Calculus BC Spring Break NO CALCULATOR!

1. What are all values of x for which the function f defined by $f(x) = x^3 + 3x^2 - 9x + 7$ is increasing?

(A) $-3 < x < 1$
(B) $-1 < x < 1$
(C) $x < -3$ or $x > 1$
(D) $x < -1$ or $x > 3$
(E) All real numbers

2. In the xy-plane, the graph of the parametric equations $x = 5t + 2$ and $y = 3t$, for $-3 \le t \le 3$, is a line segment with slope

(A) $\frac{3}{5}$ (B) $\frac{5}{3}$ (C) 3 (D) 5 (E) 13

3. The slope of the line tangent to the curve $y^2 + (xy + 1)^3 = 0$ at $(2, -1)$ is

(A) $-\frac{3}{2}$ (B) $-\frac{3}{4}$ (C) 0 (D) $\frac{3}{4}$ (E) $\frac{3}{2}$

4. $\int \frac{1}{x^2 - 6x + 8}\,dx =$

(A) $\frac{1}{2}\ln\left|\frac{x-4}{x-2}\right| + C$

(B) $\frac{1}{2}\ln\left|\frac{x-2}{x-4}\right| + C$

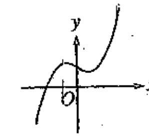
(C) $\frac{1}{2}\ln\left|(x-2)(x-4)\right| + C$

(D) $\frac{1}{2}\ln\left|(x-4)(x+2)\right| + C$

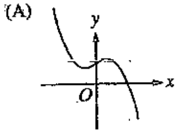
(E) $\ln\left|(x-2)(x-4)\right| + C$

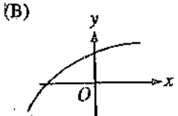
5. If f and g are twice differentiable and if $h(x) = f(g(x))$, then $h''(x) =$

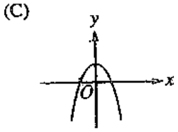
(A) $f''(g(x))[g'(x)]^2 + f'(g(x))g''(x)$
(B) $f''(g(x))g'(x) + f'(g(x))g''(x)$
(C) $f''(g(x))[g'(x)]^2$
(D) $f''(g(x))g''(x)$
(E) $f''(g(x))$

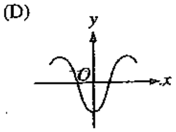


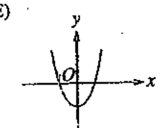
6. The graph of $y = h(x)$ is shown above. Which of the following could be the graph of $y = h'(x)$?

(A)


(B)


(C)


(D)


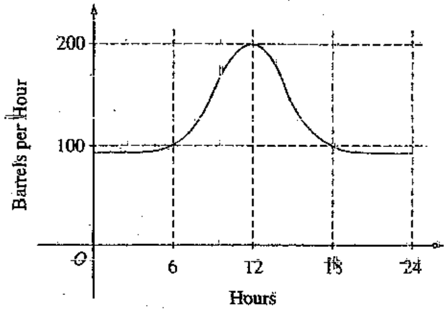
(E)


7. $\int_1^e \left(\frac{x^2-1}{x}\right) dx =$

(A) $e - \frac{1}{e}$ (B) $e^2 - e$ (C) $\frac{e^2}{2} - e + \frac{1}{2}$ (D) $e^2 - 2$ (E) $\frac{e^2}{2} - \frac{3}{2}$

8. If $\frac{dy}{dx} = \sin x \cos^2 x$ and if $y = 0$ when $x = \frac{\pi}{2}$, what is the value of y when $x = 0$?

(A) -1 (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) 1



9. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800

10. A particle moves on a plane curve so that at any time $t > 0$ its x-coordinate is $t^3 - t$ and its y-coordinate is $(2t - 1)^3$. The acceleration vector of the particle at $t = 1$ is

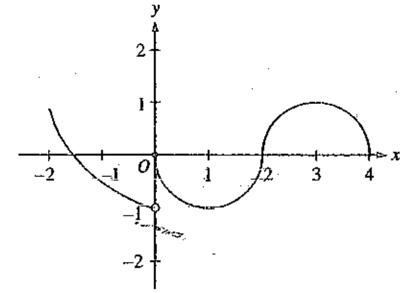
(A) (0, 1) (B) (2, 3) (C) (2, 6) (D) (6, 12) (E) (6, 24)

11. If f is a linear function and $0 < a < b$, then $\int_a^b f''(x)\,dx =$

(A) 0 (B) 1 (C) $\frac{ab}{2}$ (D) $b - a$ (E) $\frac{b^2 - a^2}{2}$

12. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ then $\lim_{x \to 2} f(x)$ is

(A) ln 2 (B) ln 8 (C) ln 16 (D) 4 (E) nonexistent



13. The graph of the function f shown in the figure above has a vertical tangent at the point (2, 0) and horizontal tangents at the points (1, −1) and (3, 1). For what values of x, $-2 < x < 4$, is f not differentiable?

(A) 0 only (B) 0 and 2 only (C) 1 and 3 only (D) 0, 1, and 3 only (E) 0, 1, 2, and 3

14. What is the approximation of the value of sin 1 obtained by using the fifth-degree Taylor polynomial about $x = 0$ for $\sin x$?

(A) $1 - \frac{1}{2} + \frac{1}{24}$

(B) $1 - \frac{1}{2} + \frac{1}{4}$

(C) $1 - \frac{1}{3} + \frac{1}{5}$

(D) $1 - \frac{1}{4} + \frac{1}{8}$

(E) $1 - \frac{1}{6} + \frac{1}{120}$

15. $\int x \cos x\,dx =$

(A) $x \sin x - \cos x + C$

(B) $x \sin x + \cos x + C$

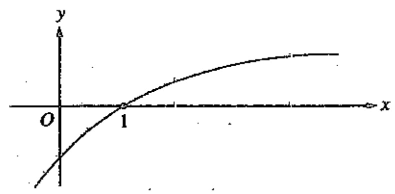
(C) $-x \sin x + \cos x + C$

(D) $x \sin x + C$

(E) $\frac{1}{2}x^2 \sin x + C$

16. If f is the function defined by $f(x) = 3x^5 - 5x^4$, what are all the x-coordinates of points of inflection for the graph of f?

(A) -1 (B) 0 (C) 1 (D) 0 and 1 (E) -1, 0, and 1



17. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f(1) < f'(1) < f''(1)$
(B) $f(1) < f''(1) < f'(1)$
(C) $f'(1) < f(1) < f''(1)$
(D) $f''(1) < f(1) < f'(1)$
(E) $f''(1) < f'(1) < f(1)$

18. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{n}{n+2}$ II. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$ III. $\sum_{n=1}^{\infty} \frac{1}{n}$

(A) None
(B) II only
(C) III only
(D) I and II only
(E) I and III only

19. The area of the region inside the polar curve $r = 4\sin\theta$ and outside the polar curve $r = 2$ is given by

(A) $\frac{1}{2}\int_0^{\pi} (4\sin\theta - 2)^2\, d\theta$ (B) $\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (4\sin\theta - 2)^2\, d\theta$ (C) $\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (4\sin\theta - 2)^2\, d\theta$

(D) $\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (16\sin^2\theta - 4)\, d\theta$ (E) $\frac{1}{2}\int_0^{\pi} (16\sin^2\theta - 4)\, d\theta$

20. When $x = 8$, the rate at which $\sqrt[3]{x}$ is increasing is $\frac{1}{k}$ times the rate at which x is increasing. What is the value of k?

(A) 3 (B) 4 (C) 6 (D) 8 (E) 12

21. The length of the path described by the parametric equations $x = \frac{1}{3}t^3$ and $y = \frac{1}{2}t^2$, where $0 \le t \le 1$, is given by

(A) $\int_0^1 \sqrt{t^2 + 1}\, dt$

(B) $\int_0^1 \sqrt{t^2 + t}\, dt$

(C) $\int_0^1 \sqrt{t^4 + t^2}\, dt$

(D) $\frac{1}{2}\int_0^1 \sqrt{4 + t^4}\, dt$

(E) $\frac{1}{6}\int_0^1 t^2\sqrt{4t^2 + 9}\, dt$

22. If $\lim_{b\to\infty} \int_1^b \frac{dx}{x^p}$ is finite, then which of the following must be true?

(A) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges

(B) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges

(C) $\sum_{n=1}^{\infty} \frac{1}{n^{p-2}}$ converges

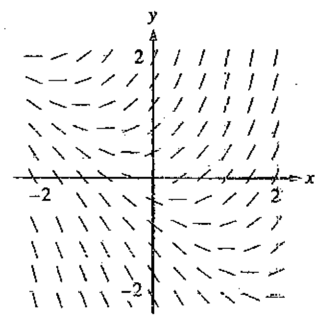
(D) $\sum_{n=1}^{\infty} \frac{1}{n^{p-1}}$ converges

(E) $\sum_{n=1}^{\infty} \frac{1}{n^{p+1}}$ diverges

23. Let f be a function defined and continuous on the closed interval $[a, b]$. If f has a relative maximum at c and $a < c < b$, which of the following statements must be true?

I. $f'(c)$ exists.
II. If $f'(c)$ exists, then $f'(c) = 0$.
III. If $f''(c)$ exists, then $f''(c) \le 0$.

(A) II only (B) III only (C) I and II only (D) I and III only (E) II and III only



24. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = 1 + x$ (B) $\frac{dy}{dx} = x^2$ (C) $\frac{dy}{dx} = x + y$ (D) $\frac{dy}{dx} = \frac{x}{y}$ (E) $\frac{dy}{dx} = \ln y$

25. $\int_0^{\infty} x^2 e^{-x^3}\,dx$ is

(A) $-\frac{1}{3}$ (B) 0 (C) $\frac{1}{3}$ (D) 1 (E) divergent

26. The population $P(t)$ of a species satisfies the logistic differential equation $\frac{dP}{dt} = P\left(2 - \frac{P}{5000}\right)$, where the initial population $P(0) = 3{,}000$ and t is the time in years. What is $\lim_{t\to\infty} P(t)$?

(A) 2,500 (B) 3,000 (C) 4,200 (D) 5,000 (E) 10,000

27. If $\sum_{n=0}^{\infty} a_n x^n$ is a Taylor series that converges to $f(x)$ for all real x, then $f'(1) =$

(A) 0 (B) a_1 (C) $\sum_{n=0}^{\infty} a_n$ (D) $\sum_{n=1}^{\infty} n a_n$ (E) $\sum_{n=1}^{\infty} n a_n^{\,n-1}$

28. $\lim_{x\to 1} \frac{\int_1^x e^{t^2}\,dt}{x^2 - 1}$ is

(A) 0 (B) 1 (C) $\frac{e}{2}$ (D) e (E) nonexistent

CALCULATOR ACTIVE!

76. For what integer k, $k > 1$, will both $\sum_{n=1}^{\infty} \frac{(-1)^{kn}}{n}$ and $\sum_{n=1}^{\infty} \left(\frac{k}{4}\right)^n$ converge?

(A) 6 (B) 5 (C) 4 (D) 3 (E) 2

77. If f is a vector-valued function defined by $f(t) = (e^{-t}, \cos t)$, then $f''(t) =$

(A) $-e^{-t} + \sin t$ (B) $e^{-t} - \cos t$ (C) $(-e^{-t}, -\sin t)$
(D) $(e^{-t}, \cos t)$ (E) $(e^{-t}, -\cos t)$

78. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?

(A) $-(0.2)\pi C$

(B) $-(0.1)C$

(C) $-\frac{(0.1)C}{2\pi}$

(D) $(0.1)^2 C$

(E) $(0.1)^2 \pi C$

79. Let f be the function given by $f(x) = \frac{(x-1)(x^2-4)}{x^2-a}$. For what positive values of a is f continuous for all real numbers x?

(A) None
(B) 1 only
(C) 2 only
(D) 4 only
(E) 1 and 4 only

80. Let R be the region enclosed by the graph of $y = 1 + \ln(\cos^4 x)$, the x-axis, and the lines $x = -\frac{2}{3}$ and $x = \frac{2}{3}$. The closest integer approximation of the area of R is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

81. If $\frac{dy}{dx} = \sqrt{1-y^2}$, then $\frac{d^2y}{dx^2} =$

(A) $-2y$ (B) $-y$ (C) $\frac{-y}{\sqrt{1-y^2}}$ (D) y (E) $\frac{1}{2}$

82. If $f(x) = g(x) + 7$ for $3 \le x \le 5$, then $\int_3^5 [f(x) + g(x)]\,dx =$

(A) $2\int_3^5 g(x)\,dx + 7$

(B) $2\int_3^5 g(x)\,dx + 14$

(C) $2\int_3^5 g(x)\,dx + 28$

(D) $\int_3^5 g(x)\,dx + 7$

(E) $\int_3^5 g(x)\,dx + 14$

83. The Taylor series for $\ln x$, centered at $x = 1$, is $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x-1)^n}{n}$. Let f be the function given by the sum of the first three nonzero terms of this series. The maximum value of $|\ln x - f(x)|$ for $0.3 \le x \le 1.7$ is

(A) 0.030 (B) 0.039 (C) 0.145 (D) 0.153 (E) 0.529

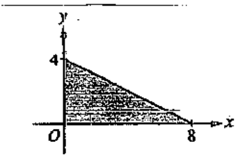
84. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{\sqrt{n}}$ converges?

(A) $-3 < x < -1$ (B) $-3 \le x < -1$ (C) $-3 \le x \le -1$ (D) $-1 \le x < 1$ (E) $-1 \le x \le 1$

x	2	5	7	8
$f(x)$	10	30	40	20

85. The function f is continuous on the closed interval [2, 8] and has values that are given in the table above. Using the subintervals [2, 5], [5, 7], and [7, 8], what is the trapezoidal approximation of $\int_2^8 f(x)\,dx$?

(A) 110 (B) 130 (C) 160 (D) 190 (E) 210

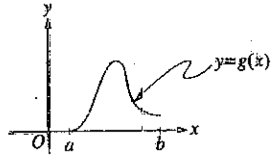


86. The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x + 2y = 8$, as shown in the figure above. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?

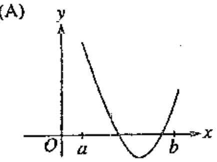
(A) 12.566 (B) 14.661 (C) 16.755 (D) 67.021 (E) 134.041

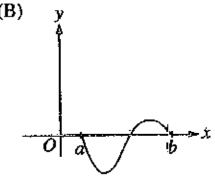
87. Which of the following is an equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$?

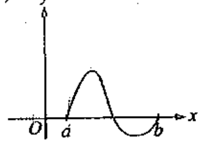
(A) $y = 8x - 5$
(B) $y = x + 7$
(C) $y = x + 0.763$
(D) $y = x - 0.122$
(E) $y = x - 2.146$

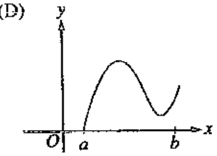


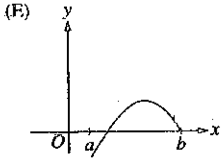
88. Let $g(x) = \int_a^x f(t)\,dt$, where $a \le x \le b$. The figure above shows the graph of g on $[a, b]$. Which of the following could be the graph of f on $[a, b]$?

(A)


(B)


(C)


(D)


(E)


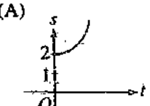
89. The graph of the function represented by the Maclaurin series

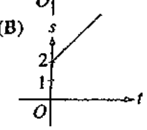
$1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots + \frac{(-1)^n x^n}{n!} + \ldots$ intersects the graph of $y = x^3$ at $x =$

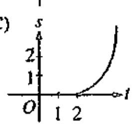
(A) 0.773 (B) 0.865 (C) 0.929 (D) 1.000 (E) 1.857

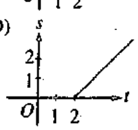
---

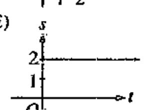
90. A particle starts from rest at the point (2, 0) and moves along the x-axis with a constant positive acceleration for time $t \ge 0$. Which of the following could be the graph of the distance $s(t)$ of the particle from the origin as a function of time t ?

(A)


(B)


(C)


(D)


(E)


t (sec)	0	2	4	6
$a(t)$ (ft/sec^2)	5	2	8	3

91. The data for the acceleration $a(t)$ of a car from 0 to 6 seconds are given in the table above. If the velocity at $t = 0$ is 11 feet per second, the approximate value of the velocity at $t = 6$, computed using a left-hand Riemann sum with three subintervals of equal length, is

(A) 26 ft/sec (B) 30 ft/sec (C) 37 ft/sec (D) 39 ft/sec (E) 41 ft/sec

---

92. Let f be the function given by $f(x) = x^2 - 2x + 3$. The tangent line to the graph of f at $x = 2$ is used to approximate values of $f(x)$. Which of the following is the greatest value of x for which the error resulting from this tangent line approximation is less than 0.5 ?

(A) 2.4 (B) 2.5 (C) 2.6 (D) 2.7 (E) 2.8

FREE RESPONSE – CALCULATOR ACTIVE

1. Let R be the region in the first quadrant bounded by the graph of $y = 8 - x^{\frac{3}{2}}$, the x-axis, and the y-axis.

(a) Find the area of the region R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x-axis, they generate solids with equal volumes. Find the value of k.

2. Let f be the function given by $f(x) = 2xe^{2x}$.

(a) Find $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$.

(b) Find the absolute minimum value of f. Justify that your answer is an absolute minimum.

(c) What is the range of f ?

(d) Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b.

3. Let f be a function that has derivatives of all orders for all real numbers. Assume $f(0) = 5$, $f'(0) = -3$, $f''(0) = 1$, and $f'''(0) = 4$.

(a) Write the third-degree Taylor polynomial for f about $x = 0$ and use it to approximate $f(0.2)$.

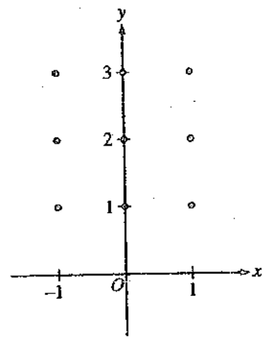
(b) Write the fourth-degree Taylor polynomial for g, where $g(x) = f(x^2)$, about $x = 0$.

(c) Write the third-degree Taylor polynomial for h, where $h(x) = \int_0^x f(t)\,dt$, about $x = 0$.

(d) Let h be defined as in part (c). Given that $f(1) = 3$, either find the exact value of $h(1)$ or explain why it cannot be determined.

4. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.

(a) On the axes provided below, sketch a slope field for the given differential equation at the nine points indicated.



(b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 3$. Use Euler's method starting at $x = 0$, with a step size of 0.1, to approximate $f(0.2)$. Show the work that leads to your answer.

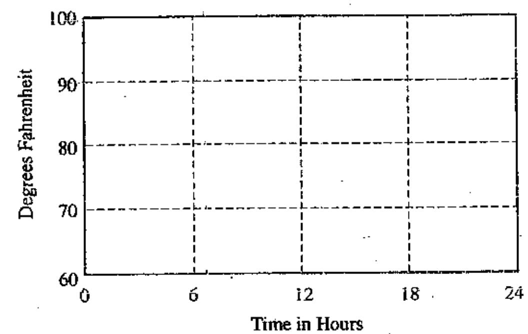
(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$. Use your solution to find $f(0.2)$.

5. The temperature outside a house during a 24-hour period is given by

$$F(t) = 80 - 10\cos\left(\frac{\pi t}{12}\right),\quad 0 \le t \le 24,$$

where $F(t)$ is measured in degrees Fahrenheit and t is measured in hours.

(a) Sketch the graph of F on the grid below.



(b) Find the average temperature, to the nearest degree Fahrenheit, between $t = 6$ and $t = 14$.

(c) An air conditioner cooled the house whenever the outside temperature was at or above 78 degrees Fahrenheit. For what values of t was the air conditioner cooling the house?

(d) The cost of cooling the house accumulates at the rate of \$0.05 per hour for each degree the outside temperature exceeds 78 degrees Fahrenheit. What was the total cost, to the nearest cent, to cool the house for this 24-hour period?

6. A particle moves along the curve defined by the equation $y = x^3 - 3x$. The x-coordinate of the particle, $x(t)$, satisfies the equation $\frac{dx}{dt} = \frac{1}{\sqrt{2t+1}}$, for $t \ge 0$ with initial condition $x(0) = -4$.

(a) Find $x(t)$ in terms of t.

(b) Find $\frac{dy}{dt}$ in terms of t.

(c) Find the location and speed of the particle at time $t = 4$.