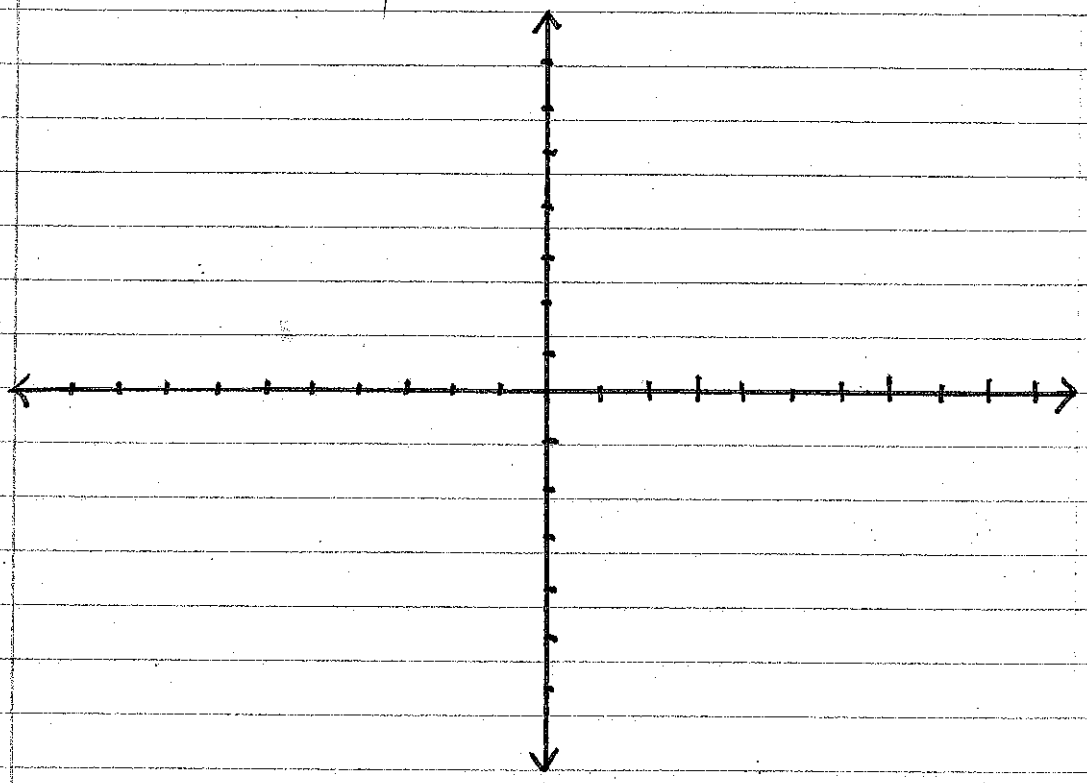
Calculus AB/AP
Curve Sketching

Name
Date

Use the information provided in the chart below to sketch a graph of the function f(x).

	$x<-4$	$x=-4$	$-4<x<-2$	$x=-2$	$-2<x<0$	$x=0$	$0<x<2$	$x=2$	$x>2$
$f(x)$		1		-2		-3		0	
$f'(x)$	+	0	−	−	−	0	+	undef.	−
$f''(x)$	−	−	−	0	+	+	+	undef.	+

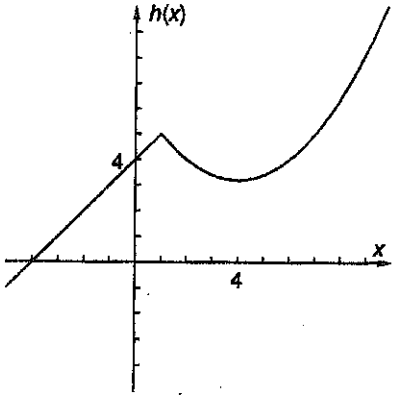


Name _____ Group _____ Date _____

Exploration 48: Derivatives and Integrals from Given Graphs

Objective: Given the graph of a function, sketch the graph of its derivative function or its integral function.

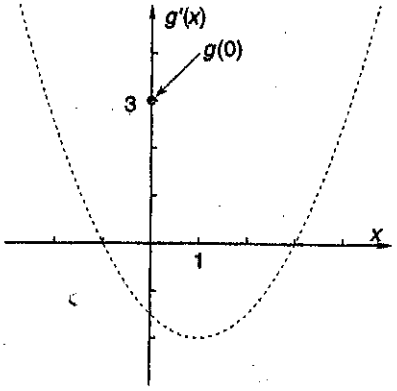
1. The graph below shows a function h. On the same axes sketch the graph of the derivative function, h'. At the cusp, $h'(1)$ is undefined. Make sure your graph accounts for this fact.



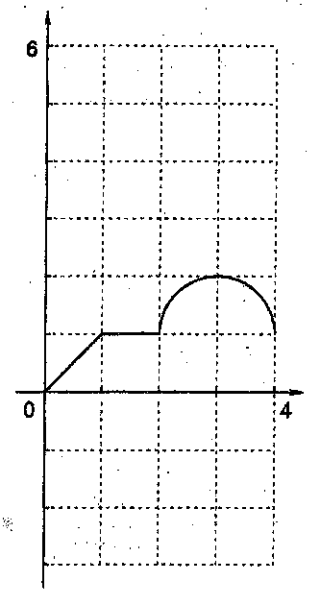
2. For h in Problem 1, is it also correct to say that $h'(1)$ is infinite? Explain.

3. For Problem 1, is h *continuous* at $x = 1$? Is h *differentiable* at $x = 1$?

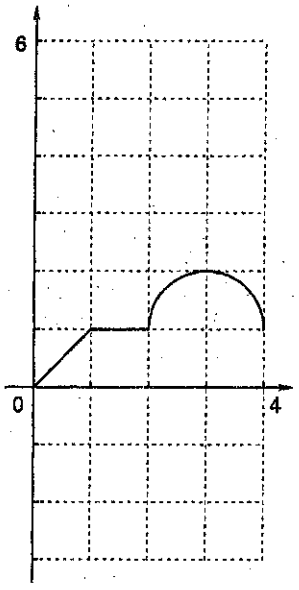
4. The graph below shows the *derivative* g' of function g. On the same axes sketch the graph of g, using as an initial condition $g(0) = 3$.



5. The graph below shows a function y. On the same axes sketch the graph of the derivative, y'.



6. The graph shows the derivative of a function, y'. On the same axes sketch the graph of the function if (0, 0) is on the graph of y.



7. What did you learn as a result of doing this Exploration that you did not know before?

HANDOUT #6 (CONTINUED)

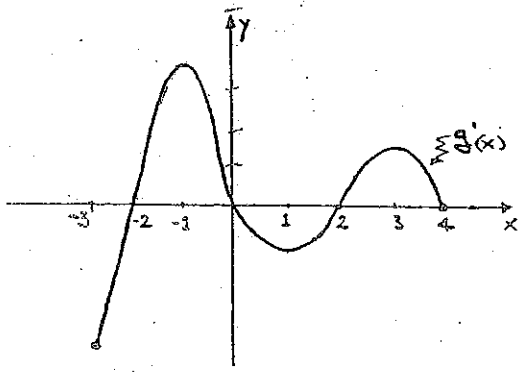
MORE SKETCHING

1. The figure below shows the graph of $g'(x)$, the derivative of a function g with domain [-3, 4].

   a) Determine the values of x for which g has a relative minimum, and a relative maximum. Explain!

   b) Determine the values of x for which g is concave down, and concave up. Explain!

   c) Based on the information given and the fact that $g(-3) = 3$ and $g(4) = 6$, sketch a graph for g.



1984 AB4/BC3

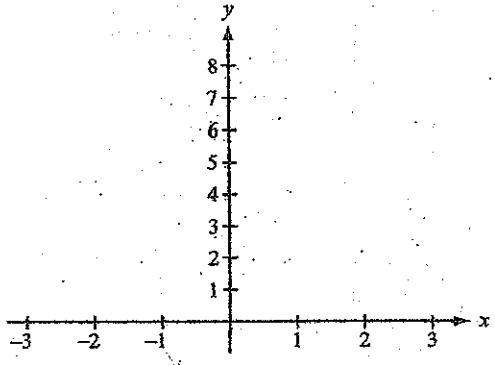
A function f is continuous on the closed interval $[-3, 3]$ such that $f(-3) = 4$ and $f(3) = 1$. The functions f' and f'' have the properties given in the table below.

x	$-3 < x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$1 < x < 3$
$f'(x)$	Positive	Fails to exist	Negative	0	Negative
$f''(x)$	Positive	Fails to exist	Positive	0	Negative

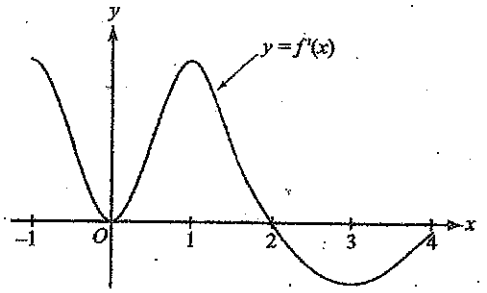
(a) What are the x-coordinates of all absolute maximum and absolute minimum points of f on the interval $[-3, 3]$? Justify your answer.

(b) What are the x-coordinates of all points of inflection of f on the interval $[-3, 3]$? Justify your answer.

(c) On the axes provided, sketch a graph that satisfies the given properties of f.



1980 BC7



Note: This is the graph of the derivative of f, NOT the graph of f.

Let f be a function that has domain the closed interval $[-1, 4]$ and range the closed interval $[-1, 2]$. Let $f(-1) = -1$, $f(0) = 0$, and $f(4) = 1$. Also let f have the derivative function f' that is continuous and that has the graph shown in the figure above.

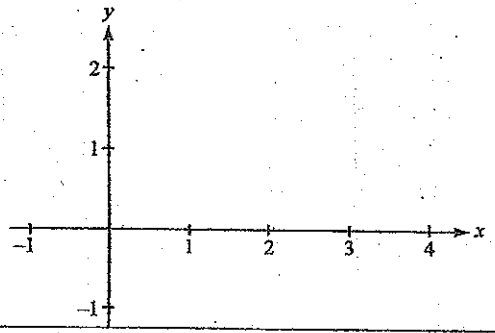
(a) Find all values of x for which f assumes a relative maximum. Justify your answer.

(b) Find all values of x for which f assumes its absolute minimum. Justify your answer.

(c) Find the intervals on which f is concave downward.

(d) Give all the values of x for which f has a point of inflection.

(e) On the axes provided, sketch the graph of f.



Note: The graph of f' has been slightly modified from the original on the 1980 exam to be consistent with the given values of f at $x = -1$, $x = 0$, and $x = 4$.

AP® CALCULUS AB
2004 SCORING GUIDELINES

Question 4

Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

(a) Show that $\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$.

(b) Show that there is a point P with x-coordinate 3 at which the line tangent to the curve at P is horizontal. Find the y-coordinate of P.

(c) Find the value of $\frac{d^2y}{dx^2}$ at the point P found in part (b). Does the curve have a local maximum, a local minimum, or neither at the point P? Justify your answer.

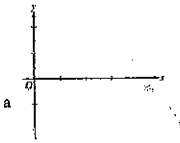
Question 4

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	−1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	−3	Negative
$f''(x)$	−2	Negative	0	Positive	DNE	Negative	0	Positive

Let f be a function that is continuous on the interval $[0, 4)$. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

(a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

(b) On the axes provided, sketch the graph of a function that has all the characteristics of f.
(Note: Use the axes provided in the pink test booklet.)

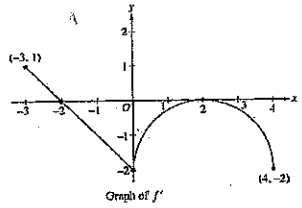


(c) Let g be the function defined by $g(x) = \int_1^x f(t)\,dt$ on the open interval $(0, 4)$. For $0 < x < 4$, find all values of x at which g has a relative extremum. Determine whether g has a relative maximum or a relative minimum at each of these values. Justify your answer.

(d) For the function g defined in part (c), find all values of x, for $0 < x < 4$, at which the graph of g has a point of inflection. Justify your answer.

Question 4

Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above.



Graph of f'

(a) On what intervals, if any, is f increasing? Justify your answer.

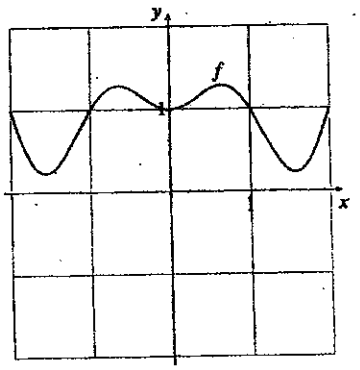
(b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

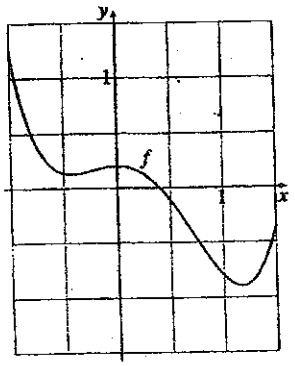
(c) Find an equation for the line tangent to the graph of f at the point $(0, 3)$.

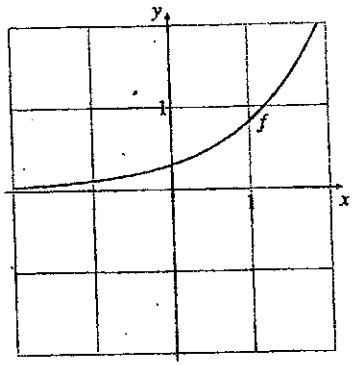
(d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.

Group Exercise 4.3: The Graph Game (Form A)

Do *not* show your partner the graphs on this sheet! Choose your favorite graph of the ones listed below. On a separate sheet of graph paper, sketch the derivative of this graph. Make sure you remember what you've learned about local maxima, local minima, and concavity when you draw your graph. Every point on the derivative graph should correspond to the slope of the original function. Give your piece of graph paper with the derivative sketch to your partner when you are done. If you finish first, try sketching the derivatives of the other functions as well.







Group Exercise 4.3: The Graph Game (Form B)

Do *not* show your partner the graphs on this sheet! Choose your favorite graph of the ones listed below. On a separate sheet of graph paper, sketch the derivative of this graph. Make sure you remember what you've learned about local maxima, local minima, and concavity when you draw your graph. Every point on the derivative graph should correspond to the slope of the original function. Give your piece of graph paper with the derivative sketch to your partner when you are done. If you finish first, try sketching the derivatives of the other functions as well.