec^2 x \tan x + \sec^2 x \tan x$

$\boxed{f''(x) = 2\sec^2 x \tan}$