A Calculus AB

Chapter Two Test Review

Complete each of the following to the best of your ability. Show enough work to justify all answers and be sure to show how you set up each derivative. Be neat and box in all final answers. Good luck.

1. Use the definition of derivative to compute $f'(x)$ if $f(x) = 1/\sqrt{x}$.
2. Given: $g(2) = 3$, $g'(2) = -2$, $h(2) = -1$, and $h'(2) = 4$. Use the derivative rules to compute each derivative.
   (a) $D_x[2g(2) - 3h(2)]$ (b) $D_x[g(2)h(2)]$ (c) $D_x[h(2)/g(2)]$
3. Differentiate each. Simplify completely.
   (a) $f(x) = x^2 - 3/x^3$ (b) $h(x) = x^{4/5} - x^{2/3}$
4. Use the product rule to compute the derivative. Simplify completely.
   $f(x) = (3x^2 - 5)(2x^4 - x)$
5. Use the quotient rule to compute the derivative. Simplify completely.
   $f(x) = (3x - 5)/(x^2 + 7)$
6. Find an equation for the tangent to $f(x) = (2x - 1)(x^2 + 3)$ at $x = -1$.
7. Determine where $f(x) = (x - 1)/(x^2 - 1)$ will have horizontal tangents.
8. An object is dropped from the top of a building and its height (feet) at time t (seconds) is given by $h(t) = -16t^2 + 1362$
   (a) Find the average velocity from $t = 1$ to $t = 3$ second
   (b) When does the ball hit the ground?
   (c) What is the instantaneous velocity of the object when it hits the ground
9. An object moves along a coordinate line so that its position S satisfies
   $S = t^3 - 9t^2 + 24t$.
   (a) Determine when the object is moving to the left and when it is moving to the right.
   (b) Determine the total distance traveled by the object on the time interval from $t = 0$ to $t = 6$.

1. $f'(x) = \lim\limits_{\Delta x \to 0} \dfrac{\frac{1}{\sqrt{x+\Delta x}} - \frac{1}{\sqrt{x}}}{\Delta x}$

$= \lim\limits_{\Delta x \to 0} \dfrac{\frac{\sqrt{x} - \sqrt{x+\Delta x}}{\sqrt{x}(\sqrt{x+\Delta x})}}{\Delta x} \cdot \dfrac{\sqrt{x} + \sqrt{x+\Delta x}}{\sqrt{x} + \sqrt{x+\Delta x}} = \dfrac{\frac{x - x - \Delta x}{\sqrt{x} + \sqrt{x+\Delta x}}}{\Delta x}$

$= \lim\limits_{\Delta x \to 0} \dfrac{x - x - \Delta x}{\Delta x\sqrt{x}(\sqrt{x+\Delta x})(\sqrt{x} + \sqrt{x+\Delta x})} \qquad \dfrac{-\cancel{\Delta x}}{\sqrt{x}+\sqrt{x+\Delta x}} \cdot \dfrac{1}{\cancel{\Delta x}}$

$= \dfrac{-1}{\sqrt{x}(\sqrt{x})(\sqrt{x}+\sqrt{x})}$

$x(2\sqrt{x})$

$= \dfrac{-1}{2x\sqrt{x}} \qquad \dfrac{-1}{2x\sqrt{x}}$

2. a) $2(-2) - 3(4)$
$-4 - 12$
-16

b) $g(2)\cdot h'(2) + h(2)g'(2)$
$3 \cdot 4 + -1(-2)$
$12 + 2$
14

c) $\dfrac{g(2)\cdot h'(2) - h(2)g'(2)}{(g(2))^2}$
$3 \cdot 4 - (-1)(-2)$
$12 - 2$
$10/9$

3. a) $f(x) = x^2 - \dfrac{3}{x^3}$
$2x + 9x^{-4}$
$f'(x) = 2x + \dfrac{9}{x^4}$

b) $h(x) = x^{4/5} - x^{2/3}$

$h'(x) = \frac{4}{5}x^{-1/5} - \frac{2}{3}x^{-1/3}$

$\frac{4}{5\sqrt[5]{x}} - \frac{2}{3\sqrt[3]{x}}$

4. $f(x) = (3x^2-5)(2x^4-x)$

$f'(x) = (3x^2-5)(8x^3-1) + (2x^4-x)(6x)$

$= 24x^5 - 3x^2 - 40x^3 + 5 + \cancel{12x^5} - 6x^2$

$= 36x^5 - 40x^3 - 9x^2 + 5$

5. $f(x) = \frac{3x-5}{x^2+7}$

$f'(x) = \frac{(x^2+7)(3) - (3x-5)(2x)}{(x^2+7)^2}$

$= \frac{3x^2 + 21 - 6x^2 + 10x}{(x^2+7)^2}$

$\frac{-3x^2 + 10x + 21}{(x^2+7)^2}$

6. $f(x) = (2x-1)(x^2+3)$

$f'(x) = (2x-1)(2x) + (x^2+3)(2)$

$4x^2 - 2x + 2x^2 + 6$

$f'(x) = 6x^2 - 2x + 6$

$m = 6(-1)^2 - 2(-1) + 6$

$= 6 + 2 + 6$

$= 14$

$y = (2(-1)-1)(-1^2+3$

$(-3)(4)$

-12

$\boxed{y+12 = 14(x+1)}$

7. $f(x) = \frac{x-1}{x^2-1}$

$f'(x) = \frac{(x^2-1)(1) - (x-1)(2x)}{(x^2-1)^2}$

$\frac{x^2-1-2x^2+2x}{(x^2-1)^2}$

$0 = \frac{-x^2+2x-1}{(x^2-1)^2}$

$0 = -\frac{(x^2-2x+1)}{(x^2-1)(x^2-1)}$

$0 = -\frac{\cancel{(x-1)}\cancel{(x-1)}}{\cancel{(x-1)}(x+1)\cancel{(x-1)}(x+1)}$

$0 = \frac{-1}{(x+1)^2}$

$0 \neq -1$

$\therefore$ no horizontal tangents

8. $h(t) = -16t^2 + 1362$

$t = 1 \quad h(1) = 1346$

$t = 3 \quad h(3) = 1218$

a) $\frac{1346-1218}{1-3} = -64$ ft/sec

b) $0 = -16t^2 + 1362$

$-1362 = -16t^2$

$85.125 = t^2$

9.226 sec $= t$

c) $v(t) = -32t$

$v(9.226) = -32(9.226$

$= -295.242$

9. $s = t^3 - 9t^2 + 24t$

$v = 3t^2 - 18t + 24$

$v = 3(t^2 - 6t + 8)$

$0 = 3(t-4)(t-2)$

$t = 4 \quad t = 2$

$v(t)$: $+$ | 2 | $-$ | 4 | $+$

moving to the right when $t < 2$ and when $t > 4$.

$s(0) = 0$ } 20

$s(2) = 20$ } 4

$s(4) = 16$ } 20

$s(6) = 36$

total distance traveled $= 20 + 4 + 20 = 44$

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a) MAX @ $t = 1$

$v(1) = \frac{1}{1+1^2} = \frac{1}{1+1} = \boxed{\frac{1}{2}}$

b)

c) $\lim_{t \to \infty} \frac{t}{1+t^2} = 0$

d)