(a) Area (hexagon) $= \dfrac{3\sqrt{3}}{2} \approx 2.5981$

Area (circle) $= \pi \approx 3.1416$

Area (circle) − Area (hexagon) ≈ 0.5435

(b) $A_n = \dfrac{n}{2}\sin\left(\dfrac{2\pi}{n}\right)$

(c)

n	6	12	24	48	96
A_n	2.5981	3.0000	3.1058	3.1326	3.1394

(d) 3.1416 or π

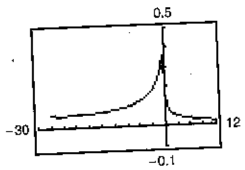
(a) $m = -\frac{12}{5}$ (b) $y = \frac{5}{12}x - \frac{169}{12}$

(c) $m_x = \dfrac{-\sqrt{169 - x^2} + 12}{x - 5}$

(d) $\frac{5}{12}$; It is the same as the slope of the tangent line found in (b).

(a) Domain: $[-27, 1) \cup (1, \infty)$

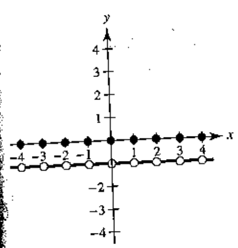
(b)



The graph has a hole at $x = 1$.

(c) $\frac{1}{14}$ (d) $\frac{1}{12}$

(a) g_1, g_4 (b) g_1 (c) g_1, g_3, g_4



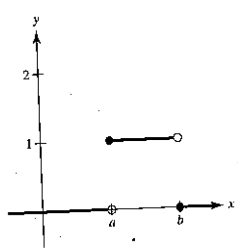
The graph jumps at every integer.

(a) $f(1) = 0,\ f(0) = 0,\ f(\frac{1}{2}) = -1,\ f(-2.7) = -1$

(b) $\lim\limits_{x \to 1^-} f(x) = -1,\ \lim\limits_{x \to 1^+} f(x) = -1,\ \lim\limits_{x \to 1/2} f(x) = -1$

(c) There is a discontinuity at each integer.

(a)



(b) (i) $\lim\limits_{x \to a^+} P_{a,b}(x) = 1$ (ii) $\lim\limits_{x \to a^-} P_{a,b}(x) = 0$

(iii) $\lim\limits_{x \to b^+} P_{a,b}(x) = 0$ (iv) $\lim\limits_{x \to b^-} P_{a,b}(x) = 1$

(c) Continuous for all positive real numbers except a and b

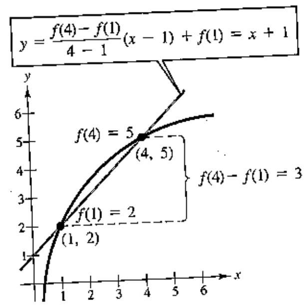
(d) The area under the curve gives a value of 1.

Chapter 2

Section 2.1 (page 101)

1. (a) $m = 0$ (b) $m = -3$

3.



5. $m = -2$ **7.** $m = 2$ **9.** $m = 3$

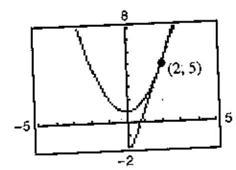
11. $f'(x) = 0$ **13.** $f'(x) = -5$ **15.** $h'(s) = \frac{2}{3}$

17. $f'(x) = 4x + 1$ **19.** $f'(x) = 3x^2 - 12$

21. $f'(x) = \dfrac{-1}{(x-1)^2}$ **23.** $f'(x) = \dfrac{1}{2\sqrt{x+1}}$

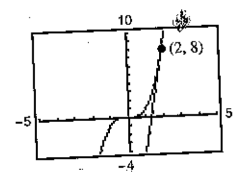
25. (a) Tangent line: $y = 4x - 3$

(b)



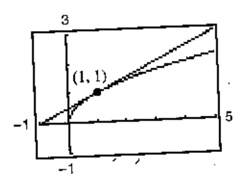
27. (a) Tangent line: $y = 12x - 16$

(b)



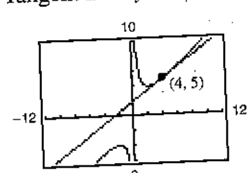
29. (a) Tangent line: $y = \frac{1}{2}x + \frac{1}{2}$

(b)



31. (a) Tangent line: $y = \frac{3}{4}x + 2$

(b)



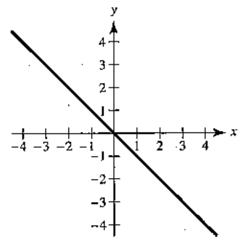
33. $y = 3x - 2;\ y = 3x + 2$

35. $y = -\frac{1}{2}x + \frac{3}{2}$

37. $g(5) = 2$; $g'(5) = -\frac{1}{2}$

39. b **40.** d **41.** a **42.** c

43. Answers will vary. Sample answer: $y = -x$



45. (a) $f'(-c) = 3$ (b) $f'(-c) = -3$

47. $y = 2x + 1$; $y = -2x + 9$

49. (a) -3

(b) 0

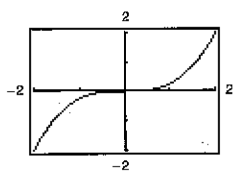
(c) The graph is moving downward to the right when $x = 1$.

(d) The graph is moving upward to the right when $x = -4$.

(e) Positive. Because $g'(x) > 0$ on $[3, 6]$, the graph of g is moving upward to the right.

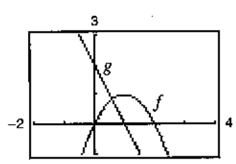
(f) No. Knowing only $g'(2)$ is not sufficient information. $g'(2)$ remains the same for any vertical translation of g.

51.



x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
$f(x)$	-2	$-\frac{27}{32}$	$-\frac{1}{4}$	$-\frac{1}{32}$	0	$\frac{1}{32}$	$\frac{1}{4}$	$\frac{27}{32}$	2
$f'(x)$	3	$\frac{27}{16}$	$\frac{3}{4}$	$\frac{3}{16}$	0	$\frac{3}{16}$	$\frac{3}{4}$	$\frac{27}{16}$	3

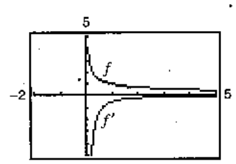
53.



$g(x) \approx f'(x)$

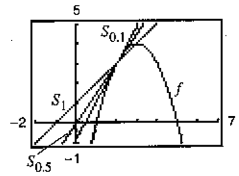
55. $f(2) = 4$; $f(2.1) = 3.99$; $f'(2) \approx -0.1$; Exact $f'(2) = 0$

57.



As x approaches infinity, the graph of f approaches a line of slope 0. Thus $f'(x)$ approaches 0.

59. (a)



(b) The graphs of S for decreasing values of Δx are secant lines approaching the tangent line to the graph of f at the point $(2, f(2))$.

61. 4 **63.** 4

65. $g(x)$ is not differentiable at $x = 0$.

67. $f(x)$ is not differentiable at $x = 6$.

69. $h(x)$ is not differentiable at $x = -5$.

71. $(-\infty, -3) \cup (-3, \infty)$ **73.** $(-\infty, -1) \cup (-1, \infty)$

75. $(-\infty, 3) \cup (3, \infty)$ **77.** $(1, \infty)$ **79.** $(-\infty, 0) \cup (0, \infty)$

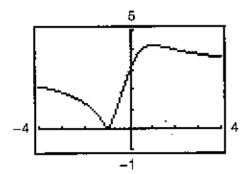
81. The derivative from the left is -1 and from the right is 1, so f is not differentiable at $x = 1$.

83. The derivative from both the right and left is 0, so $f'(1) = 0$.

85. f is differentiable at $x = 2$.

87. (a) $d = \dfrac{3|m + 1|}{\sqrt{m^2 + 1}}$

(b)



Not differentiable at $m = -1$

89. False. It is $\lim_{\Delta x \to 0} \dfrac{f(2 + \Delta x) - f(2)}{\Delta x}$.

91. False. For example: $f(x) = |x|$. The derivative from the left and the derivative from the right both exist but are not equal.

93. Proof

Section 2.2 (page 113)

1. (a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) 2 (d) 3

3. 0 **5.** $6x^5$ **7.** $-\dfrac{7}{x^8}$ **9.** $\dfrac{1}{5x^{4/5}}$ **11.** 1

13. $-4t + 3$ **15.** $2x + 12x^2$ **17.** $3t^2 - 2$

19. $\dfrac{\pi}{2}\cos\theta + \sin\theta$ **21.** $2x + \dfrac{1}{2}\sin x$ **23.** $-\dfrac{1}{x^2} - 3\cos x$

	Function	*Rewrite*	*Derivative*	*Simplify*
25.	$y = \dfrac{5}{2x^2}$	$y = \dfrac{5}{2}x^{-2}$	$y' = -5x^{-3}$	$y' = -\dfrac{5}{x^3}$
27.	$y = \dfrac{3}{(2x)^3}$	$y = \dfrac{3}{8}x^{-3}$	$y' = -\dfrac{9}{8}x^{-4}$	$y' = -\dfrac{9}{8x^4}$
29.	$y = \dfrac{\sqrt{x}}{x}$	$y = x^{-1/2}$	$y' = -\dfrac{1}{2}x^{-3/2}$,	$y' = -\dfrac{1}{2x^{3/2}}$

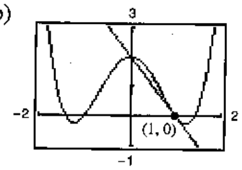
31. -6 **33.** 0 **35.** 4 **37.** 3 **39.** $2x + \dfrac{6}{x^3}$

41. $2t + \dfrac{12}{t^4}$ **43.** $\dfrac{x^3 - 8}{x^3}$ **45.** $3x^2 + 1$

47. $\dfrac{1}{2\sqrt{x}} - \dfrac{2}{x^{2/3}}$ **49.** $\dfrac{4}{5s^{1/5}} - \dfrac{2}{3s^{1/3}}$ **51.** $\dfrac{3}{\sqrt{x}} - 5 \sin x$

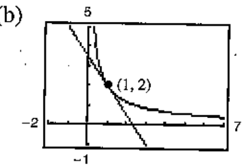
53. (a) $2x + y - 2 = 0$

(b)



55. (a) $3x + 2y - 7 = 0$

(b)

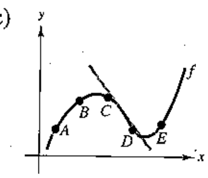


57. $(0, 2), (-2, -14), (2, -14)$ **59.** No horizontal tangents

61. (π, π) **63.** $k = 2,\ k = -10$ **65.** $k = 3$

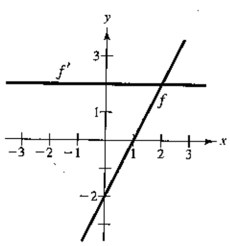
67. (a) A and B (b) Greater

(c)



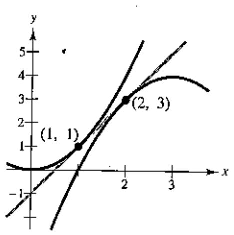
69. $g'(x) = f'(x)$

71.

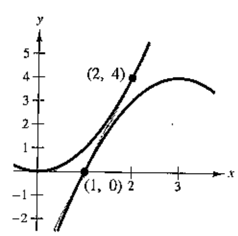


The rate of change of f is constant and therefore f' is a constant function.

73. $y = 2x - 1$

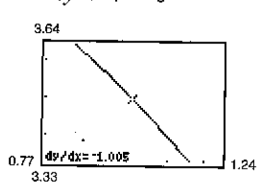


$y = 4x - 4$



75. $x - 4y + 4 = 0$

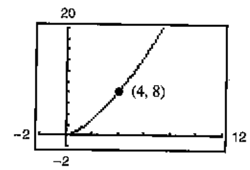
77.



$f'(1)$ appears to be close to -1.

$f'(1) = -1$

79. (a)

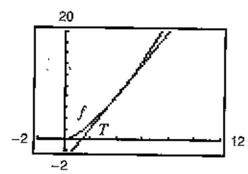


$(3.9, 7.7019)$, $S(x) = 2.981x - 3.924$

(b) $T(x) = 3(x - 4) + 8 = 3x - 4$

The slope (and equation) of the secant line approaches that of the tangent line at $(4, 8)$ as you choose points closer to $(4, 8)$.

(c) It becomes less accurate.



(d)

Δx	-3	-2	-1	-0.5	-0.1	0
$f(4 + \Delta x)$	1	2.828	5.196	6.458	7.702	8
$T(4 + \Delta x)$	-1	2	5	6.5	7.7	8

Δx	0.1	0.5	1	2	3
$f(4 + \Delta x)$	8.302	9.546	11.180	14.697	18.520
$T(4 + \Delta x)$	8.3	9.5	11	14	17

81. False: let $f(x) = x$ and $g(x) = x + 1$.

83. False: $dy/dx = 0$. **85.** True

87. Average rate: 2

Instantaneous rates:

$f'(1) = 2$

$f'(2) = 2$

89. Average rate: $\frac{1}{2}$

Instantaneous rates:

$f'(1) = 1$

$f'(2) = \frac{1}{4}$

91. (a) $s(t) = -16t^2 + 1362$

$v(t) = -32t$

(b) -48 feet per second

(c) $s'(1) = -32$ feet per second

$s'(2) = -64$ feet per second

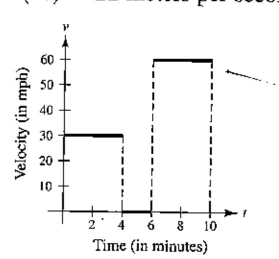
(d) $t = \dfrac{\sqrt{1362}}{4} \approx 9.226$ seconds

(e) -295.242 feet per second

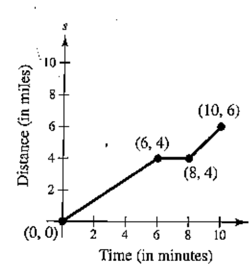
93. $v(5) = 71$ meters per second

$v(10) = 22$ meters per second

95.



97.

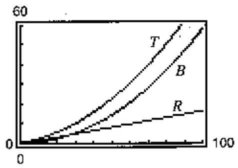


99. (a) $R(v) = 0.167v - 0.02$

(b) $B(v) = 0.006v^2 - 0.024v + 0.460$

(c) $T(v) = 0.006v^2 + 0.143v + 0.440$

(d)



(e) $T'(v) = 0.012v + 0.143$

$T'(40) = 0.623$

$T'(80) = 1.103$

$T'(100) = 1.343$

(f) Stopping distance increases at an increasing rate.

101. 8 square meters per meter change in s

103. $-\$1.91, -\$1.93$

105. (a) The rate of change of gallons of gasoline sold when the price is \$1.479

(b) In general, the rate of change when $p = 1.479$ should be negative.

107. $y = 2x^2 - 3x + 1$ **109.** $y = -9x, y = -\frac{9}{4}x - \frac{27}{4}$

111. $a = \frac{1}{3}, b = -\frac{4}{3}$ **113.** Proof

Section 2.3 (page 124)

1. $2(2x^3 - 3x^2 + x - 1)$ **3.** $\dfrac{7t^2 + 4}{3t^{2/3}}$

5. $x^2(3\cos x - x\sin x)$ **7.** $\dfrac{1 - x^2}{(x^2 + 1)^2}$

9. $\dfrac{1 - 8x^3}{3x^{2/3}(x^3 + 1)^2}$ **11.** $\dfrac{x\cos x - 2\sin x}{x^3}$

13. $f'(x) = (x^3 - 3x)(4x + 3) + (2x^2 + 3x + 5)(3x^2 - 3)$

$= 10x^4 + 12x^3 - 3x^2 - 18x - 15$

$f'(0) = -15$

15. $f'(x) = \dfrac{x^2 - 6x + 4}{(x - 3)^2}$ **17.** $f'(x) = \cos x - x\sin x$

$f'(1) = -\dfrac{1}{4}$ $f'\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{8}(4 - \pi)$

	Function	*Rewrite*	*Differentiate*	*Simplify*
19.	$y = \dfrac{x^2 + 2x}{3}$	$y = \dfrac{1}{3}(x^2 + 2x)$	$y' = \dfrac{1}{3}(2x + 2)$	$y' = \dfrac{2(x + 1)}{3}$
21.	$y = \dfrac{7}{3x^3}$	$y = \dfrac{7}{3}x^{-3}$	$y' = -7x^{-4}$	$y' = -\dfrac{7}{x^4}$
23.	$y = \dfrac{4x^{3/2}}{x}$	$y = 4x^{1/2}$, $x > 0$	$y' = 2x^{-1/2}$	$y' = \dfrac{2}{\sqrt{x}}$, $x > 0$

25. $\dfrac{(x^2 - 1)(-2 - 2x) - (3 - 2x - x^2)(2x)}{(x^2 - 1)^2} = \dfrac{2}{(x + 1)^2},\ x \neq 1$

27. $1 - \dfrac{12}{(x + 3)^2} = \dfrac{x^2 + 6x - 3}{(x + 3)^2}$

29. $\dfrac{\sqrt{x}(2) - (2x + 5)\frac{1}{2\sqrt{x}}}{x} = \dfrac{2x - 5}{2x^{3/2}}$

31. $6s^2(s^3 - 2)$ **33.** $-\dfrac{2x^2 - 2x + 3}{x^2(x - 3)^2}$

35. $(3x^3 + 4x)[(x - 5) \cdot 1 + (x + 1) \cdot 1]$

$+ [(x - 5)(x + 1)](9x^2 + 4)$

$= 15x^4 - 48x^3 - 33x^2 - 32x - 20$

37. $\dfrac{(x^2 - c^2)(2x) - (x^2 + c^2)(2x)}{(x^2 - c^2)^2} = -\dfrac{4xc^2}{(x^2 - c^2)^2}$

39. $t(t\cos t + 2\sin t)$ **41.** $-\dfrac{t\sin t + \cos t}{t^2}$

43. $-1 + \sec^2 x = \tan^2 x$ **45.** $\dfrac{1}{4t^{3/4}} + 8\sec t\tan t$

47. $\dfrac{-6\cos^2 x + 6\sin x - 6\sin^2 x}{4\cos^2 x} = \dfrac{3}{2}(-1 + \tan x\sec x - \tan^2 x)$

$= \dfrac{3}{2}\sec x(\tan x - \sec x)$

49. $\csc x\cot x - \cos x = \cos x\cot^2 x$ **51.** $x(x\sec^2 x + 2\tan x)$

53. $2x\cos x + 2\sin x - x^2\sin x + 2x\cos x$

$= 4x\cos x + (2 - x^2)\sin x$

55. $\left(\dfrac{x + 1}{x + 2}\right)(2) + (2x - 5)\left[\dfrac{(x + 2)(1) - (x + 1)(1)}{(x + 2)^2}\right]$

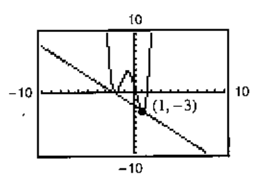
$= \dfrac{2x^2 + 8x - 1}{(x + 2)^2}$

57. $\dfrac{1 - \sin\theta + \theta\cos\theta}{(1 - \sin\theta)^2}$ **59.** $y' = \dfrac{-2\csc x\cot x}{(1 - \csc x)^2}, \quad -4\sqrt{3}$

61. $h'(t) = \dfrac{\sec t(t\tan t - 1)}{t^2}, \quad \dfrac{1}{\pi^2}$

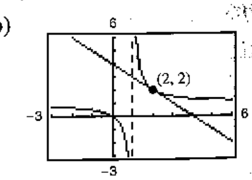
63. (a) $y = -x - 2$

(b)



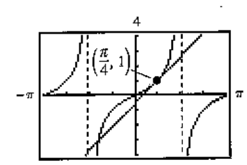
65. (a) $y = -x + 4$

(b)



67. (a) $4x - 2y - \pi + 2 = 0$

(b)



69. $(0, 0), (2, 4)$ **71.** $f(x) + 2 = g(x)$

73. $n = 1, f'(x) = x\cos x + \sin x$

$n = 2, f'(x) = x^2\cos x + 2x\sin x$

$n = 3, f'(x) = x^3\cos x + 3x^2\sin x$

$n = 4, f'(x) = x^4\cos x + 4x^3\sin x$

$f'(x) = x^n\cos x + nx^{n-1}\sin x$

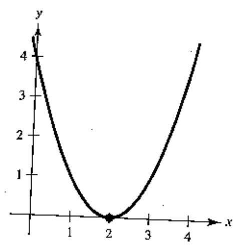
75. $\dfrac{6t+1}{2\sqrt{t}}$ square centimeters per second

77. (a) $-\$38.13$ (b) $-\$10.37$ (c) $-\$3.80$
The costs decrease with increasing order size.

79. 31.55 bacteria per hour **81.** Proof **83.** $\dfrac{3}{\sqrt{x}}$

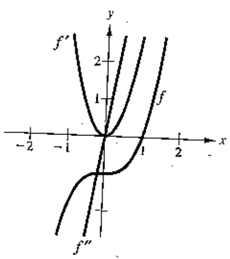
85. $\dfrac{2}{(x-1)^3}$ **87.** $-3\sin x$ **89.** $2x$ **91.** $\dfrac{1}{\sqrt{x}}$

93. Answers will vary. For example: $(x-2)^2$



95. 0 **97.** -10

99.



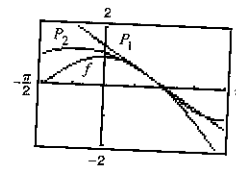
101. $v(3) = 27$ meters per second
$a(3) = -6$ meters per second per second
The speed of the object is decreasing, but the rate of that decrease is increasing.

103. (a) 2.4 ft/sec² (b) 1.2 ft/sec² (c) 0.5 ft/sec²

105. (a) $f''(x) = g(x)h''(x) + 2g'(x)h'(x) + g''(x)h(x)$
$f'''(x) = g(x)h'''(x) + 3g'(x)h''(x) + 3g''(x)h'(x) + g'''(x)h(x)$
$f^{(4)}(x) = g(x)h^{(4)}(x) + 4g'(x)h'''(x) + 6g''(x)h''(x) + 4g'''(x)h'(x) + g^{(4)}(x)h(x)$
(b) $f^{(n)}(x) = g(x)h^{(n)}(x) + \dfrac{n!}{1!(n-1)!}g'(x)h^{(n-1)}(x) + \dfrac{n!}{2!(n-2)!}g''(x)h^{(n-2)}(x) + \cdots + \dfrac{n!}{(n-1)!1!}g^{(n-1)}(x)h'(x) + g^{(n)}(x)h(x)$

107. (a) $P_1(x) = -\dfrac{\sqrt{3}}{2}\left(x - \dfrac{\pi}{3}\right) + \dfrac{1}{2}$
$P_2(x) = -\dfrac{1}{4}\left(x - \dfrac{\pi}{3}\right)^2 - \dfrac{\sqrt{3}}{2}\left(x - \dfrac{\pi}{3}\right) + \dfrac{1}{2}$

(b)



(c) P_2
(d) P_1 and P_2 become less accurate as you move farther from $x = a$.

109. False: $dy/dx = f(x)g'(x) + g(x)f'(x)$ **111.** True

113. True **115.** $f'(x) = 2|x|$; $f''(0)$ does not exist.

Section 2.4 (page 133)

	$y = f(g(x))$	$u = g(x)$	$y = f(u)$
1.	$y = (6x-5)^4$	$u = 6x - 5$	$y = u^4$
3.	$y = \sqrt{x^2-1}$	$u = x^2 - 1$	$y = \sqrt{u}$
5.	$y = \csc^3 x$	$u = \csc x$	$y = u^3$

7. $6(2x-7)^2$ **9.** $-108(4-9x)^3$

11. $\dfrac{2}{3}(9-x^2)^{-1/3}(-2x) = -\dfrac{4x}{3(9-x^2)^{1/3}}$

13. $\dfrac{1}{2}(1-t)^{-1/2}(-1) = -\dfrac{1}{2\sqrt{1-t}}$

15. $\dfrac{1}{3}(9x^2+4)^{-2/3}(18x) = \dfrac{6x}{(9x^2+4)^{2/3}}$

17. $\dfrac{1}{2}(4-x^2)^{-3/4}(-2x) = \dfrac{-x}{\sqrt[4]{(4-x^2)^3}}$

19. $-\dfrac{1}{(x-2)^2}$ **21.** $-2(t-3)^{-3}(1) = -\dfrac{2}{(t-3)^3}$

23. $-\dfrac{1}{2(x+2)^{3/2}}$

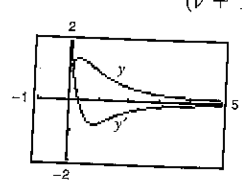
25. $x^2[4(x-2)^3(1)] + (x-2)^4(2x) = 2x(x-2)^3(3x-2)$

27. $x\left(\dfrac{1}{2}\right)(1-x^2)^{-1/2}(-2x) + (1-x^2)^{1/2}(1) = \dfrac{1-2x^2}{\sqrt{1-x^2}}$

29. $\dfrac{(x^2+1)^{1/2}(1) - x(1/2)(x^2+1)^{-1/2}(2x)}{x^2+1} = \dfrac{1}{(x^2+1)^{3/2}}$

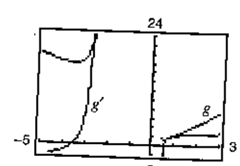
31. $\dfrac{-2(x+5)(x^2+10x-2)}{(x^2+2)^3}$ **33.** $\dfrac{-9(2v-1)^2}{(v+1)^4}$

35. $\dfrac{1-3x^2-4x^{3/2}}{2\sqrt{x}(x^2+1)^2}$



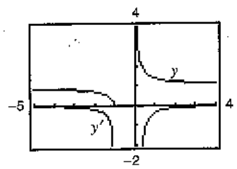
The zero of y' corresponds to the point on the graph of the function where the tangent line is horizontal.

37. $\dfrac{3t(t^2+3t-2)}{(t^2+2t-1)^{3/2}}$



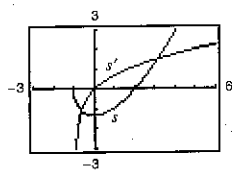
The zeros of $g'(t)$ correspond to the points on the graph of the function where the tangent line is horizontal.

39. $-\dfrac{\sqrt{\dfrac{x+1}{x}}}{2x(x+1)}$



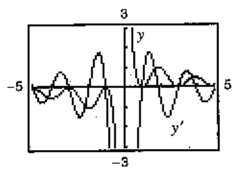
y' has no zeros.

41. $\dfrac{t}{\sqrt{1+t}}$



The zero of $s'(t)$ corresponds to the point on the graph of the function where the tangent line is horizontal.

43. $-\dfrac{\pi x \sin \pi x + \cos \pi x + 1}{x^2}$



The zeros of y' correspond to the points on the graph of the function where the tangent lines are horizontal.

45. (a) 1 (b) 2; The slope of $\sin ax$ at the origin is a.

47. $-3 \sin 3x$ **49.** $12 \sec^2 4x$ **51.** $2\pi^2 x \cos(\pi x)^2$

53. $2 \cos(4x)$ **55.** $\dfrac{-1 - \cos^2 x}{\sin^3 x}$

57. $8 \sec^2 x \tan x = \dfrac{8 \sin x}{\cos^3 x}$ **59.** $\sin 2\theta \cos 2\theta = \dfrac{1}{2} \sin 4\theta$

61. $\dfrac{6\pi \sin(\pi t - 1)}{\cos^3(\pi t - 1)}$ **63.** $\dfrac{1}{2\sqrt{x}} + 2x \cos(2x)^2$

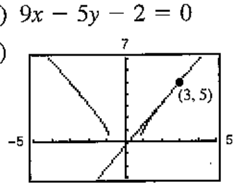
65. $-\sin x \cos(\cos x)$ **67.** $s'(t) = \dfrac{t+1}{\sqrt{t^2 + 2t + 8}}, \dfrac{3}{4}$

69. $f'(x) = \dfrac{-9x^2}{(x^3 - 4)^2}, -\dfrac{9}{25}$ **71.** $f'(t) = \dfrac{-5}{(t-1)^2}, -5$

73. $y' = -6 \sec^3(2x) \tan(2x), 0$

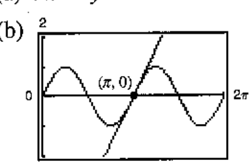
75. (a) $9x - 5y - 2 = 0$

(b)



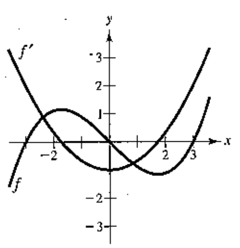
77. (a) $2x - y - 2\pi = 0$

(b)



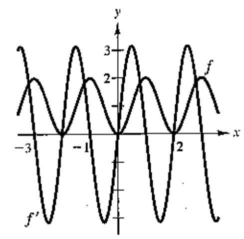
79. $12(5x^2 - 1)(x^2 - 1)$ **81.** $2(\cos x^2 - 2x^2 \sin x^2)$

83.



The zeros of f' correspond to the points where the graph of f has horizontal tangents.

85.



The zeros of f' correspond to the points where the graph of f has horizontal tangents.

87. The rate of change of g will be three times as fast as the rate of change of f.

89. (a) 24 (b) Not possible because $g'(h(5))$ is not known. (c) $\frac{4}{3}$ (d) 162

91. (a) 1.461 (b) -1.016

93. 0.2 radian, 1.45 radians per second **95.** 0.04224

97. (a) $x = -1.637t^3 + 19.31t^2 - 0.5t - 1$

(b) $\dfrac{dC}{dt} = -294.66t^2 + 2317.2t - 30$

(c) Because x, the number of units produced in t hours, is not a linear function, and therefore the cost with respect to time t is not linear.

99. (a) $f'(x) = \beta \cos \beta x$

$f''(x) = -\beta^2 \sin \beta x$

$f'''(x) = -\beta^3 \cos \beta x$

$f^{(4)}(x) = \beta^4 \sin \beta x$

(b) $f''(x) + \beta^2 f(x) = -\beta^2 \sin \beta x + \beta^2 \sin \beta x = 0$

(c) $f^{(2k)}(x) = (-1)^k \beta^{2k} \sin \beta x$

$f^{(2k-1)}(x) = (-1)^{k+1} \beta^{2k-1} \cos \beta x$

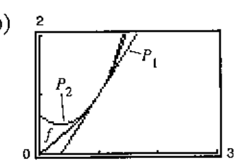
101. (a) 0 (b) $\dfrac{5}{8}$ **103.** Proof **105.** $\dfrac{2(2x-3)}{|2x-3|}, x \neq \dfrac{3}{2}$

107. $-|x| \sin x + \dfrac{x}{|x|} \cos x, \quad x \neq 0$

109. (a) $P_1(x) = \dfrac{\pi}{2}(x-1) + 1$

$P_2(x) = \dfrac{\pi^2}{8}(x-1)^2 + \dfrac{\pi}{2}(x-1) + 1$

(b)



(c) P_2

(d) P_1 and P_2 become less accurate as you move farther from $x = 1$.

111. False. $y' = -\frac{1}{2}(1 - x)^{-1/2}$ **113.** True

Section 2.5 (page 142)

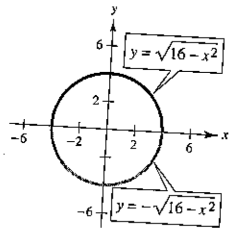
1. $-\dfrac{x}{y}$ **3.** $-\sqrt{\dfrac{y}{x}}$ **5.** $\dfrac{y - 3x^2}{2y - x}$ **7.** $\dfrac{1 - 3x^2y^3}{3x^3y^2 - 1}$

9. $\dfrac{6xy - 3x^2 - 2y^2}{4xy - 3x^2}$ **11.** $\dfrac{\cos x}{4 \sin 2y}$

13. $\dfrac{\cos x - \tan y - 1}{x \sec^2 y}$ **15.** $\dfrac{y \cos(xy)}{1 - x\cos(xy)}$

17. (a) $y_1 = \sqrt{16 - x^2}$
$y_2 = -\sqrt{16 - x^2}$

(b)

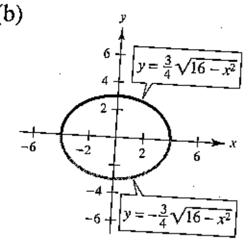


(c) $y' = \mp \dfrac{x}{\sqrt{16 - x^2}} = -\dfrac{x}{y}$

(d) $y' = -\dfrac{x}{y}$

19. (a) $y_1 = \frac{3}{4}\sqrt{16 - x^2}$
$y_2 = -\frac{3}{4}\sqrt{16 - x^2}$

(b)



(c) $y' = \mp \dfrac{3x}{4\sqrt{16 - x^2}} = -\dfrac{9x}{16y}$

(d) $y' = -\dfrac{9x}{16y}$

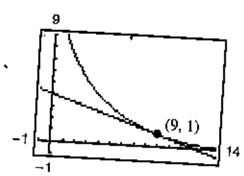
21. $-\dfrac{y}{x}, -\dfrac{1}{4}$ **23.** $\dfrac{8x}{y(x^2 + 4)^2}$, Undefined

25. $-\sqrt[3]{\dfrac{y}{x}}, -\dfrac{1}{2}$ **27.** $-\sin^2(x + y)$ or $-\dfrac{x^2}{x^2 + 1}, 0$

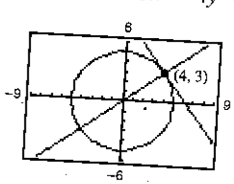
29. $-\dfrac{1}{2}$ **31.** 0 **33.** $\cos^2 y,\ -\dfrac{\pi}{2} < y < \dfrac{\pi}{2}, \dfrac{1}{1 + x^2}$

35. $-\dfrac{36}{y^3}$ **37.** $-\dfrac{16}{y^3}$ **39.** $\dfrac{3x}{4y}$

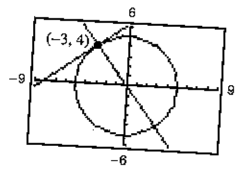
41. $x + 3y - 12 = 0$



43. At (4, 3):
Tangent line: $4x + 3y - 25 = 0$
Normal line: $3x - 4y = 0$



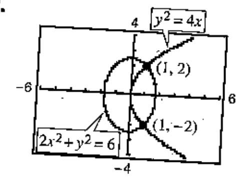
At (−3, 4):
Tangent line: $3x - 4y + 25 = 0$
Normal line: $4x + 3y = 0$



45. $x^2 + y^2 = r^2 \Rightarrow y' = -\dfrac{x}{y} \Rightarrow \dfrac{y}{x}$ = slope of normal line. Then for (x_0, y_0) on the circle, $x_0 \neq 0$, an equation of the normal line is $y = \dfrac{y_0}{x_0}x$, which passes through the origin. If $x_0 = 0$, the normal line is vertical and passes through the origin.

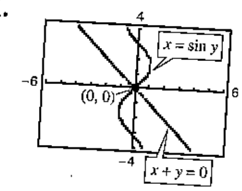
47. Horizontal tangents: $(-4, 0), (-4, 10)$
Vertical tangents: $(0, 5), (-8, 5)$

49.



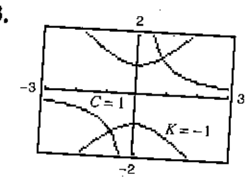
At (1, 2):
Slope of ellipse: −1
Slope of parabola: 1
At (1, −2):
Slope of ellipse: 1
Slope of parabola: −1

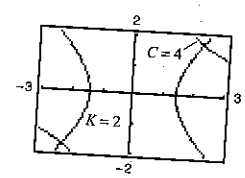
51.



At (0, 0):
Slope of line: −1
Slope of sine curve: 1

53.





Derivatives: $\dfrac{dy}{dx} = -\dfrac{y}{x}, \dfrac{dy}{dx} = \dfrac{x}{y}$

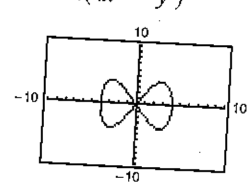
55. (a) $4y\dfrac{dy}{dx} - 12x^3 = 0$ (b) $4y\dfrac{dy}{dt} - 12x^3\dfrac{dx}{dt} = 0$

57. (a) $-\pi \sin \pi y\left(\dfrac{dy}{dx}\right) - 3\pi \cos \pi x = 0$

(b) $-\pi \sin \pi y\left(\dfrac{dy}{dt}\right) - 3\pi \cos \pi x\left(\dfrac{dx}{dt}\right) = 0$

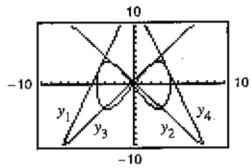
59. Answers will vary. In the explicit form of a function, the variable is explicitly written as a function of x. In an implicit equation, the function is only implied by an equation. An example of an implicit function is $x^2 + xy = 5$. In explicit form it would be $y = \dfrac{5 - x^2}{x}$.

61. (a) $x^4 = 4(4x^2 - y^2)$



(continued)

(b)



(c) $\left(\frac{8\sqrt{7}}{7}, 5\right)$

$$y_1 = \frac{1}{3}\left[\left(\sqrt{7}+7\right)x + \left(8\sqrt{7}+23\right)\right]$$

$$y_2 = -\frac{1}{3}\left[\left(-\sqrt{7}+7\right)x - \left(23-8\sqrt{7}\right)\right]$$

$$y_3 = -\frac{1}{3}\left[\left(\sqrt{7}-7\right)x - \left(23-8\sqrt{7}\right)\right]$$

$$y_4 = -\frac{1}{3}\left[\left(\sqrt{7}+7\right)x - \left(8\sqrt{7}+23\right)\right]$$

63. Proof

Section 2.6 (page 149)

1. (a) $\frac{3}{4}$ (b) 20 **3.** (a) $-\frac{5}{8}$ (b) $\frac{3}{2}$

5. (a) −4 centimeters per second
(b) 0 centimeter per second
(c) 4 centimeters per second

7. (a) 8 centimeters per second
(b) 4 centimeters per second
(c) 2 centimeters per second

9. (a) Positive (b) Negative

11. In a linear function, if x changes at a constant rate, so does y. However, unless $a = 1$, y does not change at the same rate as x.

13. $\frac{2(2x^3+3x)}{\sqrt{x^4+3x^2+1}}$

15. (a) 36π square centimeters per minute
(b) 144π square centimeters per minute

17. (a) Proof
(b) When $\theta = \frac{\pi}{6}, \frac{dA}{dt} = \frac{\sqrt{3}}{8}s^2$.
When $\theta = \frac{\pi}{3}, \frac{dA}{dt} = \frac{1}{8}s^2$.
(c) If s and $d\theta/dt$ are constant, dA/dt is proportional to $\cos\theta$.

19. (a) $\frac{2}{9\pi}$ centimeter per minute
(b) $\frac{1}{18\pi}$ centimeter per minute

21. (a) 36 square centimeters per second
(b) 360 square centimeters per second

23. $\frac{8}{405\pi}$ foot per minute

25. (a) 12.5% (b) $\frac{1}{144}$ meter per minute

27. (a) $-\frac{7}{12}$ foot per second; $-\frac{3}{2}$ feet per second; $-\frac{48}{7}$ feet per second
(b) $\frac{527}{24}$ square feet per second (c) $\frac{1}{12}$ radian per second

29. Rate of vertical change: $\frac{1}{5}$ meter per second
Rate of horizontal change: $-\frac{\sqrt{3}}{15}$ meter per second

31. (a) −750 miles per hour (b) 20 minutes

33. $-\frac{28}{\sqrt{10}} \approx -8.85$ feet per second

35. (a) $\frac{25}{3}$ feet per second (b) $\frac{10}{3}$ feet per second

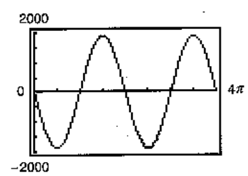
37. (a) 12 seconds
(b) $\frac{1}{2}\sqrt{3}$ meter
(c) $\frac{\sqrt{5}\,\pi}{120}$ meter per second

39. Evaporation rate proportional to
$S \Rightarrow \frac{dV}{dt} = k(4\pi r^2)$
$V = \left(\frac{4}{3}\right)\pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$
So $k = \frac{dr}{dt}$.

41. $V^{0.3}\left(1.3p\frac{dV}{dt} + V\frac{dp}{dt}\right) = 0$ **43.** $\frac{1}{20}$ radian per second

45. (a) $\frac{1}{2}$ radian per minute (b) $\frac{3}{2}$ radians per minute
(c) 1.87 radians per minute

47. (a) $\frac{dx}{dt} = -600\pi\sin\theta$
(b)



(c) $\theta = 90° + n \cdot 180°$; $\theta = 0° + n \cdot 180°$
(d) -300π centimeters per second; $-300\sqrt{3}\pi$ centimeters per second

49. $\frac{1}{25}\cos^2\theta$, $-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$

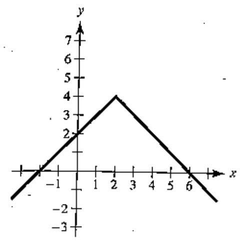
51. −0.1808 foot per second per second

53. (a) $m(s) = -0.881s^2 + 29.10s - 206.2$
(b) $(-1.762s + 29.10)\frac{ds}{dt}$ (c) 2.15 million

Review Exercises for Chapter 2 (page 153)

1. $f'(x) = 2x - 2$ **3.** $f'(x) = \frac{1}{2\sqrt{x}} = \frac{\sqrt{x}}{2x}$

5. f is differentiable at all $x \neq -1$.

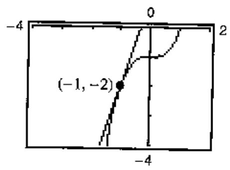
7. 

(a) Yes

(b) No, because the derivatives from the left and right are not equal.

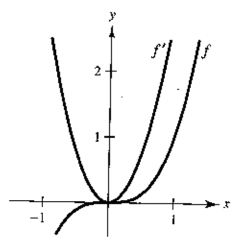
9. $-\frac{3}{2}$

11. (a) $y = 3x + 1$

(b) 

13. 8

15. $f'(x) = 2x - 2$



$f' > 0$ where the slopes of tangent lines to the graph of f are positive.

17. 0 19. $8x^7$ 21. $12t^3$ 23. $3x(x - 2)$

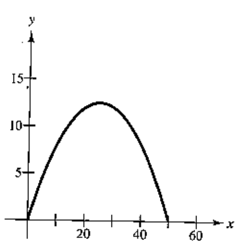
25. $\dfrac{3}{\sqrt{x}} + \dfrac{1}{x^{2/3}}$ 27. $-\dfrac{4}{3t^3}$ 29. $2 - 3\cos\theta$

31. $-3\sin\theta - \dfrac{\cos\theta}{4}$

33. (a) 50 vibrations per second per pound

(b) 33.33 vibrations per second per pound

35. 414.74 meters or 1,354 feet

37. (a) 

(b) 50

(c) $x = 25$

(d) $y' = 1 - 0.04x$

x	0	10	25	30	50
y'	1	0.6	0	-0.2	-1

(e) $y'(25) = 0$

39. (a) $x'(t) = 2t - 3$ (b) $(-\infty, 1.5)$ (c) $x = -\frac{1}{4}$ (d) 1

41. $2(6x^3 - 9x^2 + 16x - 7)$ 43. $\sqrt{x}\cos x + \dfrac{1}{2\sqrt{x}}\sin x$

45. $2 + \dfrac{2}{x^3}$ 47. $-\dfrac{x^2 + 1}{(x^2 - 1)^2}$ 49. $\dfrac{6x}{(4 - 3x^2)^2}$

51. $\dfrac{2x\cos x + x^2 \sin x}{\cos^2 x}$ 53. $3x^2 \sec x \tan x + 6x \sec x$

55. $-x\sec^2 x - \tan x$ 57. $-x\sin x$ 59. $6t$

61. $6\sec^2\theta\tan\theta$

63. $y'' + y = -(2\sin x + 3\cos x) + (2\sin x + 3\cos x) = 0$

65. $\dfrac{-3x^2}{2\sqrt{1 - x^3}}$ 67. $\dfrac{2(x - 3)(-x^2 + 6x + 1)}{(x^2 + 1)^3}$

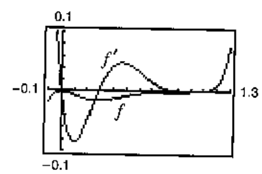
69. $s(s^2 - 1)^{3/2}(8s^3 - 3s + 25)$ 71. $-9\sin(3x + 1)$

73. $-\csc 2x \cot 2x$ 75. $\frac{1}{2}(1 - \cos 2x) = \sin^2 x$

77. $\sin^{1/2} x \cos x - \sin^{5/2} x \cos x = \cos^3 x\sqrt{\sin x}$

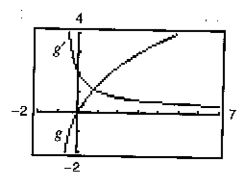
79. $\dfrac{(x + 2)[\pi\cos(\pi x)] - \sin(\pi x)}{(x + 2)^2}$

81. $t(t - 1)^4(7t - 2)$



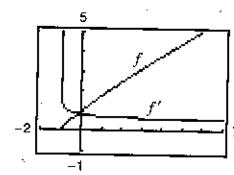
The zeros of f' correspond to the points on the graph of the function where the tangent line is horizontal.

83. $\dfrac{x + 2}{(x + 1)^{3/2}}$



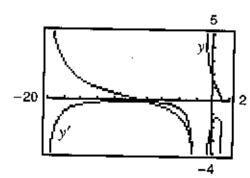
g' is not equal to zero for any x.

85. $\dfrac{5}{6(t + 1)^{1/6}}$



f' has no zeros.

87. $-\dfrac{\sec^2\sqrt{1 - x}}{2\sqrt{1 - x}}$



y' has no zeros.

89. $4 - 4\sin 2x$ 91. $2\csc^2 x \cot x$

93. $\dfrac{2(t + 2)}{(1 - t)^4}$ 95. $18\sec^2(3\theta)\tan(3\theta) + \sin(\theta - 1)$

97. (a) -18.667 degrees per hour

(b) -7.284 degrees per hour

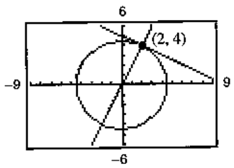
(c) -3.240 degrees per hour

(d) -0.747 degree per hour

99. $-\dfrac{2x+3y}{3(x+y^2)}$ **101.** $\dfrac{2y\sqrt{x}-y\sqrt{y}}{2x\sqrt{y}-x\sqrt{x}}$ **103.** $\dfrac{y\sin x+\sin y}{\cos x - x\cos y}$

105. Tangent line: $x+2y-10=0$

Normal line: $2x-y=0$

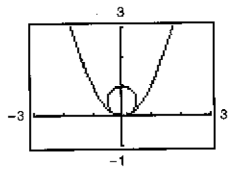


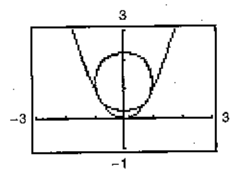
107. (a) $2\sqrt{2}$ units/sec (b) 4 units/sec (c) 8 units/sec

109. $\frac{2}{25}$ meter per minute **111.** -38.34 meters per second

P.S. Problem Solving (page 156)

1. (a) $r=\frac{1}{2}$ (b) Center: $\left(0,\frac{5}{4}\right)$





3. (a) $P_1(x)=1$

(b) $P_2(x)=1-\frac{1}{2}x^2$

(c)

x	-1.0	-0.1	-0.001	0	0.001
$\cos x$	0.5403	0.9950	1.000	1	1
$P_2(x)$	0.5000	0.9950	1.000	1	1

x	0.1	1.0
$\cos x$	0.9950	0.5403
$P_2(x)$	0.9950	0.5000

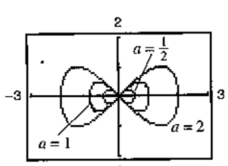
$P_2(x)$ is a good approximation of $f(x)=\cos x$ when x is very close to 0.

(d) $P_3(x)=x-\frac{1}{6}x^3$

5. $p(x)=2x^3+4x^2-5$

7. (a) Graph $\begin{cases} y_1=\dfrac{1}{a}\sqrt{x^2(a^2-x^2)} \\ y_2=-\dfrac{1}{a}\sqrt{x^2(a^2-x^2)} \end{cases}$ as separate equations.

(b) Answers will vary.



The intercepts will always be $(0,0)$, $(a,0)$, and $(-a,0)$, and the maximum and minimum y-values appear to be $\pm\frac{1}{2}a$.

(c) $\left(\dfrac{a\sqrt{2}}{2},\dfrac{a}{2}\right)$, $\left(\dfrac{a\sqrt{2}}{2},-\dfrac{a}{2}\right)$, $\left(-\dfrac{a\sqrt{2}}{2},\dfrac{a}{2}\right)$, $\left(-\dfrac{a\sqrt{2}}{2},-\dfrac{a}{2}\right)$

9. (a) When the man is 90 feet from the light, the tip of his shadow is $112\frac{1}{2}$ feet from the light. The tip of the child's shadow is $111\frac{1}{9}$ feet from the light, so the man's shadow extends $1\frac{7}{18}$ feet beyond the child's shadow.

(b) When the man is 60 feet from the light, the tip of his shadow is 75 feet from the light. The tip of the child's shadow is $77\frac{7}{9}$ feet from the light, so the child's shadow extends $2\frac{7}{9}$ feet beyond the man's shadow.

(c) $d=80$ feet

(d) Let x be the distance of the man from the light and s be the distance from the light to the tip of the shadow.

If $0<x<80$, $\dfrac{ds}{dt}=-\dfrac{50}{9}$.

If $x>80$, $\dfrac{ds}{dt}=-\dfrac{25}{4}$.

There is a discontinuity at $x=80$.

11. Proof. The graph of L is a line passing through the origin $(0,0)$.

13. (a)

$z°$	0.1	0.01	0.0001
$\dfrac{\sin z}{z}$	0.01745241	0.0174532924	0.0174532925

(b) $\dfrac{\pi}{180}$ (c) $\dfrac{\pi}{180}\cos z$ (d) $\dfrac{\pi}{180}C(z)$

(e) Answers will vary.

15. (a) j would be the rate of change of acceleration/deceleration.

(b) $j=0$. Deceleration is constant, so there is no change in deceleration.

Chapter 3

Section 3.1 (page 165)

1. $f'(0)=0$ **3.** $f'(3)=0$

5. $f'(-2)$ is undefined. **7.** 2, absolute maximum

9. 1, absolute maximum; 2, absolute minimum; 3, absolute maximum

11. $x=0,\ x=2$ **13.** $t=\dfrac{8}{3}$ **15.** $x=\dfrac{\pi}{3},\pi,\dfrac{5\pi}{3}$

17. Minimum: $(2,2)$
Maximum: $(-1,8)$

19. Minimum: $(0,0)$ and $(3,0)$
Maximum: $\left(\frac{3}{2},\frac{9}{4}\right)$

21. Minimum: $\left(-1,-\frac{5}{2}\right)$
Maximum: $(2,2)$

23. Minimum: $(0,0)$
Maximum: $(-1,5)$

25. Minimum: $(0,0)$
Maximum: $\left(-1,\frac{1}{4}\right)$ and $\left(1,\frac{1}{4}\right)$

27. Minimum: $(1,-1)$
Maximum: $\left(0,-\frac{1}{2}\right)$

29. Minimum: $\left(\dfrac{1}{6},\dfrac{\sqrt{3}}{2}\right)$
Maximum: $(0,1)$

31. Minimum: $(2,3)$
Maximum: $\left(1,\sqrt{2}+3\right)$

33. (a) Minimum: $(0,-3)$;
Maximum: $(2,1)$
(b) Minimum: $(0,-3)$
(c) Maximum: $(2,1)$
(d) No extrema

35. (a) Minimum: $(1,-1)$;
Maximum: $(-1,3)$
(b) Minimum: $(3,3)$
(c) Minimum: $(1,-1)$
(d) Minimum: $(1,-1)$