Answers and Answer Explanations

Using the table below, score your test. Determine how many questions you answered correctly and how many you answered incorrectly. Additional information about scoring is at the end of the Practice Test.

1. E	2. B	3. C	4. D	5. C
6. E	7. E	8. C	9. B	10. C
11. A	12. A	13. A	14. A	15. A
16. D	17. C	18. D	19. A	20. C
21. A	22. C	23. B	24. D	25. D
26. B	27. C	28. E	29. B	30. D
31. D	32. C	33. C	34. E	35. B
36. A	37. C	38. B	39. D	40. E
41. D	42. E	43. A	44. B	45. D

MULTIPLE-CHOICE QUESTIONS

***Note:* Asterisks (*) indicate BC questions and solutions.**

1. ANSWER: **(E)** $f'(x) = 2x\,(-\sin x) + \cos x\,(2) = -2x\sin x + 2\cos x$
(*Calculus* 7th ed. pages 117–123 / 8th ed. pages 119–125)

2. ANSWER: **(B)** $f'(x)$ increasing $\Rightarrow f''(x) > 0 \Rightarrow f(x)$ is concave up.
$f'(x)$ decreasing $\Rightarrow f''(x) < 0 \Rightarrow f(x)$ is concave down.
(B) is concave up for $x < 2$ and concave down for $x > 2$.
(*Calculus* 7th ed. pages 202–207 / 8th ed. pages 209–214)

3. ANSWER: **(C)** If the interval from 0 to 1 is partitioned into n subintervals, then each one has width $\Delta x = \frac{1}{n}$ and their x-coordinates are $\frac{0}{n}, \frac{1}{n}, \frac{2}{n}, \cdots, \frac{k}{n}, \cdots, \frac{n}{n}$. Thus $x_k = \frac{k}{n}$. Recall that a definite integral is defined as the limit of a Riemann sum, $\lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\,\Delta x$. In this problem, $f(x) = \sqrt{x}$. Therefore
$$\lim_{n\to\infty}\sum_{k=1}^{n}\sqrt{\frac{k}{n}}\cdot\frac{1}{n} = \lim_{n\to\infty}\sum_{k=1}^{n}\sqrt{x_k}\,\Delta x = \int_0^1 \sqrt{x}\,dx = \frac{2}{3}x^{\frac{3}{2}}\Big|_0^1 = \frac{2}{3}.$$
(*Calculus* 7th ed. pages 265–272 / 8th ed. pages 271–278)

*4. **ANSWER: (D)** Using Euler's method, $y_{n+1} \approx y_n + \left.\frac{dy}{dx}\right|_{(x_n, y_n)} \cdot \Delta x$

$x_0 = 2$ $\quad y_0 = 5$ $\quad \left.\frac{dy}{dx}\right|_{(2,\,5)} = 5 - 4 + 3 = 4$

$x_1 = 2.5$ $\quad y_1 = 5 + 4(0.5) = 7$ $\quad \left.\frac{dy}{dx}\right|_{(2.5,\,7)} = 7 - 5 + 3 = 5$

$x_2 = 3$ $\quad y_2 = 7 + 5(0.5) = 9.5$ $\quad$ Therefore $f(3) \approx 9.5$.

(*Calculus* 7th ed. pages A2–A3 / 8th ed. pages 404–408)

5. **ANSWER: (C)** $y(5) = 8\sqrt{3 \cdot 5 + 1} = 32$

$y' = \frac{8 \cdot 3}{2\sqrt{3x+1}} = \frac{12}{\sqrt{3x+1}} \Rightarrow y'(5) = \frac{12}{\sqrt{3 \cdot 5 + 1}} = 3$

The equation of the tangent line is $y - 32 = 3(x - 5)$, or $y = 3x + 17$.

(*Calculus* 7th ed. pages 94–101 / 8th ed. pages 96–103)

*6. **ANSWER: (E)** $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{16}{2\sqrt{t}}}{\frac{10}{1+t}} = \frac{16(1+t)}{20\sqrt{t}}$. Therefore

$\left.\frac{dy}{dx}\right|_{t=4} = \frac{80}{40} = 2.$

(*Calculus* 7th ed. pages 675–680 / 8th ed. pages 719–724)

7. **ANSWER: (E)** $\int_0^1 \frac{3}{x}\,dx = \lim_{b \to 0} \int_b^1 \frac{3}{x}\,dx = \lim_{b \to 0} 3 \ln|x|\Big|_b^1 =$

$3 \ln|1| - \lim_{b \to 0} 3 \ln|b| = 0 - (-\infty) = \infty$

(*Calculus* 7th ed. pages 540–546 / 8th ed. pages 578–584)

8. **ANSWER: (C)** Since $f(x)$ is strictly increasing, left end points produce inscribed rectangles and an underapproximation. Right end points produce circumscribed rectangles and an overapproximation. Since $f(x)$ is concave down, a trapezoidal approximation consists of line segments which are below $f(x)$, producing an underapproximation. So I and II are false and III is true, making (C) correct.

(*Calculus* 7th ed. pages 300–304 / 8th ed. pages 309–313)

9. **ANSWER: (B)** Separating variables,

$\int \frac{dy}{y} = \int \left(1 + \frac{1}{x^2}\right) dx \Rightarrow \ln|y| = x - \frac{1}{x} + C_1 \Rightarrow y = e^{x - \frac{1}{x} + C_1} \Rightarrow y = Ce^{x - \frac{1}{x}}$

(*Calculus* 7th ed. pages 369–376 / 8th ed. pages 421–428)

10. ANSWER: (C) Differentiating both sides implicitly,
$2x \cdot 2y \dfrac{dy}{dx} + 2y^2 = 6x - 3y^2 \dfrac{dy}{dx}$.

At point (1, 1), this equation is $4\dfrac{dy}{dx} + 2 = 6 - 3\dfrac{dy}{dx}$. Therefore
$\left.\dfrac{dy}{dx}\right|_{(1,1)} = \dfrac{6-2}{4+3} = \dfrac{4}{7}$.
(*Calculus* 7th ed. pages 137–141 / 8th ed. pages 141–145)

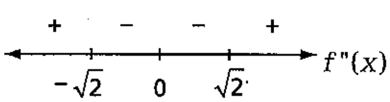
11. ANSWER: (A) $f(x) = \int 12x^2 \sin(2x^3 - 16)\, dx$. Let
$u = 2x^3 - 16 \Rightarrow du = 6x^2 dx$.
$f(x) = \int 2 \sin u \, du = -2 \cos u + C = -2 \cos(2x^3 - 16) + C$.
$5 = -2\cos(2 \cdot 2^3 - 16) + C \Rightarrow 5 = -2\cos(0) + C = -2 + C \Rightarrow C = 7$.
$f(x) = -2\cos(2x^3 - 16) + 7$.
(*Calculus* 7th ed. pages 242–249 / 8th ed. pages 248–255)

*12. ANSWER: (A) The Taylor expansion for a function about $x = a$ is
defined as $\sum_{k=0}^{\infty} \dfrac{f^{(k)}(a)(x-a)^k}{k!}$. Therefore,
$\dfrac{f''(3)(x-3)^2}{2!} = -\dfrac{7(x-3)^2}{3}$. Solving, $f''(3) = -\dfrac{7}{3} \cdot 2! = -\dfrac{14}{3}$.

Equivalently, differentiating the given polynomial twice and substituting $x = 3$ produces $f''(3) = -14/3$.
(*Calculus* 7th ed. pages 605–612 / 8th ed. pages 648–655)

13. ANSWER: (A) $f'(x) = 6x^5 - 20x^3 \Rightarrow f''(x) = 30x^4 - 60x^2 = 30x^2(x^2 - 2)$

$30x^2(x^2 - 2) = 0 \Rightarrow x = 0, \ \pm\sqrt{2}$



The sign of $f''(x)$ changes at $x = \pm\sqrt{2}$ only, so these are the locations of the inflection points.
(*Calculus* 7th ed. pages 184–188 / 8th ed. pages 190–194)

*14. ANSWER: (A) $\dfrac{\cos(0) - e^0}{\ln(1+0)} = \dfrac{1-1}{0} = \dfrac{0}{0}$. This is a quotient indeterminate form, so L'Hôpital's rule applies.
$\lim\limits_{x\to 0} \dfrac{\cos x - e^x}{\ln(1+x)} = \lim\limits_{x\to 0} \dfrac{-\sin x - e^x}{\dfrac{1}{1+x}} = \dfrac{-0-1}{\dfrac{1}{1+0}} = -1$
(*Calculus* 7th ed. pages 530–536 / 8th ed. pages 567–573)

*15. **ANSWER: (A)** I: The series is geometric with $r = 3/4$, so it converges.

II: $\lim_{k\to\infty} \frac{k^2}{(2k+1)^2} = \frac{1}{4} \neq 0$, so the series diverges by the nth-Term Test.

III: $|\sec k| \geq 1$, so $\frac{|\sec k|}{k} \geq \frac{1}{k}$. Since $\sum_{k=1}^{\infty} \frac{1}{k}$ is a divergent p-series (harmonic series, $p = 1$), the series $\sum_{k=1}^{\infty} \frac{|\sec k|}{k}$ diverges by the Direct Comparison Test. Therefore (A) is correct.

(*Calculus* 7th ed. pages 567–572 , 583–586 / 8th ed. pages 606–611 , 624–627)

16. **ANSWER: (D)** Separating variables,

$$\int \frac{dy}{y-2} = \int k\,dt \Rightarrow \ln|y-2| = kt + C_1 \Rightarrow y = e^{kt+C_1} + 2 \Rightarrow y = Ce^{kt} + 2.$$

(*Calculus* 7th ed. pages 369–376 / 8th ed. pages 421–428)

17. **ANSWER: (C)** By the Second Fundamental Theorem, the domain is the largest continuous interval of $f(t)$ containing the lower limit of the integral. Since the upper limit is a function of x, solve the inequality $-1 < 2x - 1 < 5$. The solution is $0 < x < 3$.

(*Calculus* 7th ed. pages 275–283 / 8th ed. pages 282–290)

18. **ANSWER: (D)**

$$F'(x) = f(2x-1)\cdot 2 \Rightarrow F'(2) = 2f(2\cdot 2-1) = 2f(3) = 2\cdot 2 = 4$$

(*Calculus* 7th ed. pages 275–283 / 8th ed. pages 282–290)

19. **ANSWER: (A)** $f'(x) = \frac{3}{\sqrt{1-x^2}} \Rightarrow f'(0) = \frac{3}{1} = 3 \Rightarrow -\frac{1}{f'(0)} = -\frac{1}{3}$

(*Calculus* 7th ed. pages 137–141 / 8th ed. pages 141–145)

*20. **ANSWER: (C)** $\frac{dx}{dt} = \sec^2 t$ and $\frac{dy}{dt} = \frac{1}{2}e^{\frac{1}{2}t}$

$$\text{Length} = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = \int_0^1 \sqrt{\sec^4 t + \frac{1}{4}e^t}\,dt$$

(*Calculus* 7th ed. pages 675–680 / 8th ed. pages 719–724)

*21. **ANSWER: (A)** The Maclaurin series for e^x is $\sum_{k=0}^{\infty} \frac{x^k}{k!}$. So the series for e^{x^2} is $\sum_{k=0}^{\infty} \frac{(x^2)^k}{k!} = \sum_{k=0}^{\infty} \frac{x^{2k}}{k!}$.

$$\text{Area} = \int_0^1 e^{x^2}dx = \int_0^1 \left(\sum_{k=0}^{\infty} \frac{x^{2k}}{k!}\right) dx \approx \int_0^1 \left(\frac{x^0}{0!} + \frac{x^2}{1!} + \frac{x^4}{2!} + \frac{x^6}{3!}\right) dx$$

$$= \frac{x^1}{1\cdot 0!} + \frac{x^3}{3\cdot 1!} + \frac{x^5}{5\cdot 2!} + \frac{x^7}{7\cdot 3!}\Bigg|_0^1 = 1 + \frac{1}{3} + \frac{1}{10} + \frac{1}{42}$$

(*Calculus* 7th ed. pages 632–640 / 8th ed. pages 676–684)

22. **ANSWER: (C)** The slopes are positive in quadrants I and II and negative in quadrants III and IV. This indicates no change in sign on opposite sides of the y-axis, thus x has an even power. There is a change in sign on opposite sides of the x-axis, thus y has an odd power. Therefore (C) is correct.
(*Calculus* 7th ed. pages A2–A3 / 8th ed. pages 404–408)

*23. **ANSWER: (B)** The answer can be determined in two ways: 1. Find the series for $f(x) = x \sin x$ and differentiate. 2. Differentiate $f(x) = x \sin x$ and find its series.

$$1: \quad f(x) = x\sin x \approx x\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}\right)$$

$$= x^2 - \frac{x^4}{3!} + \frac{x^6}{5!} - \frac{x^8}{7!}. \quad f'(x) \approx 2x - \frac{4x^3}{3!} + \frac{6x^5}{5!} - \frac{8x^7}{7!}$$

$$2: \quad f'(x) = x\cos x + \sin x \approx x\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}\right) + x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$$

$$= x - \frac{x^3}{2!} + \frac{x^5}{4!} - \frac{x^7}{6!} + x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$$

$$= 2x - \frac{4x^3}{3!} + \frac{6x^5}{5!} - \frac{8x^7}{7!}$$

(*Calculus* 7th ed. pages 625–629 / 8th ed. pages 669–673)

*24. **ANSWER: (D)** $r = \dfrac{1}{\cos\left(\frac{1}{2}\theta\right)} = \sec\left(\frac{1}{2}\theta\right)$

$$\text{Polar area} = \frac{1}{2}\int_{\theta_1}^{\theta_2} r^2\, d\theta = \frac{1}{2}\int_0^{\frac{\pi}{2}} \sec^2 \frac{1}{2}\theta\, d\theta$$

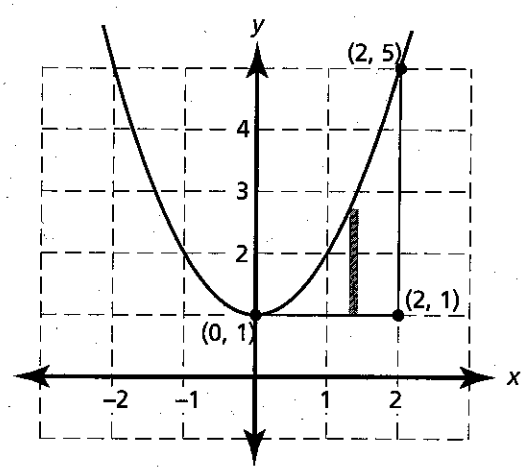
$$= 2\cdot\frac{1}{2}\tan\frac{1}{2}\theta\Big|_0^{\frac{\pi}{2}}$$

$$= \tan\frac{\pi}{4} - \tan 0$$

$$= 1 - 0 = 1$$

(*Calculus* 7th ed. pages 694–699 / 8th ed. pages 739–744)

25. **ANSWER: (D)** By the Washer Method, Volume $= \pi\int_0^2\left[\left(1+x^2\right)^2 - 1^2\right]dx$.
(*Calculus* 7th ed. pages 421–427 / 8th ed. pages 456–462)



*26. **ANSWER: (B)** $\int \frac{2x-3}{x^2+9x+18}\,dx = \int \frac{2x-3}{(x+6)(x+3)}\,dx$. Integrate by partial fractions.

$$\frac{2x-3}{(x+6)(x+3)} = \frac{A}{x+6} + \frac{B}{x+3} \Rightarrow 2x-3 = A(x+3)+B(x+6)$$

Let $x=-6$: $2(-6)-3 = A(-6+3) \Rightarrow -15 = 3A \Rightarrow A = 5$

Let $x=-3$: $2(-3)-3 = B(-3+6) \Rightarrow -9 = 3B \Rightarrow B = -3$

Therefore,

$$\int \frac{2x-3}{(x+6)(x+3)}\,dx = \int \left(\frac{5}{x+6} - \frac{3}{x+3}\right) dx$$
$$= 5\ln|x+6| - 3\ln|x+3| + C$$
$$= \ln\left|\frac{(x+6)^5}{(x+3)^3}\right| + C.$$

(*Calculus* 7th ed. pages 515–521 / 8th ed. pages 552–558)

*27. **ANSWER: (C)** To get the velocity vector, integrate the coordinates of the acceleration vector.

$\int -\pi \sin \pi t\,dt = \cos \pi t + C_1$. $\cos(\pi \cdot 0) + C_1 = 1 \Rightarrow 1 + C_1 = 1 \Rightarrow C_1 = 0$.

$\int (2t+1)\,dt = t^2 + t + C_2$. $0^2 + 0 + C_2 = 0 \Rightarrow C_2 = 0$. The velocity vector is $(\cos \pi t,\ t^2+t)$. Therefore the speed of the particle when $t = 2$ is

$\sqrt{\cos^2(\pi \cdot 2) + (2^2+2)^2} = \sqrt{1^2+6^2} = \sqrt{37}$.

(*Calculus* 7th ed. pages 675–680 / 8th ed. pages 719–724)

*28. **ANSWER: (E)** This is a variation on a p-series, $\sum_{k=1}^{\infty} \frac{1}{k^p}$, so the Limit Comparison Test should be used. If $\lim_{k\to\infty} \frac{u_k}{v_k}$ is finite and positive, then the original series and the comparison series will both converge or both diverge. A p-series converges if $p > 1$. Compare to $\sum_{k=1}^{\infty} \frac{1}{k^{2a-5}}$, because the difference in degree (denominator minus numerator) of the original series is $2a-3-2 = 2a-5$.

$\lim_{k\to\infty} \frac{\frac{k^2}{k^{2a-3}+4}}{\frac{1}{k^{2a-5}}} = \lim_{k\to\infty} \frac{k^2 \cdot k^{2a-5}}{k^{2a-3}+4} = \lim_{k\to\infty} \frac{k^{2a-3}}{k^{2a-3}+4} = 1$, which is finite and positive. $\sum_{k=1}^{\infty} \frac{1}{k^{2a-5}}$ converges if $2a-5 > 1 \Rightarrow 2a > 6 \Rightarrow a > 3$.

(*Calculus* 7th ed. pages 577–580 / 8th ed. pages 617–620)

29. **ANSWER: (B)** Using the Trapezoid Rule,

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}(y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n)$$
$$= \frac{10-4}{2\cdot 3}(24 + 2\cdot 37 + 2\cdot 47 + 58) = 1(250) = 250$$

(*Calculus* 7th ed. pages 300–304 / 8th ed. pages 309–313)

30. **ANSWER: (D)** $\lim\limits_{x\to -2} f(x) = +\infty$, which is nonexistent; the graph is closed at $x = 1$, thus $f(1)$ exists; $f(x)$ is not continuous at $x = 1$, and therefore cannot be differentiable at $x = 1$; $\lim\limits_{x\to\infty} f(x) = 0$, indicated by the horizontal asymptote $y = 0$. Thus (A), (B), (C), and (E) are all false. $\lim\limits_{x\to 1^+} f(x)$ is a finite value, even though it is not the same value as $f(1)$. Therefore (D) is the true statement.
(*Calculus* 7th ed. pages 68–76 / 8th ed. pages 70–78)

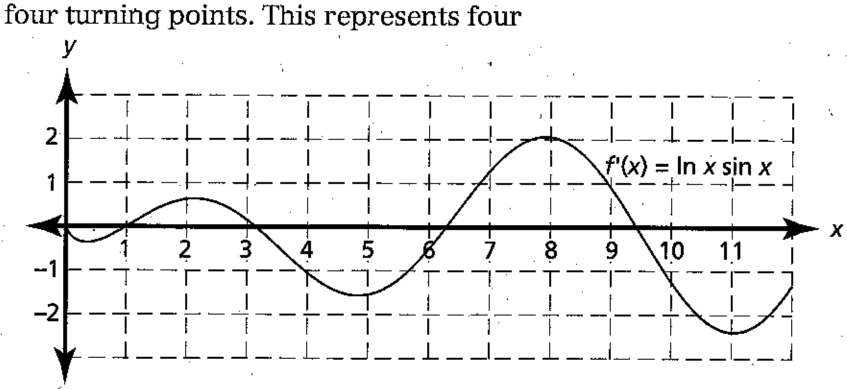
31. **ANSWER: (D)** $f(3) = 0 + \int_{-3}^{3} f'(x)\,dx$ and represents the accumulated area under the curve from $x = -3$ to $x = 3$. The net signed areas of the triangles are $\frac{1}{2}\cdot 3\cdot 2 + \frac{1}{2}\cdot 2\cdot 2 - \frac{1}{2}.1\cdot 1 = 3 + 2 - \frac{1}{2} = 4.5$.
(*Calculus* 7th ed. pages 275–283 / 8th ed. pages 282–290)

*32. **ANSWER: (C)** $dy/dx = \dfrac{dy/dt}{dx/dt} \Rightarrow dx/dt = \dfrac{dy/dt}{dy/dx}$. For the given function, $\dfrac{dy}{dx} = x\cos x + \sin x \Rightarrow \left.\dfrac{dy}{dx}\right|_{x=3} = 3\cos 3 + \sin 3$. Therefore,

$$\left.\frac{dx}{dt}\right|_{x=3} = \frac{-2}{3\cos 3 + \sin 3} \approx 0.707.$$

(*Calculus* 7th ed. pages 675–680 / 8th ed. pages 719–724)

33. **ANSWER: (C)** $1.6 \le x \le 11.6$. In looking at the graph of $f'(x)$ on the given window, there are four turning points. This represents four points at which $f''(x)$ (or the slope of $f'(x)$) is equal to 0 and changes sign from either positive to negative or negative to positive. So these represent four changes in concavity, hence four inflection points.
(*Calculus* 7th ed. pages 184–188 / 8th ed. pages 190–194)



34. **ANSWER: (E)** The average value of a function in an interval is the value of the definite integral divided by its length, that is, $f_{ave} = \dfrac{1}{b-a}\int_a^b f(x)\,dx$. A look at the graphs of $y = \cos x$, $y = \cos 2x$, and $y = \sin x$ reveals that the areas under the curves can be easily

compared without computation. (A) and (C) have positive areas, (B) and (D) are zero, and (E) is negative. Therefore (E) is the only choice to have a negative average value, so it is the smallest. Alternatively, calculate the five average values on the calculator and see that (E) is the smallest.
(*Calculus* 7th ed. pages 275–283 / 8th ed. pages 282–290)

35. ANSWER: **(B)** Use the graphing calculator to graph $v(t)$. Use the derivative feature to graph $v'(t)$. Then $v'(t) = a(t) = 0$ at $t = 2.35619$, $v(2.35619) = -0.670$.
(*Calculus* 7th ed. pages 675–680 / 8th ed. pages 719–724)

*36. ANSWER: **(A)** Series I and II are essentially p-series, so the convergence of the series of absolute values can be obtained by using the Limit Comparison Test. Recall that $\sum_{k=1}^{\infty} \frac{1}{k^p}$ converges if $p > 1$ and diverges if $0 < p \le 1$. Compare I to $\sum_{k=1}^{\infty} \frac{1}{k}$, which is divergent. $\lim_{k\to\infty} \left| \frac{(-1)^k \frac{k^2}{k^3+1}}{\frac{1}{k}} \right| = \lim_{k\to\infty} \left| \frac{k^3}{k^3+1} \right| = 1$, which is finite and positive, thus I does not converge absolutely. But the sequence of positive terms decreases to a limit of zero, so as an alternating series, I converges by the Alternating Series Test. Therefore, I is conditionally convergent. Compare II to $\sum_{k=1}^{\infty} \frac{1}{k^2}$. Using the same limit procedure, II converges absolutely. The sequence in III has limit 1, so it is divergent. In summary, I is the only series that converges conditionally.
(*Calculus* 7th ed. pages 590–595 / 8th ed. pages 631–636)

37. ANSWER: **(C)** $\tan\theta = \frac{h}{8} \Rightarrow h = 8\tan\theta \Rightarrow \frac{dh}{dt} = 8\sec^2\theta \frac{d\theta}{dt}$. When $h = 13$, $8^2 + 13^2 = z^2 \Rightarrow z = \sqrt{64+169} = \sqrt{233}$.
$\left.\frac{dh}{dt}\right|_{h=13} = 8 \cdot \frac{233}{64}(0.03) = 0.874$ cm/sec.
(*Calculus* 7th ed. pages 144–148 / 8th ed. pages 149–153)

38. ANSWER: **(B)** $f''(x) = e^x + 1$, which is positive everywhere. Therefore $f(x)$ is concave up everywhere, so any tangent line to $f(x)$ will be below the curve except at the point of tangency. Thus $H(a + 0.1) < f(a + 0.1)$ for all values of a. [Note: If, for example, $f''(x) = e^x - 1$, there would be a sign change in $f''(x)$, hence an inflection point on the graph of $f(x)$. In that case (C) would be the correct answer.]
(*Calculus* 7th ed. pages 228–232 / 8th ed. pages 235–239)

*39. **ANSWER: (D)** Using the Ratio Test for Absolute Convergence,

$$\lim_{k\to\infty}\left|\frac{\frac{(2x)^{k+1}}{k+2}}{\frac{(2x)^k}{k+1}}\right| = \lim_{k\to\infty}\left|\frac{(2x)^{k+1}\cdot(k+1)}{(k+2)\cdot(2x)^k}\right| < 1$$

$$\Rightarrow \lim_{k\to\infty}|2x| < 1 \Rightarrow |2x| < 1 \Rightarrow -\frac{1}{2} < x < \frac{1}{2}$$

End points must be checked separately.

$x = -\frac{1}{2}$: $\sum_{k=0}^{\infty}\frac{(-1)^k}{k+1}$ converges by the Alternating Series Test, since the series of positive terms is decreasing and $\lim_{k\to\infty}\frac{1}{k+1} = 0$.

$x = \frac{1}{2}$: $\sum_{k=0}^{\infty}\frac{1}{k+1}$ diverges, since it is a p-series (harmonic series, $p = 1$). Therefore, the interval of convergence is $-\frac{1}{2} \le x < \frac{1}{2}$.

(*Calculus* 7th ed. pages 616–622 / 8th ed. pages 659–665)

40. **ANSWER: (E)** The functions intersect at $x = -6$, 0, and 4 and enclose two regions.

Area = $\int_{-6}^{0}[f(x)-g(x)]\,dx + \int_{0}^{4}[g(x)-f(x)]\,dx = 31.5 + 10.667 =$ 42.167. Alternatively, Area = $\int_{-6}^{4}|f(x)-g(x)|\,dx = 42.167$.

(*Calculus* 7th ed. pages 412–417 / 8th ed. pages 446–451)

41. **ANSWER: (D)** The total sales figure is represented by $\int_{0}^{30}(0.32x^2 - 0.01x^3)\,dx = 855$.

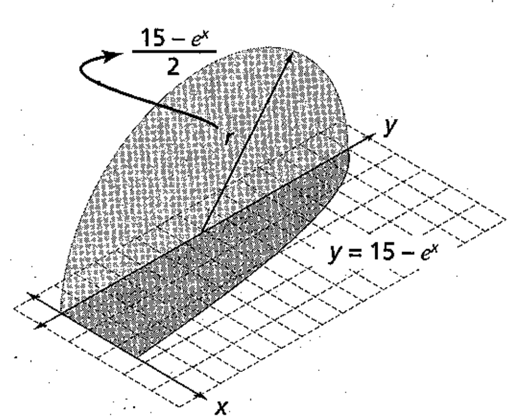
(*Calculus* 7th ed. pages 275–283 / 8th ed. pages 282–290)

42. **ANSWER: (E)** By the product and chain rules, $g'(x) = x^2\cdot 3f'(3x) + 2x\cdot f(3x)$. Therefore, $g'(-1) = (-1)^2\cdot 3f'(-3) - 2f(-3) = 3\cdot 7 - 2(-2) = 25$.

(*Calculus* 7th ed. pages 117–123, 127–133 / 8th ed. pages 119–125, 130–136)

43. **ANSWER: (A)** $15 - e^x = 0 \Rightarrow x = 2.70805$. The radius of each cross section is $\frac{15-e^x}{2}$, so the area of each cross section is $\frac{1}{2}\pi\left(\frac{15-e^x}{2}\right)^2 = \frac{\pi}{8}(15-e^x)^2$. Therefore, $V = \int_{0}^{2.70805}\frac{\pi}{8}(15-e^x)^2\,dx = 118.325$.

(*Calculus* 7th ed. pages 421–427 / 8th ed. pages 456–462)



*44. ANSWER: (B) This is the indeterminate form ∞^0. Let $y = [f(x)]^{\frac{1}{x}}$.

Then $\ln y = \frac{1}{x}\ln[f(x)] = \frac{\ln[f(x)]}{x}$.

Thus $\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\ln[f(x)]}{x}$. This is the indeterminate form $\frac{\infty}{\infty}$, so

use L'Hôpital's rule. $\lim_{x\to\infty} \frac{\ln[f(x)]}{x} = \lim_{x\to\infty} \frac{\frac{f'(x)}{f(x)}}{1} = \frac{\frac{3}{\infty}}{1} = 0$. Therefore,

$\lim_{x\to\infty} \ln y = 0 \Rightarrow \lim_{x\to\infty} y = e^0 = 1$.

(*Calculus* 7th ed. pages 530–536 / 8th ed. pages 567–573)

45. **(D)** The Mean Value Theorem guarantees at least one value c in the interval $0.5 < x < 3.5$ such that $f'(c) = \frac{f(3.5) - f(0.5)}{3.5 - 0.5}$. For the given function, $f'(x) = \frac{1}{x} - 1$. Therefore $f'(c) = \frac{1}{c} - 1 = \frac{0.75276 - 1.80685}{3.5 - 0.5} =$

$-\frac{1.05409}{3} = -0.35136$. So $\frac{1}{c} - 1 = -0.35136 \Rightarrow c = 1.542$.

(*Calculus* 7th ed. pages 168–171 / 8th ed. pages 172–175)

FREE-RESPONSE QUESTIONS

*1. A particle moves in the xy-plane with position vector $(x(t), y(t))$ such that $x(t) = t^3 - 6t^2 + 9t + 1$ and $y(t) = -t^2 + 6t + 2$ in the time interval $0 \le t \le 5$.

a. At what time t is the particle at rest? Justify your answer.
b. Give the velocity vector at $t = 5$.
c. How fast is the particle moving when $t = 5$?
d. Is the speed of the particle increasing or decreasing when $t = 5$? Justify your answer.
e. What is the average speed of the particle for the time interval $0 \le t \le 5$?

	Solution	Possible points
a.	$x'(t) = 3t^2 - 12t + 9 = 3(t-1)(t-3) = 0$ $t = 1$ or 3 $y'(t) = -2t + 6 = -2(t-3) = 0$ $t = 3$ The particle is at rest at $t = 3$ because both $x'(3) = 0$ and $y'(3) = 0$.	3: {1: $x'(t)$ and $y'(t)$ 1: zeros of $x'(t)$ and $y'(t)$ 1: answer with reason
b.	$x'(5) = 3 \cdot 25 - 12 \cdot 5 + 9 = 24$ $y'(5) = -10 + 6 = -4$ The velocity vector at $t = 5$ is $(24, -4)$.	1: answer