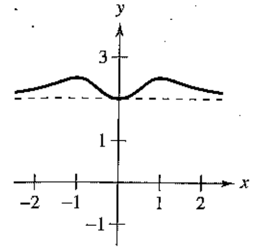
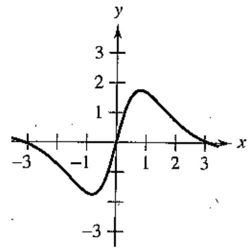
EXERCISES FOR SECTION 3.5

In Exercises 1–6, match the function with one of the graphs [(a), (b), (c), (d), (e), or (f)] using horizontal asymptotes as an aid.

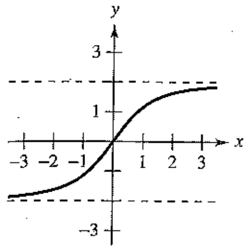
(a)



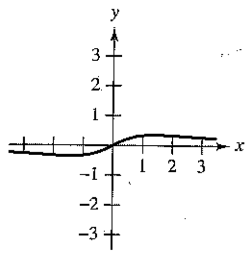
(b)



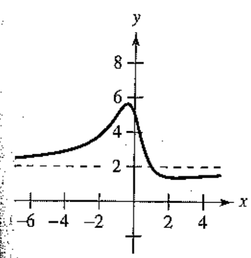
(c)



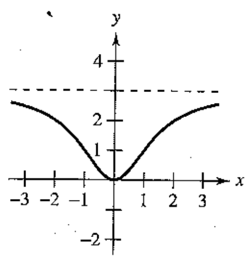
(d)



(e)



(f)



1. $f(x) = \dfrac{3x^2}{x^2 + 2}$ **2.** $f(x) = \dfrac{2x}{\sqrt{x^2 + 2}}$

3. $f(x) = \dfrac{x}{x^2 + 2}$ **4.** $f(x) = 2 + \dfrac{x^2}{x^4 + 1}$

5. $f(x) = \dfrac{4 \sin x}{x^2 + 1}$ **6.** $f(x) = \dfrac{2x^2 - 3x + 5}{x^2 + 1}$

Numerical and Graphical Analysis **In Exercises 7–12, use a graphing utility to complete the table and estimate the limit as x approaches infinity. Then use a graphing utility to graph the function and estimate the limit graphically.**

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$							

7. $f(x) = \dfrac{4x + 3}{2x - 1}$ **8.** $f(x) = \dfrac{2x^2}{x + 1}$

9. $f(x) = \dfrac{-6x}{\sqrt{4x^2 + 5}}$ **10.** $f(x) = \dfrac{8x}{\sqrt{x^2 - 3}}$

11. $f(x) = 5 - \dfrac{1}{x^2 + 1}$ **12.** $f(x) = 4 + \dfrac{3}{x^2 + 2}$

In Exercises 13 and 14, find $\lim_{x\to\infty} h(x)$, if possible.

13. $f(x) = 5x^3 - 3x^2 + 10$

(a) $h(x) = \dfrac{f(x)}{x^2}$ (b) $h(x) = \dfrac{f(x)}{x^3}$

(c) $h(x) = \dfrac{f(x)}{x^4}$

14. $f(x) = 5x^2 - 3x + 7$

(a) $h(x) = \dfrac{f(x)}{x}$ (b) $h(x) = \dfrac{f(x)}{x^2}$

(c) $h(x) = \dfrac{f(x)}{x^3}$

In Exercises 15–18, find each of the limits, if possible.

15. (a) $\lim_{x\to\infty} \dfrac{x^2 + 2}{x^3 - 1}$ (b) $\lim_{x\to\infty} \dfrac{x^2 + 2}{x^2 - 1}$ (c) $\lim_{x\to\infty} \dfrac{x^2 + 2}{x - 1}$

16. (a) $\lim_{x\to\infty} \dfrac{3 - 2x}{3x^3 - 1}$ (b) $\lim_{x\to\infty} \dfrac{3 - 2x}{3x - 1}$ (c) $\lim_{x\to\infty} \dfrac{3 - 2x^2}{3x - 1}$

17. (a) $\lim_{x\to\infty} \dfrac{5 - 2x^{3/2}}{3x^2 - 4}$ (b) $\lim_{x\to\infty} \dfrac{5 - 2x^{3/2}}{3x^{3/2} - 4}$ (c) $\lim_{x\to\infty} \dfrac{5 - 2x^{3/2}}{3x - 4}$

18. (a) $\lim_{x\to\infty} \dfrac{5x^{3/2}}{4x^2 + 1}$ (b) $\lim_{x\to\infty} \dfrac{5x^{3/2}}{4x^{3/2} + 1}$ (c) $\lim_{x\to\infty} \dfrac{5x^{3/2}}{4\sqrt{x} + 1}$

In Exercises 19–32, find the limit.

19. $\lim_{x\to\infty} \dfrac{2x - 1}{3x + 2}$ **20.** $\lim_{x\to\infty} \dfrac{3x^3 + 2}{9x^3 - 2x^2 + 7}$

21. $\lim_{x\to\infty} \dfrac{x}{x^2 - 1}$ **22.** $\lim_{x\to\infty} \left(4 + \dfrac{3}{x}\right)$

23. $\lim_{x\to-\infty} \dfrac{5x^2}{x + 3}$ **24.** $\lim_{x\to-\infty} \left(\dfrac{1}{2}x - \dfrac{4}{x^2}\right)$

25. $\lim_{x\to-\infty} \dfrac{x}{\sqrt{x^2 - x}}$ **26.** $\lim_{x\to-\infty} \dfrac{x}{\sqrt{x^2 + 1}}$

27. $\lim_{x\to-\infty} \dfrac{2x + 1}{\sqrt{x^2 - x}}$ **28.** $\lim_{x\to-\infty} \dfrac{-3x + 1}{\sqrt{x^2 + x}}$

29. $\lim_{x\to\infty} \dfrac{\sin 2x}{x}$ **30.** $\lim_{x\to\infty} \dfrac{x - \cos x}{x}$

31. $\lim_{x\to\infty} \dfrac{1}{2x + \sin x}$ **32.** $\lim_{x\to\infty} \cos \dfrac{1}{x}$

In Exercises 33 and 34, use a graphing utility to graph the function and verify that it has two horizontal asymptotes.

33. $f(x) = \dfrac{|x|}{x + 1}$ **34.** $f(x) = \dfrac{3x}{\sqrt{x^2 + 2}}$

In Exercises 35 and 36, find the limit. (*Hint:* Let $x = 1/t$ and find the limit as $t \to 0^+$.)

35. $\lim_{x\to\infty} x \sin \frac{1}{x}$ **36.** $\lim_{x\to\infty} x \tan \frac{1}{x}$

In Exercises 37–40, find the limit. (*Hint:* Treat the expression as a fraction whose denominator is 1, and rationalize the numerator.) Use a graphing utility to verify your result.

37. $\lim_{x\to-\infty} \left(x + \sqrt{x^2+3}\right)$ **38.** $\lim_{x\to\infty} \left(2x - \sqrt{4x^2+1}\right)$

39. $\lim_{x\to\infty} \left(x - \sqrt{x^2+x}\right)$ **40.** $\lim_{x\to-\infty} \left(3x + \sqrt{9x^2-x}\right)$

Numerical, Graphical, and Analytic Analysis **In Exercises 41–44, use a graphing utility to complete the table and estimate the limit as x approaches infinity. Then use a graphing utility to graph the function and estimate the limit graphically. Finally, find the limit analytically and compare your results with the estimates.**

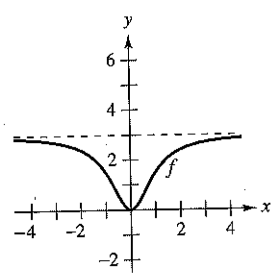
x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$							

41. $f(x) = x - \sqrt{x(x-1)}$ **42.** $f(x) = x^2 - x\sqrt{x(x-1)}$

43. $f(x) = x \sin \frac{1}{2x}$ **44.** $f(x) = \frac{x+1}{x\sqrt{x}}$

Getting at the Concept

45. The graph of a function f is shown below. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.



(a) Sketch f'.

(b) Use the graphs to estimate $\lim_{x\to\infty} f(x)$ and $\lim_{x\to\infty} f'(x)$.

(c) Explain the answers you gave in part (b).

46. Sketch a graph of a differentiable function f that satisfies the following conditions and has $x = 2$ as its only critical number.

$f'(x) < 0$ for $x < 2$

$f'(x) > 0$ for $x > 2$

$\lim_{x\to-\infty} f(x) = \lim_{x\to\infty} f(x) = 6$

47. Is it possible to sketch a graph of a function that satisfies the conditions of Exercise 46 and has *no* points of inflection? Explain.

Getting at the Concept *(continued)*

48. If f is a continuous function such that $\lim_{x\to\infty} f(x) = 5$, find, if possible, $\lim_{x\to-\infty} f(x)$ for each specified condition.

(a) The graph of f is symmetric to the y-axis.

(b) The graph of f is symmetric to the origin.

In Exercises 49–66, sketch the graph of the equation. Look for extrema, intercepts, symmetry, and asymptotes as necessary. Use a graphing utility to verify your result.

49. $y = \frac{2+x}{1-x}$ **50.** $y = \frac{x-3}{x-2}$

51. $y = \frac{x}{x^2-4}$ **52.** $y = \frac{2x}{9-x^2}$

53. $y = \frac{x^2}{x^2+9}$ **54.** $y = \frac{x^2}{x^2-9}$

55. $y = \frac{2x^2}{x^2-4}$ **56.** $y = \frac{2x^2}{x^2+4}$

57. $xy^2 = 4$ **58.** $x^2y = 4$

59. $y = \frac{2x}{1-x}$ **60.** $y = \frac{2x}{1-x^2}$

61. $y = 2 - \frac{3}{x^2}$ **62.** $y = 1 + \frac{1}{x}$

63. $y = 3 + \frac{2}{x}$ **64.** $y = 4\left(1 - \frac{1}{x^2}\right)$

65. $y = \frac{x^3}{\sqrt{x^2-4}}$ **66.** $y = \frac{x}{\sqrt{x^2-4}}$

In Exercises 67–76, use a computer algebra system to analyze the graph of the function. Label any extrema and/or asymptotes that exist.

67. $f(x) = 5 - \frac{1}{x^2}$

68. $f(x) = \frac{x^2}{x^2-1}$

69. $f(x) = \frac{x}{x^2-4}$

70. $f(x) = \frac{1}{x^2-x-2}$

71. $f(x) = \frac{x-2}{x^2-4x+3}$

72. $f(x) = \frac{x+1}{x^2+x+1}$

73. $f(x) = \frac{3x}{\sqrt{4x^2+1}}$

74. $g(x) = \frac{2x}{\sqrt{3x^2+1}}$

75. $g(x) = \sin\left(\frac{x}{x-2}\right), \quad 3 < x < \infty$

76. $f(x) = \frac{2 \sin 2x}{x}$

In Exercises 77 and 78, (a) use a graphing utility to graph f and g in the same viewing window, (b) verify algebraically that f and g represent the same function, and (c) zoom out sufficiently far so that the graph appears as a line. What equation does this line appear to have? (Note that the points at which the function is not continuous are not readily seen when you zoom out.)

77. $f(x) = \dfrac{x^3 - 3x^2 + 2}{x(x-3)}$

$g(x) = x + \dfrac{2}{x(x-3)}$

78. $f(x) = -\dfrac{x^3 - 2x^2 + 2}{2x^2}$

$g(x) = -\dfrac{1}{2}x + 1 - \dfrac{1}{x^2}$

79. ***Average Cost*** A business has a cost of $C = 0.5x + 500$ for producing x units. The average cost per unit is

$$\overline{C} = \frac{C}{x}.$$

Find the limit of $\overline{C}$ as x approaches infinity.

80. ***Engine Efficiency*** The efficiency of an internal combustion engine is

$$\text{Efficiency } (\%) = 100\left[1 - \frac{1}{(v_1/v_2)^c}\right]$$

where v_1/v_2 is the ratio of the uncompressed gas to the compressed gas and c is a positive constant dependent on the engine design. Find the limit of the efficiency as the compression ratio approaches infinity.

81. A line with slope m passes through the point $(0, 4)$.

(a) Write the distance d between the line and the point $(3, 1)$ as a function of m.

(b) Use a graphing utility to graph the equation in part (a).

(c) Find $\lim_{m\to\infty} d(m)$ and $\lim_{m\to-\infty} d(m)$. Interpret the results geometrically.

82. ***Modeling Data*** The table shows the world record times for running one mile, where t represents the year with $t = 0$ corresponding to 1900, and y is the time in minutes and seconds.

t	23	33	45	54	58	66	79	85	99
y	4:10.4	4:07.6	4:01.3	3:59.4	3:54.5	3:51.3	3:48.9	3:46.3	3:43.1

A model for the data is

$$y = \frac{3.351t^2 + 42.461t - 543.730}{t^2}$$

where the seconds have been changed to a decimal part of a minute.

(a) Use a graphing utility to plot the data and graph the model.

(b) Does there appear to be a limiting time for running one mile? Explain.

83. ***Modeling Data*** A heat probe is attached to the heat exchanger of a heating system. The temperature T (degrees Celsius) is recorded t seconds after the furnace is started. The results for the first 2 minutes are recorded in the table.

t	0	15	30	45	60	75	90	105	120
T	25.2°	36.9°	45.5°	51.4°	56.0°	59.6°	62.0°	64.0°	65.2°

(a) Use the regression capabilities of a graphing utility to find a model of the form $T_1 = at^2 + bt + c$ for the data.

(b) Use a graphing utility to graph T_1.

(c) A rational model for the data is

$$T_2 = \frac{1451 + 86t}{58 + t}.$$

Use a graphing utility to graph the model.

(d) Find $T_1(0)$ and $T_2(0)$.

(e) Find $\lim_{t\to\infty} T_2$.

(f) Interpret the result in part (e) in the context of the problem. Is it possible to do this type of analysis using T_1? Explain.

84. ***Modeling Data*** The average typing speed S of a typing student after t weeks of lessons is shown in the table.

t	5	10	15	20	25	30
S	28	56	79	90	93	94

A model for the data is $S = \dfrac{100t^2}{65 + t^2}$, $t > 0$.

(a) Use a graphing utility to plot the data and graph the model.

(b) Does there appear to be a limiting typing speed? Explain.

85. In your own words, state the guidelines for finding the limit of a rational function. Give examples.

86. Prove that if $p(x) = a_nx^n + \cdots + a_1x + a_0$ and $q(x) = b_mx^m + \cdots + b_1x + b_0$ $(a_n \neq 0, b_m \neq 0)$, then

$$\lim_{x\to\infty} \frac{p(x)}{q(x)} = \begin{cases} 0, & n < m \\ \dfrac{a_n}{b_m}, & n = m \\ \pm\infty, & n > m. \end{cases}$$

True or False? **In Exercises 87 and 88, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

87. If $f'(x) > 0$ for all real numbers x, then f increases without bound.

88. If $f''(x) < 0$ for all real numbers x, then f decreases without bound.