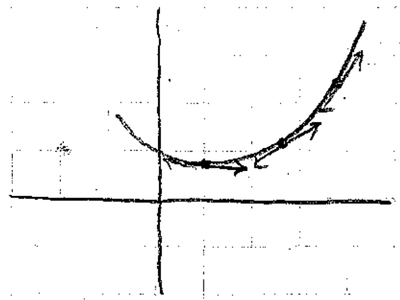
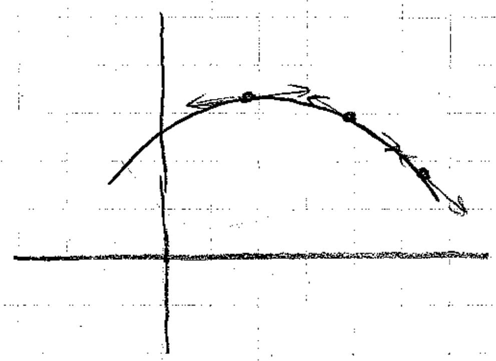
Concavity 10/29/01

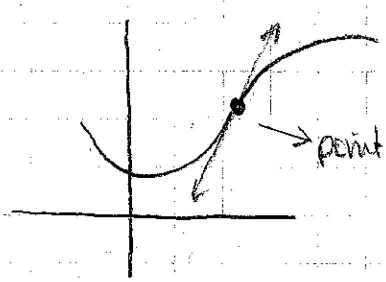


(f')
concave up - slope of tangent line is increasing
(graph is above tangent lines)



(f')
concave down - slope of tangent line is decreasing.
(graph is below tangent lines)

Point of Inflection: point where the concavity of a graph changes. (A graph crosses its tangent line at a point of inflection.)



At each point of inflection $f''(c)=0$ or f'' is undefined at $x=c$.

Ex: Determine the open intervals on which the graph of $f(x) = 6(x^2+3)^{-1}$ is concave up or concave down.

① Check continuity of $f(x)$

$$f(x) = \frac{6}{x^2+3} \longrightarrow \text{everywhere continuous}$$

② Find $f''(x)$ and inflection points

$$f'(x) = \frac{(x^2+3)(0) - 6(2x)}{(x^2+3)^2}$$

$$f'(x) = \frac{-12x}{(x^2+3)^2}$$

$$f''(x) = \frac{(x^2+3)^2(-12) - (-12x)(2(x^2+3))(2x)}{(x^2+3)^4}$$

$$f''(x) = \frac{-12(x^2+3)^2 + 48x^2(x^2+3)}{(x^2+3)^4}$$

$$f''(x) = \frac{-12\cancel{(x^2+3)}\left[x^2+3-4x^2\right]}{(x^2+3)^4}$$

$$f''(x) = \frac{-12\,[-3x^2+3]}{(x^2+3)^3}$$

$$f''(x) = \frac{36\,(x^2-1)}{(x^2+3)^3}$$ never undefined

$0 = 36(x^2-1)$

$0 = x^2-1$

$1 = x^2$

$\pm 1 = x$

Inflection points: 1, -1

③ Make # line, test values

$f(x)$

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$(-2)\ \ -1\ \ (0)\ \ 1\ \ (2)$ $\quad f''(x)$

$$f''(-2) = \frac{36(-2^2-1)}{(-2^2+3)^3} = \frac{36(3)}{7^3} \qquad f''(0) = \frac{36(0^2-1)}{(0^2+3)^3} = \frac{-36}{3^3} \qquad f''(2) = \frac{36(2^2-1)}{(2^2+3)^3} = \frac{36(3)}{7^3}$$

GP

Determine the points of inflection & discuss the concavity of the graph of $f(x)=x^4-4x^3$

① continuous everywhere

② $f(x) = x^4-4x^3$

$f'(x) = 4x^3-12x^2$

$f''(x) = 12x^2-24x$ ⟿ never undefined

$0 = 12x^2-24x$

$0 = 12x(x-2)$

$0 = x \quad x = 2$ ⟿ inflection points

③

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$(-1)\ \ 0\ \ (1)\ \ 2\ \ (3)$

$f''(-1) = 12(-1)^2 - 24(-1)$

$12 + 24$

$f''(1) = 12(1)^2 - 24(1)$

$12 - 24$

$f''(3) = 12(3)^2 - 24(3)$

$108 - 72$

Determine the points of inflection & discuss the concavity of the graph of $f(x) = \frac{x^2+1}{x^2-4}$

① $f(x) = \frac{x^2+1}{(x-2)(x+2)}$ ⟿ not continuous at $x=2$, $x=-2$. (add these #'s to # line)

$$f'(x) = \frac{(x^2-4)(2x) - (x^2+1)(2x)}{(x^2-4)^2}$$

$$f'(x) = \frac{\cancel{2x^3} - 8x - \cancel{2x^3} - 2x}{(x^2-4)^2}$$

$$f'(x) = -\frac{10x}{(x^2-4)^2}$$

$$f''(x) = \frac{(x^2-4)^2(-10) - (-10x)(2(x^2-4)(2x))}{(x^2-4)^4}$$

$$= \frac{-10(x^2-4)^2 + 40x^2(x^2-4)}{(x^2-4)^4}$$

$$= \frac{-10\cancel{(x^2-4)}\left[x^2-4 - 4x^2\right]}{\cancel{(x^2-4)^4}}$$

$$= \frac{-10(-3x^2-4)}{(x^2-4)^3}$$

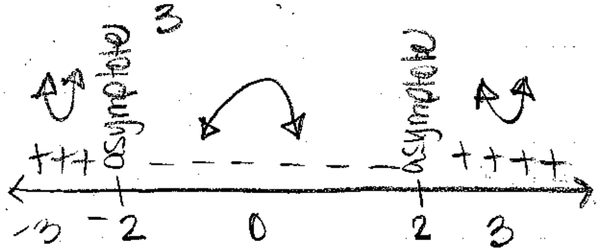
$$= \frac{10(3x^2+4)}{(x^2-4)^3}$$ ⟿ undefined when $x = 2, -2$

$0 = 10(3x^2+4)$

$0 = 3x^2+4$

$-4 = 3x^2$

$-\frac{4}{3} = x^2$



$f''(-3) = \frac{10(3(-3)^2+4)}{((-3)^2-4)^3}$

$= \frac{10(+)}{+^3}$

$f''(0) = \frac{10(3(0)^2+4)}{(0^2-4)^3}$

$= \frac{+}{-}$

$f''(3) = \frac{10(3(3)^2+4)}{(3^2-4)^3}$

$= \frac{+}{+}$