Curve Sketching

10/31/01

p.208 #1, 25, 11
& from chart

Review discontinuities

① Removable & non-removable
Find any discontinuities in f(x)

$$f(x) = \frac{x+2}{x^2-3x-10}$$

$$f(x) = \frac{x+2}{(x+2)(x-5)}$$

hole @ $x = -2$
v. asymptote @ $x = 5$

② Other types of asymptotes

$$\lim_{x\to\infty} \frac{5x^3+1}{10x^3-3x^2+7}$$

RULES:
① Bottom heavy: lim = 0
h.a. at $y=0$
② Equal: lim = $\frac{\text{l.c.}}{\text{l.c.}}$
h.a. @ ratio of l.c.
③ Top heavy: Divide to find slant asymptote
no h.a.

Curve Sketching

Often need to know what a graph looks like w/o being able to see the whole graph – Ex: calculator not always show appropriate window.

To analyze a graph you need:
① Domain
② x & y intercepts
③ symmetry (Even, odd neither)
} Precalculus

④ asymptotes (horizontal, vertical, slant)
⑤ continuous?
⑥ differentiable?
⑦ Extrema
⑧ Points of inflection
⑨ Concavity

Analyze the graph of $f(x) = \frac{2(x^2-9)}{x^2-4}$ FACTOR IF POSSIBLE!!

① Domain: $x \in \mathbb{R}; x \neq 2, -2$ (all $\mathbb{R}$ unless fraction or $\sqrt{}$, etc..)

② x-intercepts ($y=0$): $0 = \frac{2(x^2-9)}{x^2-4}$

$0 = 2(x^2-9)$

$0 = x^2-9$

$9 = x^2$

$\pm 3 = x$

$(3,0)\ (-3,0)$

y-intercepts ($x=0$): $y = \frac{2(0^2-9)}{0^2-4}$

$y = \frac{-18}{-4}$

$y = 18/4$

$y = 9/2$

$(0, 9/2)$

③ Symmetry: Even, odd, neither? (Plug in $-x$ for x & simplify)

$f(x) = \frac{2(x^2-9)}{x^2-4}$

$f(-x) = \frac{2((-x)^2-9)}{(-x)^2-4}$

$f(-x) = \frac{2(x^2-9)}{x^2-4}$

$= \Rightarrow$ Even $\Rightarrow$ y-axis symmetry

④ Asymptotes: v.a. $x=2$

$x=-2$

h.a. $y = \frac{2}{1} = 2$ (equal heavy)

s.a. NONE

⑤ not continuous - non-removable at $x=2, -2$

⑥ Not differentiable at $x=2, -2$

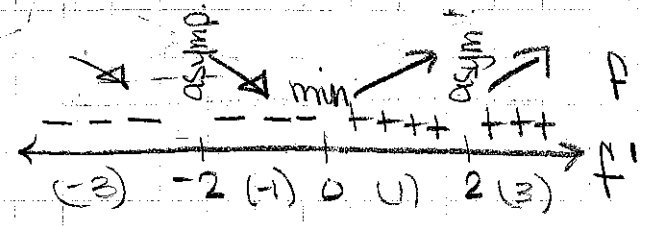
⑦ $f'(x) = \frac{(x^2-4)(4x) - (2x^2-18)(2x)}{(x^2-4)^2}$

$f'(x) = \frac{4x^3 - 16x - 4x^3 + 36x}{(x^2-4)^2}$

$f'(x) = \frac{20x}{(x^2-4)^2}$ $\rightsquigarrow$ undefined at $x=2, -2$

$0 = 20x$

$0 = x$ $\rightsquigarrow$ critical value



$f(0) = \frac{2(0^2-9)}{0^2-4}$ minimum at $(0, 9/2)$

$= 9/2$

(8.9)

$f''(x) = \frac{(x^2-4)^2(20) - (20x)(2(x^2-4)(2x))}{(x^2-4)^4}$

$= \frac{20(x^2-4)^{\cancel{2}} - 80x^2(x^2-4)}{(x^2-4)^{\cancel{4}\,3}}$

$= \frac{20(x^2-4) - 80x^2}{(x^2-4)^3}$

$= \frac{20x^2 - 80 - 80x^2}{(x^2-4)^3}$

$f''(x) = \frac{-60x^2 - 80}{(x^2-4)^3}$

$f''(x) = \frac{-20(3x^2+4)}{(x^2-4)^3}$ ⟶ undefined at $x = 2, -2$

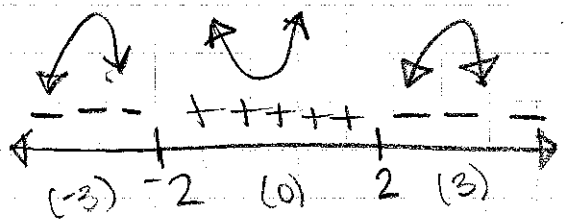
$0 = -20(3x^2+4)$

$0 = 3x^2 + 4$

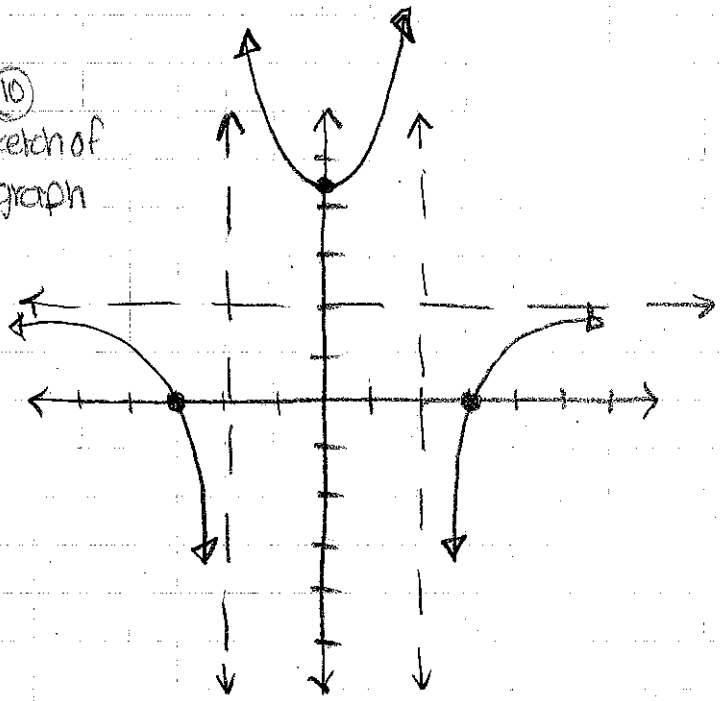
$-\frac{4}{3} = \frac{3x^2}{3}$

$-\frac{4}{3} = x^2$

$\sqrt{-\frac{4}{3}} = x$



(10) Sketch of graph



HW: p. 202 #1, 7, 33