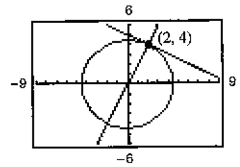
99. $-\dfrac{2x + 3y}{3(x + y^2)}$ **101.** $\dfrac{2y\sqrt{x} - y\sqrt{y}}{2x\sqrt{y} - x\sqrt{x}}$ **103.** $\dfrac{y \sin x + \sin y}{\cos x - x \cos y}$

105. Tangent line: $x + 2y - 10 = 0$

Normal line: $2x - y = 0$

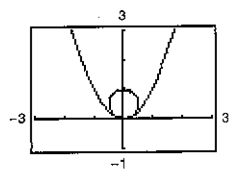


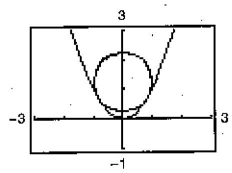
107. (a) $2\sqrt{2}$ units/sec (b) 4 units/sec (c) 8 units/sec

109. $\frac{2}{25}$ meter per minute **111.** -38.34 meters per second

P.S. Problem Solving (page 156)

1. (a) $r = \frac{1}{2}$ (b) Center: $\left(0, \frac{5}{4}\right)$





3. (a) $P_1(x) = 1$

(b) $P_2(x) = 1 - \frac{1}{2}x^2$

(c)

x	-1.0	-0.1	-0.001	0	0.001
$\cos x$	0.5403	0.9950	1.000	1	1
$P_2(x)$	0.5000	0.9950	1.000	1	1

x	0.1	1.0
$\cos x$	0.9950	0.5403
$P_2(x)$	0.9950	0.5000

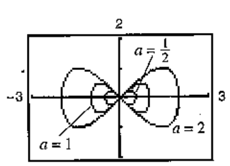
$P_2(x)$ is a good approximation of $f(x) = \cos x$ when x is very close to 0.

(d) $P_3(x) = x - \frac{1}{6}x^3$

5. $p(x) = 2x^3 + 4x^2 - 5$

7. (a) Graph $\begin{cases} y_1 = \dfrac{1}{a}\sqrt{x^2(a^2 - x^2)} \\ y_2 = -\dfrac{1}{a}\sqrt{x^2(a^2 - x^2)} \end{cases}$ as separate equations.

(b) Answers will vary.



The intercepts will always be $(0, 0)$, $(a, 0)$, and $(-a, 0)$, and the maximum and minimum y-values appear to be $\pm\frac{1}{2}a$.

(c) $\left(\dfrac{a\sqrt{2}}{2}, \dfrac{a}{2}\right), \left(\dfrac{a\sqrt{2}}{2}, -\dfrac{a}{2}\right), \left(-\dfrac{a\sqrt{2}}{2}, \dfrac{a}{2}\right), \left(-\dfrac{a\sqrt{2}}{2}, -\dfrac{a}{2}\right)$

9. (a) When the man is 90 feet from the light, the tip of his shadow is $112\frac{1}{2}$ feet from the light. The tip of the child's shadow is $111\frac{1}{9}$ feet from the light, so the man's shadow extends $1\frac{7}{18}$ feet beyond the child's shadow.

(b) When the man is 60 feet from the light, the tip of his shadow is 75 feet from the light. The tip of the child's shadow is $77\frac{7}{9}$ feet from the light, so the child's shadow extends $2\frac{7}{9}$ feet beyond the man's shadow.

(c) $d = 80$ feet

(d) Let x be the distance of the man from the light and s be the distance from the light to the tip of the shadow.

If $0 < x < 80$, $\dfrac{ds}{dt} = -\dfrac{50}{9}$.

If $x > 80$, $\dfrac{ds}{dt} = -\dfrac{25}{4}$.

There is a discontinuity at $x = 80$.

11. Proof. The graph of L is a line passing through the origin $(0, 0)$.

13. (a)

$z°$	0.1	0.01	0.0001
$\dfrac{\sin z}{z}$	0.01745241	0.0174532924	0.0174532925

(b) $\dfrac{\pi}{180}$ (c) $\dfrac{\pi}{180}\cos z$ (d) $\dfrac{\pi}{180}C(z)$

(e) Answers will vary.

15. (a) j would be the rate of change of acceleration/deceleration.

(b) $j = 0$. Deceleration is constant, so there is no change in deceleration.

Chapter 3

Section 3.1 (page 165)

1. $f'(0) = 0$ **3.** $f'(3) = 0$

5. $f'(-2)$ is undefined. **7.** 2, absolute maximum

9. 1, absolute maximum; 2, absolute minimum; 3, absolute maximum

11. $x = 0,\ x = 2$ **13.** $t = \dfrac{8}{3}$ **15.** $x = \dfrac{\pi}{3}, \pi, \dfrac{5\pi}{3}$

17. Minimum: $(2, 2)$
Maximum: $(-1, 8)$

19. Minimum: $(0, 0)$ and $(3, 0)$
Maximum: $\left(\frac{3}{2}, \frac{9}{4}\right)$

21. Minimum: $\left(-1, -\frac{5}{2}\right)$
Maximum: $(2, 2)$

23. Minimum: $(0, 0)$
Maximum: $(-1, 5)$

25. Minimum: $(0, 0)$
Maximum: $\left(-1, \frac{1}{4}\right)$ and $\left(1, \frac{1}{4}\right)$

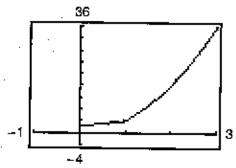
27. Minimum: $(1, -1)$
Maximum: $\left(0, -\frac{1}{2}\right)$

29. Minimum: $\left(\dfrac{1}{6}, \dfrac{\sqrt{3}}{2}\right)$
Maximum: $(0, 1)$

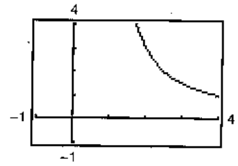
31. Minimum: $(2, 3)$
Maximum: $\left(1, \sqrt{2} + 3\right)$

33. (a) Minimum: $(0, -3)$; Maximum: $(2, 1)$
(b) Minimum: $(0, -3)$
(c) Maximum: $(2, 1)$
(d) No extrema

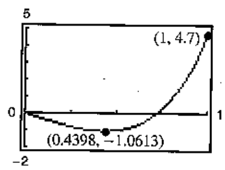
35. (a) Minimum: $(1, -1)$; Maximum: $(-1, 3)$
(b) Minimum: $(3, 3)$
(c) Minimum: $(1, -1)$
(d) Minimum: $(1, -1)$

37. 

Minimum: (0, 2)
Maximum: (3, 36)

39. 

Minimum: (4, 1)

41. (a) 

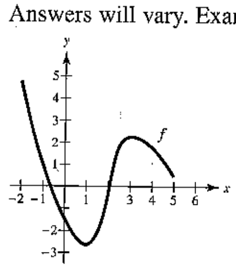
(b) Minimum: (0.4398, −1.0613)

43. Maximum: $\left|f''\left(\sqrt[3]{-10+\sqrt{108}}\right)\right| = f''\left(\sqrt{3}-1\right) \approx 1.47$

45. Maximum: $\left|f^{(4)}(0)\right| = \frac{56}{81}$

47. Because f is continuous on $\left[0, \frac{\pi}{4}\right]$, but not continuous on $[0, \pi]$.

49. Answers will vary. Example:



51. (a) Yes (b) No **53.** (a) No (b) Yes

55. Maximum: $P(12) = 72$

No. P is decreasing for $I \geq 12$.

57. 0.9553 radian

59. (a) $y = \dfrac{3}{40{,}000}x^2 - \dfrac{3}{200}x + \dfrac{75}{4}$

(b)

x	−500	−400	−300	−200	−100	0
d	0	0.75	3	6.75	12	18.75

x	100	200	300	400	500
d	12	6.75	3	0.75	0

(c) Lowest point ≈ (100, 18); No

61. True **63.** True

Section 3.2 (page 172)

1. $f(0) = f(2) = 0$; f is not differentiable on (0, 2).

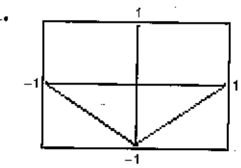
3. (2, 0), (−1, 0); $f'\left(\frac{1}{2}\right) = 0$ **5.** (0, 0), (−4, 0); $f'\left(-\frac{8}{3}\right) = 0$

7. $f'(1) = 0$ **9.** $f'\left(\dfrac{6-\sqrt{3}}{3}\right) = 0$; $f'\left(\dfrac{6+\sqrt{3}}{3}\right) = 0$

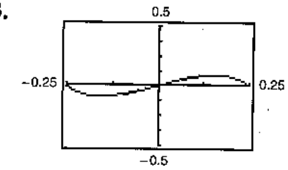
11. Not differentiable at $x = 0$ **13.** $f'\left(-2+\sqrt{5}\right) = 0$

15. $f'\left(\dfrac{\pi}{2}\right) = 0$; $f'\left(\dfrac{3\pi}{2}\right) = 0$ **17.** $f'(0.249) \approx 0$

19. Not continuous on $[0, \pi]$

21. 

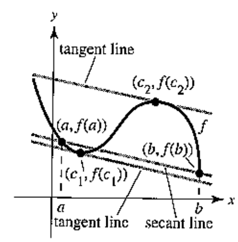
Rolle's Theorem does not apply.

23. 

$f'(\pm 0.1533) = 0$

25. (a) $f(1) = f(2) = 64$

(b) Velocity = 0 for some t in (−1, 2); $t = \frac{3}{2}$ seconds

27. 

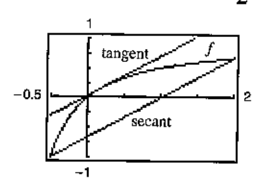
29. The function is discontinuous on [0, 6].

31. $f'\left(-\frac{1}{2}\right) = -1$ **33.** $f'\left(\frac{8}{27}\right) = 1$

35. $f'\left(-\dfrac{1}{4}\right) = -\dfrac{1}{3}$ **37.** $f'\left(\dfrac{\pi}{2}\right) = 0$

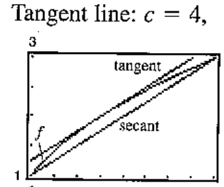
39. Secant line: $2x - 3y - 2 = 0$

Tangent line: $c = \dfrac{-2+\sqrt{6}}{2}$, $2x - 3y + 5 - 2\sqrt{6} = 0$



41. Secant line: $x - 4y + 3 = 0$

Tangent line: $c = 4$, $x - 4y + 4 = 0$



43. (a) −14.7 meters per second (b) 1.5 seconds

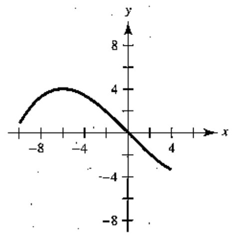
45. No. Let $f(x) = x^2$ on [−1, 2].

47. By the Mean Value Theorem, there is a time when the speed of the plane must equal the average speed of 454.5 miles per hour. The speed was 400 miles per hour when the plane was accelerating to 454.5 miles per hour and decelerating from 454.5 miles per hour.

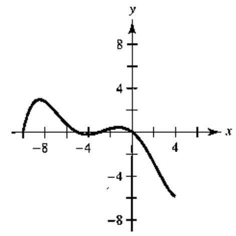
49. (a) f is continuous and changes signs in $[-10, 4]$ (Intermediate Value Theorem).

(b) There exist real numbers a and b such that $-10 < a < b < 4$ and $f(a) = f(b) = 2$. Therefore, f' has a zero in the interval by Rolle's Theorem.

(c)

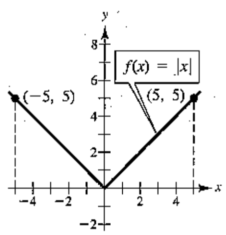


(d)



(e) No, by Theorem 2.1.

51.



53. False. f is not continuous on $[-1, 1]$.

55. True **57.** Proof **59.** Proof **61.** Proof

Section 3.3 (page 181)

1. Increasing on $(3, \infty)$; Decreasing on $(-\infty, 3)$

3. Increasing on $(-\infty, -2)$ and $(2, \infty)$; Decreasing on $(-2, 2)$

5. Increasing on $(-\infty, 0)$; Decreasing on $(0, \infty)$

7. Increasing on $(1, \infty)$; Decreasing on $(-\infty, 1)$

9. Increasing on $(-2\sqrt{2}, 2\sqrt{2})$
Decreasing on $(-4, -2\sqrt{2}), (2\sqrt{2}, 4)$

11. Critical number: $x = 3$
Increasing on $(3, \infty)$
Decreasing on $(-\infty, 3)$
Relative minimum: $(3, -9)$

13. Critical number: $x = 1$
Increasing on $(-\infty, 1)$
Decreasing on $(1, \infty)$
Relative maximum: $(1, 5)$

15. Critical numbers: $x = -2, 1$
Increasing on $(-\infty, -2)$ and $(1, \infty)$
Decreasing on $(-2, 1)$
Relative maximum: $(-2, 20)$
Relative minimum: $(1, -7)$

17. Critical numbers: $x = 0, 2$
Increasing on $(0, 2)$
Decreasing on $(-\infty, 0), (2, \infty)$
Relative maximum: $(2, 4)$
Relative minimum: $(0, 0)$

19. Critical numbers: $x = -1, 1$
Increasing on $(-\infty, -1)$ and $(1, \infty)$
Decreasing on $(-1, 1)$
Relative maximum: $\left(-1, \frac{4}{5}\right)$
Relative minimum: $\left(1, -\frac{4}{5}\right)$

21. Critical number: $x = 0$
Increasing on $(-\infty, \infty)$
No relative extrema

23. Critical number: $x = 1$
Increasing on $(1, \infty)$
Decreasing on $(-\infty, 1)$
Relative minimum: $(1, 0)$

25. Critical number: $x = 5$
Increasing on $(-\infty, 5)$
Increasing on $(-\infty, 5)$
Relative maximum: $(5, 5)$

27. Critical numbers: $x = -1, 1$
Discontinuity: $x = 0$
Increasing on $(-\infty, -1)$ and $(1, \infty)$
Decreasing on $(-1, 0)$ and $(0, 1)$
Relative maximum: $(-1, -2)$
Relative minimum: $(1, 2)$

29. Critical number: $x = 0$
Discontinuities: $x = -3, 3$
Increasing on $(-\infty, -3)$ and $(-3, 0)$
Decreasing on $(0, 3)$ and $(3, \infty)$
Relative maximum: $(0, 0)$

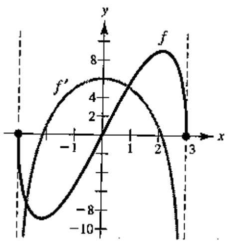
31. Critical numbers: $x = -3, 1$
Discontinuity: $x = -1$
Increasing on $(-\infty, -3)$ and $(1, \infty)$
Decreasing on $(-3, -1)$ and $(-1, 1)$
Relative maximum: $(-3, -8)$
Relative minimum: $(1, 0)$

33. Critical numbers: $x = \frac{\pi}{6}, \frac{5\pi}{6}$
Increasing on $\left(0, \frac{\pi}{6}\right), \left(\frac{5\pi}{6}, 2\pi\right)$
Decreasing on $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$
Relative maximum: $\left(\frac{\pi}{6}, \frac{[\pi + 6\sqrt{3}]}{12}\right)$
Relative minimum: $\left(\frac{5\pi}{6}, \frac{[5\pi - 6\sqrt{3}]}{12}\right)$

35. Critical numbers: $x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$
Increasing on $\left(0, \frac{\pi}{2}\right), \left(\frac{7\pi}{6}, \frac{3\pi}{2}\right), \left(\frac{11\pi}{6}, 2\pi\right)$
Decreasing on $\left(\frac{\pi}{2}, \frac{7\pi}{6}\right), \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$
Relative maxima: $\left(\frac{\pi}{2}, 2\right), \left(\frac{3\pi}{2}, 0\right)$
Relative minima: $\left(\frac{7\pi}{6}, -\frac{1}{4}\right), \left(\frac{11\pi}{6}, -\frac{1}{4}\right)$

37. (a) $f'(x) = \dfrac{2(9 - 2x^2)}{\sqrt{9 - x^2}}$

(b)



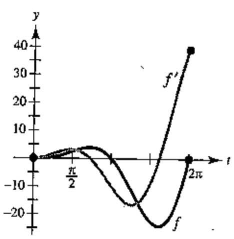
(c) $x = \pm\dfrac{3\sqrt{2}}{2}$

(d) $f' > 0$ on $\left(-\dfrac{3\sqrt{2}}{2}, \dfrac{3\sqrt{2}}{2}\right)$

$f' < 0$ on $\left(-3, -\dfrac{3\sqrt{2}}{2}\right), \left(\dfrac{3\sqrt{2}}{2}, 3\right)$

39. (a) $f'(t) = t(t \cos t + 2 \sin t)$

(b)



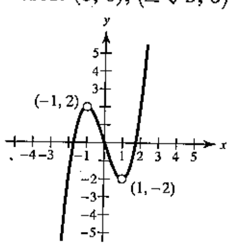
(c) Critical numbers: $t = 2.2889, 5.0870$

(d) $f' > 0$ on $(0, 2.2889), (5.0870, 2\pi)$

$f' < 0$ on $(2.2889, 5.0870)$

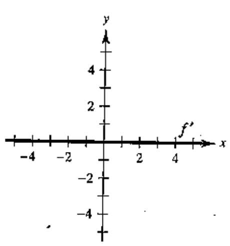
41. $f(x)$ is symmetric with respect to the origin.

Zeros: $(0, 0), (\pm\sqrt{3}, 0)$

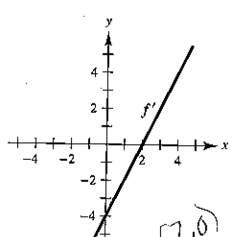


$g(x)$ is continuous on $(-\infty, \infty)$ and $f(x)$ has holes at $x = 1$ and $x = -1$.

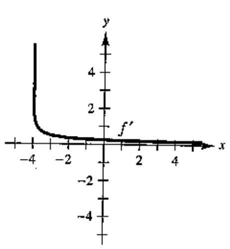
43.



45.

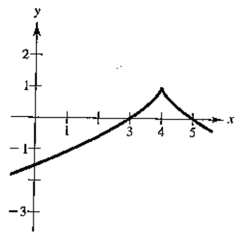


47.

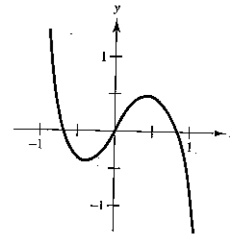


49. $g'(0) < 0$ **51.** $g'(-6) < 0$ **53.** $g'(0) > 0$

55.



57.



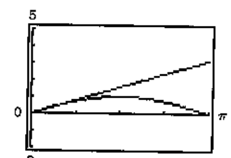
Minimum at the approximate critical number $x = -0.40$
Maximum at the approximate critical number $x = 0.48$

59. (a)

x	0.5	1	1.5	2	2.5	3
$f(x)$	0.5	1	1.5	2	2.5	3
$g(x)$	0.48	0.84	1.00	0.91	0.60	0.14

$f(x) > g(x)$

(b)



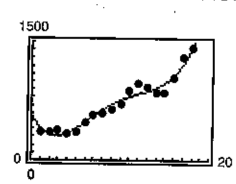
$f(x) > g(x)$

(c) Proof

61. $r = \dfrac{2R}{3}$ **63.** Maximum when $R_2 = R_1$

65. (a) $B = 0.11980t^4 - 4.4879t^3 + 56.991t^2 - 223.02t + 580.0$

(b)



(c) $(2.8, 311.2)$

67. (a) 3

(b) $a_3(0)^3 + a_2(0)^2 + a_1(0) + a_0 = 0$
$a_3(2)^3 + a_2(2)^2 + a_1(2) + a_0 = 2$
$3a_3(0)^2 + 2a_2(0) + a_1 = 0$
$3a_3(2)^2 + 2a_2(2) + a_1 = 0$

(c) $f(x) = -\frac{1}{2}x^3 + \frac{3}{2}x^2$

69. (a) 4

(b) $a_4(0)^4 + a_3(0)^3 + a_2(0)^2 + a_1(0) + a_0 = 0$
$a_4(2)^4 + a_3(2)^3 + a_2(2)^2 + a_1(2) + a_0 = 4$
$a_4(4)^4 + a_3(4)^3 + a_2(4)^2 + a_1(4) + a_0 = 0$
$4a_4(0)^3 + 3a_3(0)^2 + 2a_2(0) + a_1 = 0$
$4a_4(2)^3 + 3a_3(2)^2 + 2a_2(2) + a_1 = 0$

(c) $f(x) = \frac{1}{4}x^4 - 2x^3 + 4x^2$

71. True **73.** False. Let $f(x) = x^3$.

75. False. Let $f(x) = x^3$. There is a critical number at $x = 0$, but not a relative extremum.

77. Proof **79.** Proof

Section 3.4 (page 189)

1. Concave upward: $(-\infty, \infty)$

3. Concave upward: $(-\infty, -2), (2, \infty)$
Concave downward: $(-2, 2)$

5. Concave upward: $(-\infty, -1), (1, \infty)$
Concave downward: $(-1, 1)$

7. Concave upward: $(-\infty, 1)$
Concave downward: $(1, \infty)$

9. Concave upward: $\left(-\frac{\pi}{2}, 0\right)$
Concave downward: $\left(0, \frac{\pi}{2}\right)$

11. Point of inflection: $(2, 8)$
Concave downward: $(-\infty, 2)$
Concave upward: $(2, \infty)$

13. Points of inflection: $\left(\pm\frac{2}{\sqrt{3}}, -\frac{20}{9}\right)$
Concave upward: $\left(-\infty, -\frac{2}{\sqrt{3}}\right), \left(\frac{2}{\sqrt{3}}, \infty\right)$
Concave downward: $\left(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right)$

15. Points of inflection: $(2, -16), (4, 0)$
Concave upward: $(-\infty, 2), (4, \infty)$
Concave downward: $(2, 4)$

17. Concave upward: $(-3, \infty)$

19. Points of inflection: $\left(-\sqrt{3}, -\frac{\sqrt{3}}{4}\right), (0, 0), \left(\sqrt{3}, \frac{\sqrt{3}}{4}\right)$
Concave upward: $(-\sqrt{3}, 0), (\sqrt{3}, \infty)$
Concave downward: $(-\infty, -\sqrt{3}), (0, \sqrt{3})$

21. Point of inflection: $(2\pi, 0)$
Concave upward: $(2\pi, 4\pi)$
Concave downward: $(0, 2\pi)$

23. Concave upward: $(0, \pi), (2\pi, 3\pi)$
Concave downward: $(\pi, 2\pi), (3\pi, 4\pi)$

25. Points of inflection: $(\pi, 0), (1.823, 1.452), (4.46, -1.452)$
Concave upward: $(1.823, \pi), (4.46, 2\pi)$
Concave downward: $(0, 1.823), (\pi, 4.46)$

27. Relative minimum: $(3, -25)$

29. Relative minimum: $(5, 0)$

31. Relative maximum: $(0, 3)$
Relative minimum: $(2, -1)$

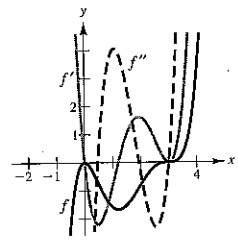
33. Relative maximum: $(2.4, 268.74)$
Relative minimum: $(0, 0)$

35. Relative minimum: $(0, -3)$

37. Relative maximum: $(-2, -4)$
Relative minimum: $(2, 4)$

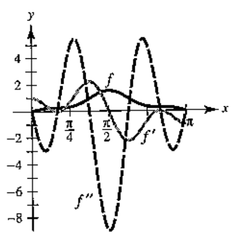
39. No relative extrema, because f is nonincreasing.

41. (a) $f'(x) = 0.2x(x - 3)^2(5x - 6)$
$f''(x) = 0.4(x - 3)(10x^2 - 24x + 9)$
(b) Relative maximum: $(0, 0)$
Relative minimum: $(1.2, -1.6796)$
Points of inflection: $(0.4652, -0.7048)$, $(1.9348, -0.9048)$, $(3, 0)$
(c)



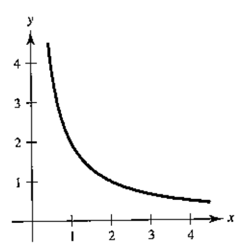
f is increasing when f' is positive, and decreasing when f' is negative. f is concave upward when f'' is positive, and concave downward when f'' is negative.

43. (a) $f'(x) = \cos x - \cos 3x + \cos 5x$
$f''(x) = -\sin x + 3 \sin 3x - 5 \sin 5x$
(b) Relative maximum: $(\pi/2, 1.53333)$
Points of inflection:
$(0.5236, 0.2667), (1.1731, 0.9637)$,
$(1.9685, 0.9637), (2.6180, 0.2667)$
(c)

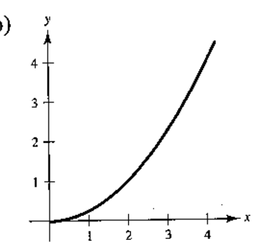


f is increasing when f' is positive, and decreasing when f' is negative. f is concave upward when f'' is positive, and concave downward when f'' is negative.

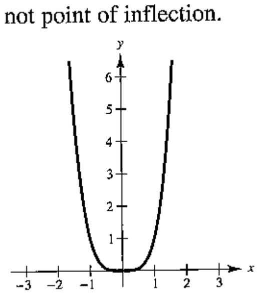
45. (a)



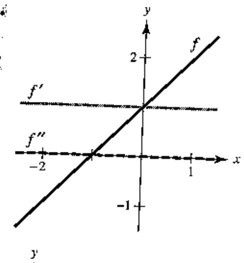
(b)



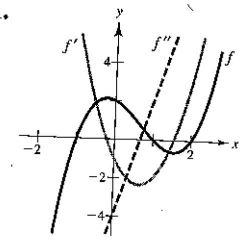
47. Answers will vary. Example: $f(x) = x^4$ $f''(0) = 0$, but $(0, 0)$ is not point of inflection.



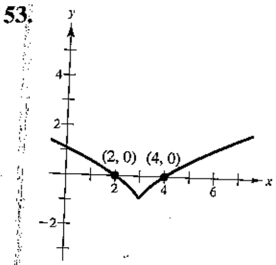
49.



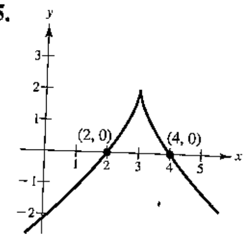
51.



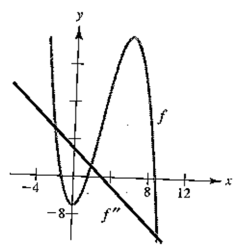
53.



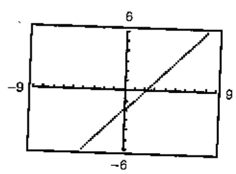
55.

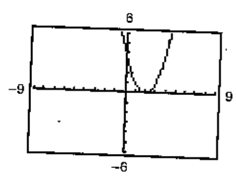


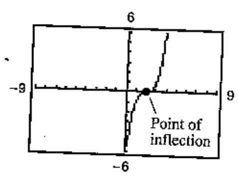
57. Example:

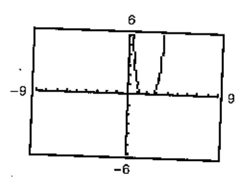


59. (a) $f(x) = (x - 2)^n$ has a point of inflection at $(2, 0)$ if n is odd and $n \geq 3$.









(b) Proof

61. $f(x) = \frac{1}{2}x^3 - 6x^2 + \frac{45}{2}x - 24$

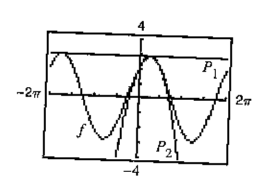
63. (a) $f(x) = \frac{1}{32}x^3 + \frac{3}{16}x^2$ (b) Two miles from touchdown

65. $x = \left(\dfrac{15 - \sqrt{33}}{16}\right)L \approx 0.578L$ **67.** $x = 100$ units

69. $t = \sqrt{\frac{8}{3}} \approx 1.633$ years

71. $P_1(x) = 2\sqrt{2}$

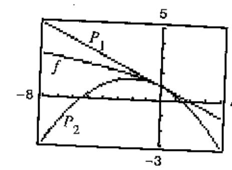
$P_2(x) = 2\sqrt{2} - \sqrt{2}\left(x - \dfrac{\pi}{4}\right)^2$



The values of f, P_1, and P_2 and their first derivatives are equal when $x = \pi/4$. The approximations worsen as you move away from $x = \pi/4$.

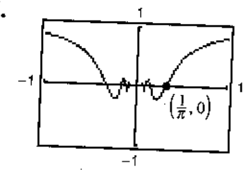
73. $P_1(x) = 1 - \dfrac{x}{2}$

$P_2(x) = 1 - \dfrac{x}{2} - \dfrac{x^2}{8}$



The values of f, P_1, and P_2 and their first derivatives are equal when $x = 0$. The approximations worsen as you move away from $x = 0$.

75.



77. Proof **79.** True

81. False. The maximum value is $\sqrt{13} \approx 3.60555$.

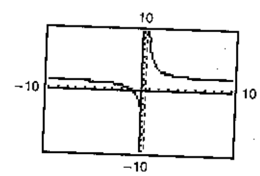
83. False. f is concave upward at $x = c$ if $f''(c) > 0$.

Section 3.5 (page 199)

1. f **2.** c **3.** d **4.** a **5.** b **6.** e

7.

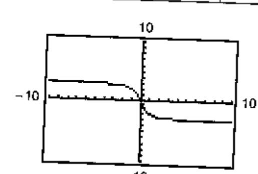
x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	7	2.2632	2.0251	2.0025	2.0003	2.0000	2.0000



$\lim_{x \to \infty} \dfrac{4x + 3}{2x - 1} = 2$

9.

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	-2	-2.9814	-2.9998	-3.0000	-3.0000	-3.0000	-3.0000

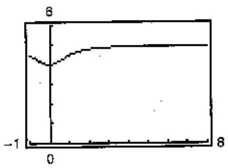


$\lim_{x \to \infty} \dfrac{-6x}{\sqrt{4x^2 + 5}} = -3$

11.

x	10^0	10^1	10^2	10^3	10^4
$f(x)$	4.5000	4.9901	4.9999	5.0000	5.0000

x	10^5	10^6
$f(x)$	5.0000	5.0000



$\lim_{x\to\infty}\left(5 - \frac{1}{x^2+1}\right) = 5$

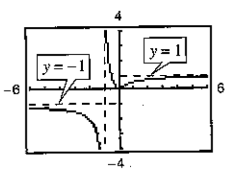
13. (a) ∞ (b) 5 (c) 0 **15.** (a) 0 (b) 1 (c) ∞

17. (a) 0 (b) $-\frac{2}{3}$ (c) $-\infty$

19. $\frac{2}{3}$ **21.** 0 **23.** $-\infty$

25. -1 **27.** -2 **29.** 0 **31.** 0

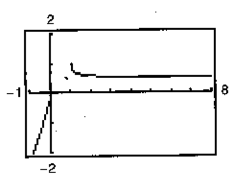
33.



35. 1 **37.** 0 **39.** $-\frac{1}{2}$

41.

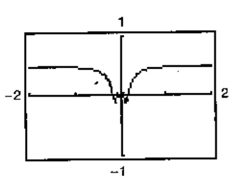
x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	1.000	0.513	0.501	0.500	0.500	0.500	0.500



$\lim_{x\to\infty}\left[x - \sqrt{x(x-1)}\right] = \frac{1}{2}$

43.

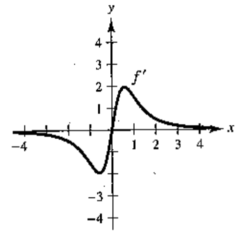
x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	0.479	0.500	0.500	0.500	0.500	0.500	0.500



The graph has a hole at $x = 0$.

$\lim_{x\to\infty} x \sin\frac{1}{2x} = \frac{1}{2}$

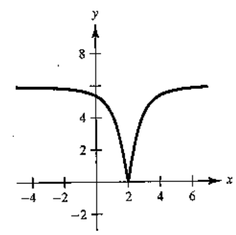
45. (a)



(b) $\lim_{x\to\infty} f(x) = 3, \ \lim_{x\to\infty} f'(x) = 0$

(c) $y = 3$ is a horizontal asymptote. The rate of increase of the function approaches 0 as the graph approaches $y = 3$.

47. Yes. For example, let $f(x) = \frac{6|x-2|}{\sqrt{(x-2)^2+1}}$.



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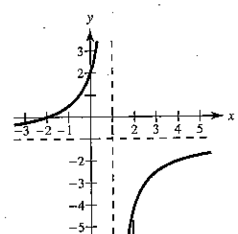

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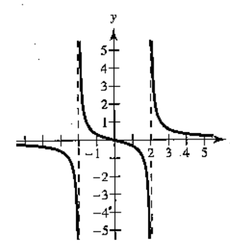

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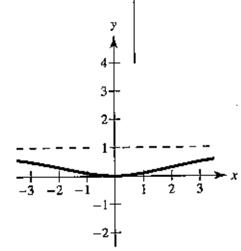

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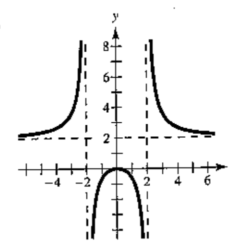

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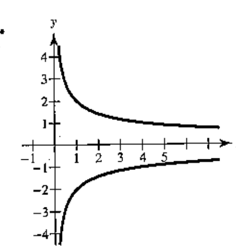

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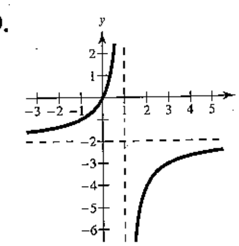

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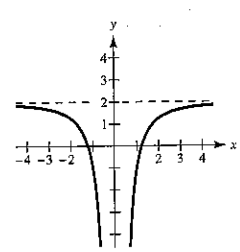

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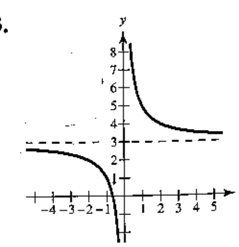

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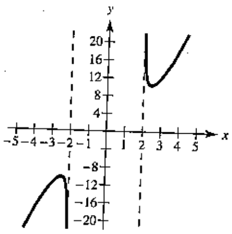

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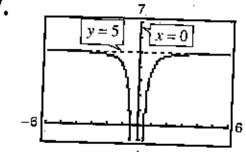

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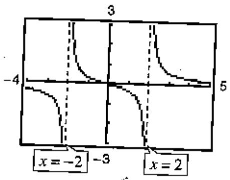

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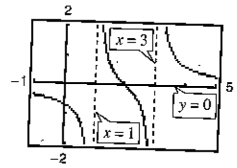
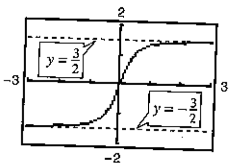
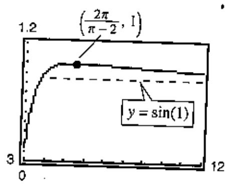


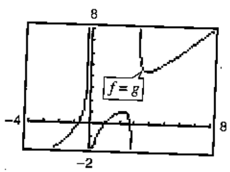
73.



75.

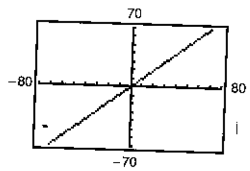


77. (a)



(b) Proof

(c)

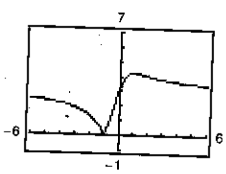


The slant asymptote $y = x$

79. $\frac{1}{2}$

81. (a) $d = \dfrac{|3m + 3|}{\sqrt{m^2 + 1}}$

(b)

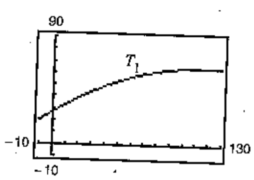


(c) $\lim_{m \to \infty} d(m) = 3$; $\lim_{m \to -\infty} d(m) = 3$

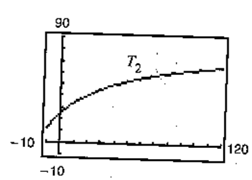
As $m \to \infty$, the line approaches the vertical line $x = 0$. Therefore, the distance approaches 3.

83. (a) $T_1 = -0.003t^2 + 0.68t + 26.6$

(b)



(c)



(d) $T_1(0) \approx 26.6°$, $T_2(0) \approx 25.0°$ (e) 86

(f) The limiting temperature is 86°.
No. T_1 has no horizontal asymptote.

85. Answers will vary. See "Guidelines for Finding Limits at Infinity of Rational Functions" on page 195. Examples:

(a) $\lim_{x \to \infty} \left(\dfrac{5 - 2x}{3x^2 - 4}\right) = 0$ since the degree of the numerator is less than the degree of the denominator.

(b) $\lim_{x \to \infty} \left(\dfrac{2x - 1}{3x + 2}\right) = \dfrac{2}{3}$ since the degree of the numerator is equal to the degree of the denominator.

(c) $\lim_{x \to \infty} \left(\dfrac{x^2 + 2}{x - 1}\right) = \infty$ since the degree of the numerator is greater than the degree of the denominator.

87. False. Let $f(x) = \dfrac{2x}{\sqrt{x^2 + 2}}$.

$f'(x) > 0$ for all real numbers.

Section 3.6 (page 208)

1. d **2.** c **3.** a **4.** b

5. (a) $f'(x) = 0$ for $x = \pm 2$

$f'(x) > 0$ for $(-\infty, -2), (2, \infty)$

$f'(x) < 0$ for $(-2, 2)$

(b) $f''(x) = 0$ for $x = 0$

$f''(x) > 0$ for $(0, \infty)$

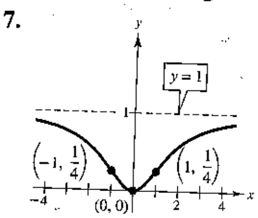
$f''(x) < 0$ for $(-\infty, 0)$

(c) $(0, \infty)$

(d) f' is minimum for $x = 0$.

f is decreasing at the fastest rate.

7.



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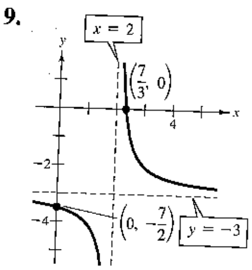
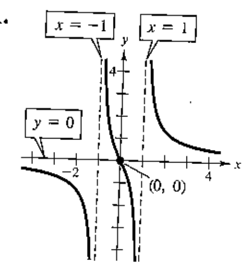


11.



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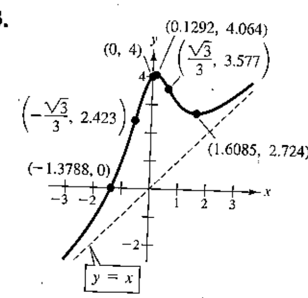

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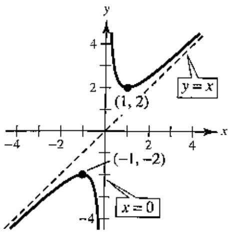
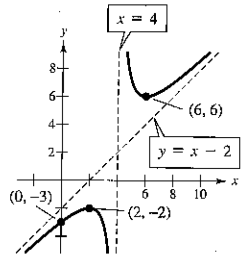


17.



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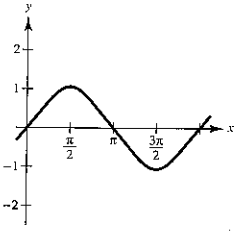

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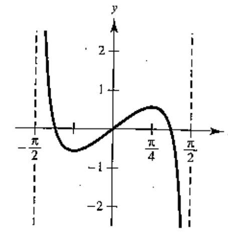
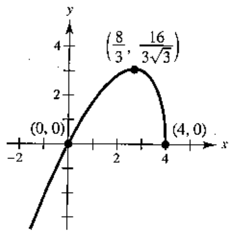


19.



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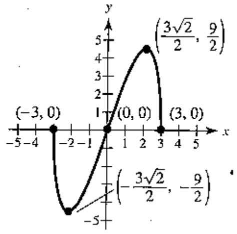

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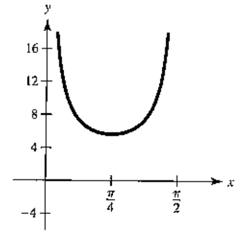

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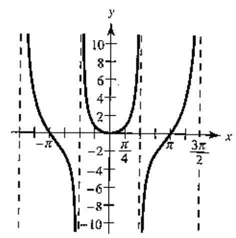

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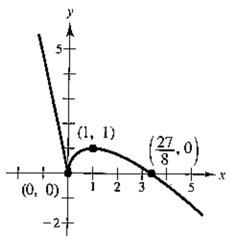

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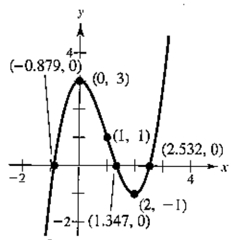

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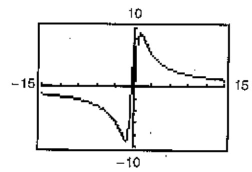


Minimum: $(-1.10, -9.05)$

Maximum: $(1.10, 9.05)$

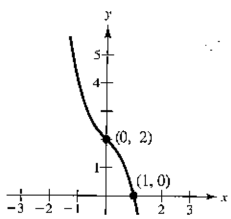
Points of inflection:

$(-1.84, -7.86)$, $(1.84, 7.86)$

Vertical asymptote: $x = 0$

Horizontal asymptote: $y = 0$

27.



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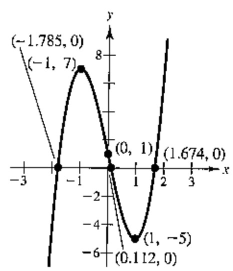

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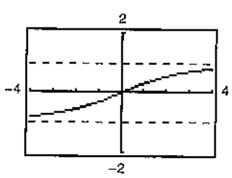
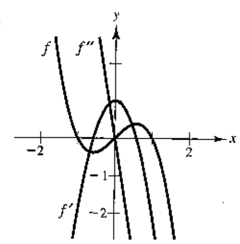


Point of inflection: $(0, 0)$

Horizontal asymptotes: $y = \pm 1$

51.



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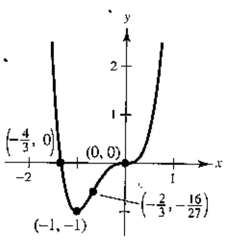

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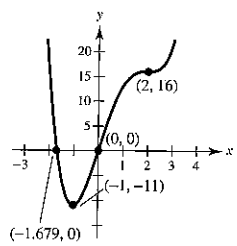

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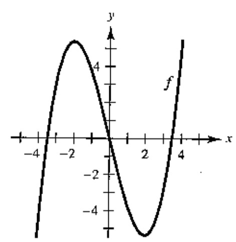

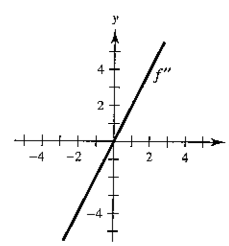

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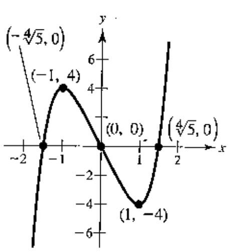
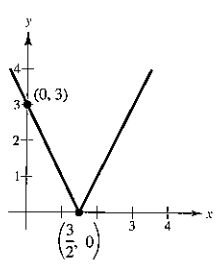
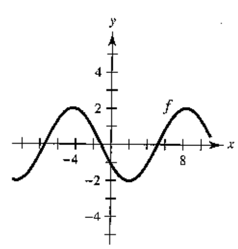
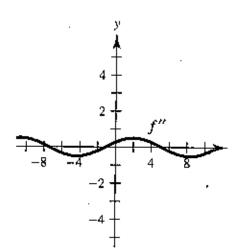


37.



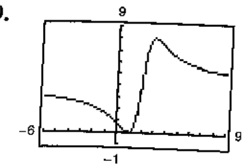
55.





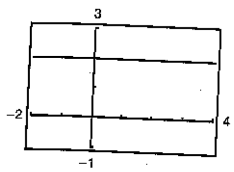
57. f is decreasing on $(2, 8)$ and therefore $f(3) > f(5)$.

59.



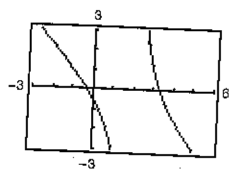
The graph crosses the horizontal asymptote $y = 4$. The graph of f does not cross its vertical asymptote $x = c$ because $f(c)$ does not exist.

61.



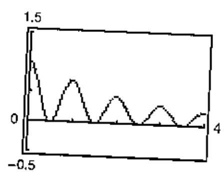
The graph has a hole at $x = 3$. The rational function is not reduced to lowest terms.

63.



The graph appears to approach the line $y = -x + 1$, which is the slant asymptote.

65. (a)



The graph has a hole at $x = 0$ and at $x = 4$.

Visual approximate critical numbers: $\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \frac{7}{2}$

(b) $f'(x) = \dfrac{-\cos^2(\pi x)}{(x^2 + 1)^{3/2}} - \dfrac{2\pi \sin(\pi x)\cos(\pi x)}{\sqrt{x^2 + 1}}$

Approximate critical numbers: $\frac{1}{2}, 0.97, \frac{3}{2}, 1.98, \frac{5}{2}, 2.98, \frac{7}{2}$

The critical numbers where maxima occur appear to be integers in part (a), but approximating them using f' you see that they are not integers.

67. Answers will vary. Example: $y = \dfrac{1}{x - 5}$

69. Answers will vary. Example: $y = \dfrac{3x^2 - 13x - 9}{x - 5}$

71. (a) Rate of change of f changes as a varies. If the sign of a is changed, the graph is reflected through the x-axis.

(b) The locations of the vertical asymptote and the minimum (if $a > 0$) or maximum (if $a < 0$) are changed.

73. (a) If n is even, f is symmetric with respect to the y-axis. If n is odd, f is symmetric with respect to the origin.

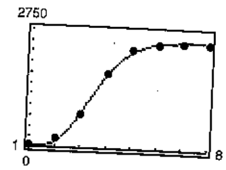
(b) $n = 0, 1, 2, 3$ (c) $n = 4$

(d) When $n = 5$, the slant asymptote is $y = 3x$.

(e)

n	0	1	2	3	4	5
M	1	2	3	2	1	0
N	2	3	4	5	2	3

75. (a)



(b) 2434

(c) The number of bacteria reached its maximum early on the seventh day.

(d) The rate of increase in the number of bacteria was greatest approximately in the middle of the third day.

(e) $\dfrac{13{,}250}{7}$

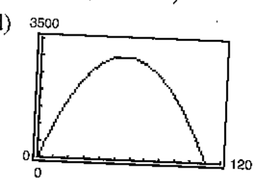
Section 3.7 (page 216)

1. (a) and (b)

First Number, x	*Second Number*	*Product P*
10	110 − 10	10(110 − 10) = 1000
20	110 − 20	20(110 − 20) = 1800
30	110 − 30	30(110 − 30) = 2400
40	110 − 40	40(110 − 40) = 2800
50	110 − 50	50(110 − 50) = 3000
60	110 − 60	60(110 − 60) = 3000
70	110 − 70	70(110 − 70) = 2800
80	110 − 80	80(110 − 80) = 2400
90	110 − 90	90(110 − 90) = 1800
100	110 − 100	100(110 − 100) = 1000

(c) $P = x(110 - x)$

(d)



(e) 55 and 55

3. $\sqrt{192}$ and $\sqrt{192}$ **5.** 1 and 1 **7.** $l = w = 25$ meters

9. $l = w = 8$ feet **11.** $\left(\frac{7}{2}, \sqrt{\frac{7}{2}}\right)$ **13.** $(1, 1)$

15. $x = \dfrac{Q_0}{2}$ **17.** 600×300 meters

19. (a) Proof

(b) $V_1 = 99$ cubic inches

$V_2 = 125$ cubic inches

$V_3 = 117$ cubic inches

(c) $5 \times 5 \times 5$ inches

21. (a) $V = x(s - 2x)^2, 0 < x < \frac{s}{2}$

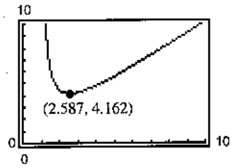
Maximum: $V\left(\frac{s}{6}\right) = \frac{2s^3}{27}$

(b) Increased by a factor of 8

23. Rectangular portion: $\frac{16}{\pi + 4} \times \frac{32}{\pi + 4}$ feet

25. (a) $L = \sqrt{x^2 + 4 + \frac{8}{x - 1} + \frac{4}{(x - 1)^2}}, \quad x > 1$

(b)



Minimum when $x \approx 2.587$

(c) (0, 0), (2, 0), (0, 4)

27. Width: $\frac{5\sqrt{2}}{2}$; Length: $5\sqrt{2}$

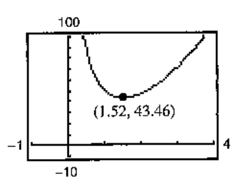
29. Dimensions of page: $(2 + \sqrt{30})$ inches $\times$ $(2 + \sqrt{30})$ inches

31. (a) and (b)

Radius, r	*Height*	*Surface Area, S*
0.2	$\frac{22}{\pi(0.2)^2}$	$2\pi(0.2)\left[0.2 + \frac{22}{\pi(0.2)^2}\right] \approx 220.3$
0.4	$\frac{22}{\pi(0.4)^2}$	$2\pi(0.4)\left[0.4 + \frac{22}{\pi(0.4)^2}\right] \approx 111.0$
0.6	$\frac{22}{\pi(0.6)^2}$	$2\pi(0.6)\left[0.6 + \frac{22}{\pi(0.6)^2}\right] \approx 75.6$
0.8	$\frac{22}{\pi(0.8)^2}$	$2\pi(0.8)\left[0.8 + \frac{22}{\pi(0.8)^2}\right] \approx 59.0$
1.0	$\frac{22}{\pi(1.0)^2}$	$2\pi(1.0)\left[1.0 + \frac{22}{\pi(1.0)^2}\right] \approx 50.3$
1.2	$\frac{22}{\pi(1.2)^2}$	$2\pi(1.2)\left[1.2 + \frac{22}{\pi(1.2)^2}\right] \approx 45.7$
1.4	$\frac{22}{\pi(1.4)^2}$	$2\pi(1.4)\left[1.4 + \frac{22}{\pi(1.4)^2}\right] \approx 43.7$
1.6	$\frac{22}{\pi(1.6)^2}$	$2\pi(1.6)\left[1.6 + \frac{22}{\pi(1.6)^2}\right] \approx 43.6$
1.8	$\frac{22}{\pi(1.8)^2}$	$2\pi(1.8)\left[1.8 + \frac{22}{\pi(1.8)^2}\right] \approx 44.8$
2.0	$\frac{22}{\pi(2.0)^2}$	$2\pi(2.0)\left[2.0 + \frac{22}{\pi(2.0)^2}\right] \approx 47.1$

(c) $S = 2\pi r\left(r + \frac{22}{\pi r^2}\right)$

(d)



(e) $r = \sqrt[3]{\frac{11}{\pi}}, h = 2r$

Minimum area of 43.46 square inches when $r = 1.52$

33. 18 × 18 × 36 inches

35. $\frac{32\pi r^3}{81}$

37. Answers will vary. If area is expressed as a function of either length or width, the feasible domain is the interval (0, 10). No dimensions will yield a minimum area because the second derivative on this open interval is always negative.

39. $r = \sqrt[3]{\frac{9}{\pi}} \approx 1.42$ cm

41. Side of square: $\frac{10\sqrt{3}}{9 + 4\sqrt{3}}$

Side of triangle: $\frac{30}{9 + 4\sqrt{3}}$

43. $w = 8\sqrt{3}$ inches, $h = 8\sqrt{6}$ inches

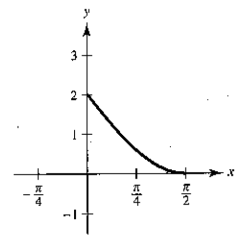
45. $\theta = \frac{\pi}{4}$

47. $h = \sqrt{2}$ feet

49. One mile from the nearest point on the coast

51. Proof

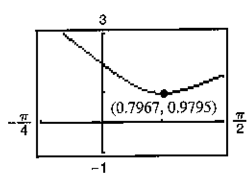
53.



(a) Origin to y-intercept: 2

Origin to x-intercept: $\frac{\pi}{2}$

(b) $d = \sqrt{x^2 + (2 - 2\sin x)^2}$



(c) Minimum distance is 0.9795 when $x \approx 0.7967$.

55. $F = \frac{kW}{\sqrt{k^2 + 1}}$; $\theta = \arctan k$

57. (a)

Base 1	Base 2	Altitude	Area
8	8 + 16 cos 10°	8 sin 10°	≈22.1
8	8 + 16 cos 20°	8 sin 20°	≈42.5
8	8 + 16 cos 30°	8 sin 30°	≈59.7
8	8 + 16 cos 40°	8 sin 40°	≈72.7
8	8 + 16 cos 50°	8 sin 50°	≈80.5
8	8 + 16 cos 60°	8 sin 60°	≈83.1

(b)

Base 1	Base 2	Altitude	Area
8	$8 + 16\cos 10°$	$8\sin 10°$	≈ 22.1
8	$8 + 16\cos 20°$	$8\sin 20°$	≈ 42.5
8	$8 + 16\cos 30°$	$8\sin 30°$	≈ 59.7
8	$8 + 16\cos 40°$	$8\sin 40°$	≈ 72.7
8	$8 + 16\cos 50°$	$8\sin 50°$	≈ 80.5
8	$8 + 16\cos 60°$	$8\sin 60°$	≈ 83.1
8	$8 + 16\cos 70°$	$8\sin 70°$	≈ 80.7
8	$8 + 16\cos 80°$	$8\sin 80°$	≈ 74.0
8	$8 + 16\cos 90°$	$8\sin 90°$	≈ 64.0

The maximum cross-sectional area is approximately 83.1 square feet.

(c) $A = (a + b)\dfrac{h}{2}$

$= [8 + (8 + 16\cos\theta)]\dfrac{8\sin\theta}{2}$

$= 64(1 + \cos\theta)\sin\theta,\ 0° < \theta < 90°$

(d) $\dfrac{dA}{d\theta} = 64(1 + \cos\theta)\cos\theta + (-64\sin\theta)\sin\theta$

$= 64(\cos\theta + \cos^2\theta - \sin^2\theta)$

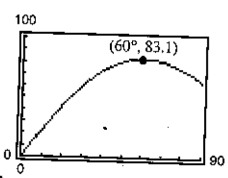
$= 64(2\cos^2\theta + \cos\theta - 1)$

$= 64(2\cos\theta - 1)(\cos\theta + 1)$

$= 0$ when $\theta = 60°, 180°, 300°$

The maximum occurs when $0 = 60°$.

(e)



59. 4045 units

61. $y = \frac{64}{141}x$; $S_1 = 6.1$ miles

63. $y = \frac{3}{10}x$; $S_3 = 4.50$ miles

Section 3.8 (page 226)

1.

n	x_n	$f(x_n)$	$f'(x_n)$	$\dfrac{f(x_n)}{f'(x_n)}$	$x_n - \dfrac{f(x_n)}{f'(x_n)}$
1	1.7000	−0.1100	3.4000	−0.0324	1.7324
2	1.7324	0.0012	3.4648	0.0003	1.7321

3.

n	x_n	$f(x_n)$	$f'(x_n)$	$\dfrac{f(x_n)}{f'(x_n)}$	$x_n - \dfrac{f(x_n)}{f'(x_n)}$
1	3	0.1411	−0.9900	−0.1425	3.1425
2	3.1425	−0.0009	−1.0000	0.0009	3.1416

5. 0.682 **7.** 1.146, 7.854 **9.** −1.442

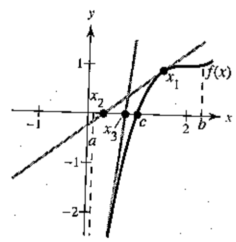
11. 0.900, 1.100, 1.900 **13.** −0.489 **15.** 0.569

17. 4.493 **19.** $x_{i+1} = \dfrac{x_i^2 + a}{2x_i}$ **21.** 2.646

23. 1.565 **25.** 3.141 **27.** $f'(x_1) = 0$

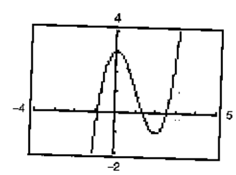
29. $2 = x_1 = x_3 = \ldots$
$1 = x_2 = x_4 = \ldots$

31. If f is a function continuous on $[a, b]$ and differentiable on (a, b), where $c \in [a, b]$ and $f(c) = 0$, Newton's Method uses tangent lines to approximate c. First, estimate an initial x_1 close to c. (See graph.) Then determine x_2 by $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$. Calculate a third estimate by $x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)}$. Continue this process until $|x_n - x_{n+1}|$ is within the desired accuracy and let x_{n+1} be the final approximation of c.



33. 0.74

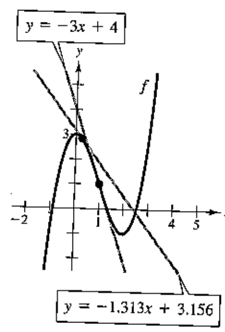
35. (a)



(b) 1.347

(c) 2.532

(d)

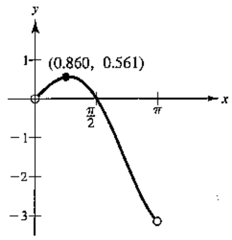


x-intercept of $y = -3x + 4$ is $\frac{4}{3}$.
x-intercept of $y = -1.313x + 3.156$ is approximately 2.404.

(e) If the initial estimate $x = x_1$ is not sufficiently close to the desired zero of a function, the x-intercept of the corresponding tangent line to the function may approximate a second zero of the function.

37. Proof

39. 0.860



41. (1.939, 0.240) **43.** $x \approx 1.563$ miles **45.** \$384,356

47. False: let $f(x) = \dfrac{x^2 - 1}{x - 1}$. **49.** True **51.** $x \approx 11.803$

Section 3.9 (page 233)

1. $T(x) = 4x - 4$

x	1.9	1.99	2	2.01	2.1
$f(x)$	3.610	3.960	4	4.040	4.410
$T(x)$	3.600	3.960	4	4.040	4.400

3. $T(x) = 80x - 128$

x	1.9	1.99	2	2.01	2.1
$f(x)$	24.761	31.208	32	32.808	40.841
$T(x)$	24.000	31.200	32	32.800	40.000

5. $T(x) = (\cos 2)(x - 2) + \sin 2$

x	1.9	1.99	2	2.01	2.1
$f(x)$	0.946	0.913	0.909	0.905	0.863
$T(x)$	0.951	0.913	0.909	0.905	0.868

7. $\Delta y = 0.6305$; $dy = 0.6000$ **9.** $\Delta y = -0.039$; $dy = -0.040$

11. $6x\,dx$ **13.** $-\dfrac{3}{(2x-1)^2}\,dx$ **15.** $\dfrac{1 - 2x^2}{\sqrt{1 - x^2}}\,dx$

17. $(2 + 2\cot x + 2\cot^3 x)\,dx$ **19.** $-\pi \sin\left(\dfrac{6\pi x - 1}{2}\right)dx$

21. (a) 0.9 (b) 1.04 **23.** (a) 1.05 (b) 0.98

25. (a) 8.035 (b) 7.95 **27.** (a) 8 (b) 8

29. $\pm\frac{3}{8}$ square inch **31.** $\pm 7\pi$ square inches

33. (a) $\frac{2}{3}\%$ (b) 1.25%

35. (a) $\pm 2.88\pi$ cubic inches (b) $\pm 0.96\pi$ square inches
(c) 1%, $\frac{2}{3}\%$

37. 80π cubic centimeters

39. (a) $\frac{1}{4}\%$ (b) 216 seconds = 3.6 minutes

41. (a) 0.87% (b) 2.16% **43.** 4961 feet

45. $f(x) = \sqrt{x}$, $dy = \dfrac{1}{2\sqrt{x}}\,dx$

$$f(99.4) \approx \sqrt{100} + \frac{1}{2\sqrt{100}}(-0.6) = 9.97$$

Calculator: 9.97

47. $f(x) = \sqrt[4]{x}$, $dy = \dfrac{1}{4x^{3/4}}\,dx$

$$f(624) \approx \sqrt[4]{625} + \frac{1}{4(625)^{3/4}}(-1) = 4.998$$

Calculator: 4.998

49. $f(x) = \sqrt{x}$; $dy = \dfrac{1}{2\sqrt{x}}\,dx$

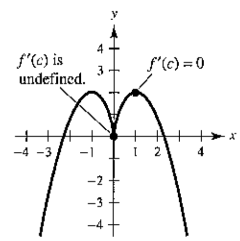
$$f(4.02) \approx \sqrt{4} + \frac{1}{2\sqrt{4}}(0.02) = 2 + \frac{1}{4}(0.02)$$

51. The value of dy becomes closer to the value of Δy as Δx decreases.

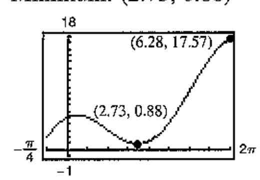
53. True **55.** True

Review Exercises for Chapter 3 (page 235)

1. Let f be defined at c. If $f'(c) = 0$ or if f' is undefined at c, then c is a critical number of f.

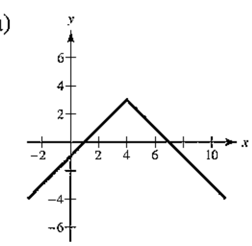


3. Maximum: $(2\pi, 17.57)$
Minimum: (2.73, 0.88)



5. $f'\left(\frac{1}{3}\right) = 0$

7. (a)



(b) f is not differentiable at $x = 4$.

9. $f'\left(\dfrac{2744}{729}\right) = \dfrac{3}{7}$ **11.** $f'(0) = 1$ **13.** $c = \dfrac{x_1 + x_2}{2}$

15. Critical numbers: $x = 1, \frac{7}{3}$
Increasing on $(-\infty, 1)$, $\left(\frac{7}{3}, \infty\right)$
Decreasing on $\left(1, \frac{7}{3}\right)$

17. Critical number: $x = 1$
Increasing on $(1, \infty)$
Decreasing on $(0, 1)$

19. Minimum: $(2, -12)$

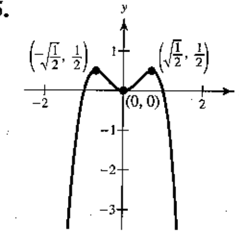
21. (a) $y = \frac{1}{4}$ inch; $v = 4$ inches per second

(b) Proof

(c) Period: $\frac{\pi}{6}$; Frequency: $\frac{6}{\pi}$

23. $\left(\frac{\pi}{2}, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, \frac{3\pi}{2}\right)$

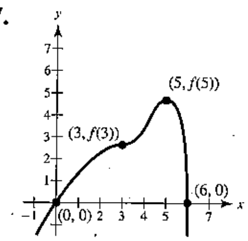
25.



Relative maxima: $\left(\frac{\sqrt{2}}{2}, \frac{1}{2}\right), \left(-\frac{\sqrt{2}}{2}, \frac{1}{2}\right)$

Relative minimum: $(0, 0)$

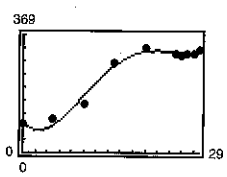
27.



29. Increasing and concave down

31. (a) $D = 0.00340t^4 - 0.2352t^3 + 4.942t^2 - 20.86t + 94.4$

(b)



(c) Maximum occurs in 1991; Minimum occurs in 1972.

(d) 1979

33. $\frac{2}{3}$ **35.** 0

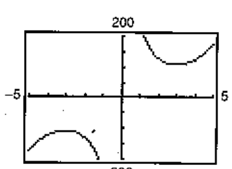
37. Vertical asymptote: $x = 4$

Horizontal asymptote: $y = 2$

39. Vertical asymptote: $x = 0$

Horizontal asymptote: $y = -2$

41.

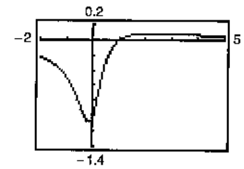


Vertical asymptote: $x = 0$

Relative minimum: $(3, 108)$

Relative maximum: $(-3, -108)$

43.

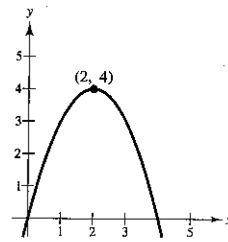


Horizontal asymptote: $y = 0$

Relative minimum: $(-0.155, -1.077)$

Relative maximum: $(2.155, 0.077)$

45.



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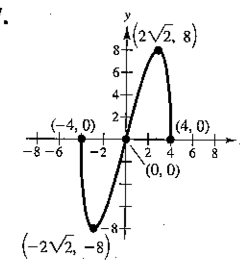

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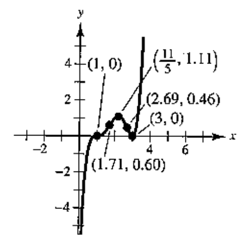
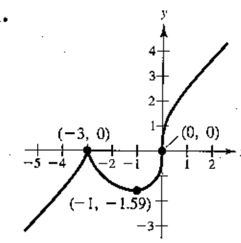


51.



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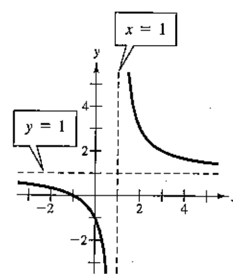

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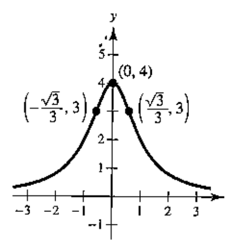

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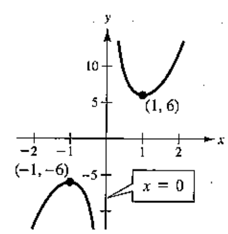

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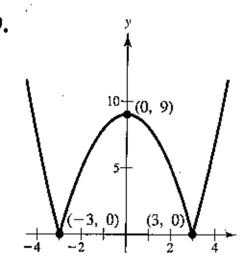
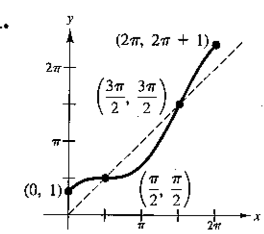


61.



63. Maximum: $(1, 3)$

Minimum: $(1, 1)$

65. $t \approx 4.92 \approx$ 4:55 P.M.; $d \approx 64$ kilometers

67. $(0, 0), (5, 0), (0, 10)$ **69.** Proof **71.** 14.05 feet

73. $3(3^{2/3} + 2^{2/3})^{3/2} \approx 21.07$ feet

75. $v \approx 54.77$ miles per hour

77. $-1.532, -0.347, 1.879$ **79.** $-1.164, 1.453$

81. $dy = (1 - \cos x + x \sin x)\,dx$

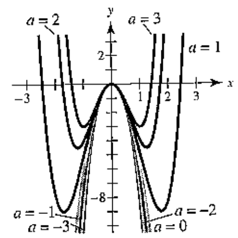
83. $dS = \pm 1.8\pi$ square centimeters, $\frac{dS}{S} \times 100 \approx \pm 0.56\%$

$dV = \pm 8.1\pi$ cubic centimeters, $\frac{dV}{V} \times 100 \approx \pm 0.83\%$

P.S. Problem Solving (page 238)

1. Proof

3. (a)



Relative maximum for all a at $(0, 0)$

Two relative minima for $a = 1, 2, 3$

(b) $p = ax^4 - 6x^2$

$p' = 4ax^3 - 12x$ has critical points at $x = 0$ and $x = \pm\sqrt{3/a}, a > 0$

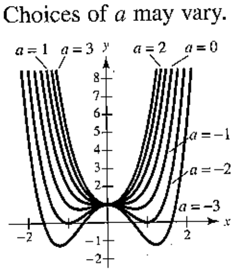
$p'' = 12ax^2 - 12, p''(0) = -12$

Therefore, by the Second Derivative Test, p has a relative maximum for all a at $x = 0$.

(c) $p''(\pm\sqrt{3/a}) = 24$. Therefore, by the Second Derivative Test, p has a relative minimum when $x = \pm\sqrt{3/a}, a > 0$.

(d) Relative extrema of p occur at $x = 0, \pm\sqrt{3/a}, a > 0$. If $x = 0$, $p(x) = 0$ and $(0, 0)$ also lies on the graph of $y = -3x^2$. If $x = \pm\sqrt{3/a}$, $p(x) = -9/a$ and $(\pm\sqrt{3/a}, -9/a)$ also lies on the graph of $y = -3x^2$.

5. Choices of a may vary.



(a) One relative minimum at $(0, 1)$ for $a \geq 0$

(b) One relative maximum at $(0, 1)$ for $a < 0$

(c) Two relative minima for $a < 0$ when $x = \pm\sqrt{-\frac{a}{2}}$

(d) If $a < 0$, there are three critical points; if $a \geq 0$, there is only one critical point.

7. All c where c is a real number

9. Proof **11.** $\phi \approx 42.1°$ or 0.736 radians

13. $\theta = \frac{\pi}{2} + 2n\pi$ and $\theta = \frac{3\pi}{2} + 2n\pi$, where n is an integer.

15. Rectangle: $\frac{3}{2} \times 2$

Circle: $r = 1$

Semicircle: $r = \frac{12}{7}$

Calculus was helpful for the rectangle.

17. Greatest slope at $\left(-\frac{\sqrt{3}}{3}, \frac{3}{4}\right)$; Least slope at $\left(\frac{\sqrt{3}}{3}, \frac{3}{4}\right)$

19. (a) Proof (b) Proof

Chapter 4

Section 4.1 (page 249)

1. Proof **3.** Proof **5.** $y = t^3 + C$

7. $y = \frac{2}{5}x^{5/2} + C$

	Original Integral	*Rewrite*	*Integrate*	*Simplify*
9.	$\int \sqrt[3]{x}\,dx$	$\int x^{1/3}\,dx$	$\frac{x^{4/3}}{4/3} + C$	$\frac{3}{4}x^{4/3} + C$
11.	$\int \frac{1}{x\sqrt{x}}\,dx$	$\int x^{-3/2}\,dx$	$\frac{x^{-1/2}}{-1/2} + C$	$-\frac{2}{\sqrt{x}} + C$
13.	$\int \frac{1}{2x^3}\,dx$	$\frac{1}{2}\int x^{-3}\,dx$	$\frac{1}{2}\left(\frac{x^{-2}}{-2}\right) + C$	$-\frac{1}{4x^2} + C$

15. $\frac{1}{2}x^2 + 3x + C$ **17.** $x^2 - x^3 + C$ **19.** $\frac{1}{4}x^4 + 2x + C$

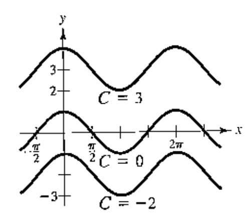
21. $\frac{2}{5}x^{5/2} + x^2 + x + C$ **23.** $\frac{3}{5}x^{5/3} + C$ **25.** $-\frac{1}{2x^2} + C$

27. $\frac{2}{15}x^{1/2}(3x^2 + 5x + 15) + C$ **29.** $x^3 + \frac{1}{2}x^2 - 2x + C$

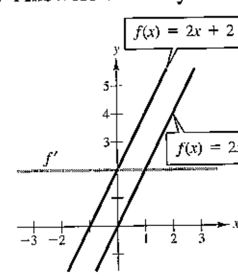
31. $\frac{2}{7}y^{7/2} + C$ **33.** $x + C$ **35.** $-2\cos x + 3\sin x + C$

37. $t + \csc t + C$ **39.** $\tan\theta + \cos\theta + C$ **41.** $\tan y + C$

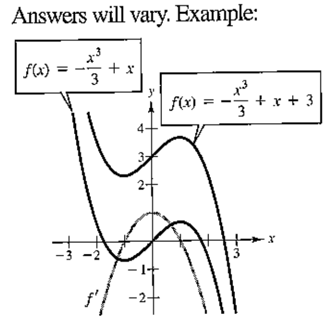
43.



45. Answers will vary. Example:



47. Answers will vary. Example:



49. $y = x^2 - x + 1$ **51.** $y = \sin x + 4$