ERCISES FOR SECTION 6.4

Exercises 1 and 2, find the distance between the points by (a) the Distance Formula and (b) integration.

0), (5, 12) **2.** (1, 2), (7, 10)

Exercises 3–10, find the arc length of the graph of the function over the indicated interval.

$y = \frac{2}{3}x^{3/2} + 1$, $[0, 1]$ **4.** $y = 2x^{3/2} + 3$, $[0, 9]$

$y = \frac{3}{2}x^{2/3}$, $[1, 8]$ **6.** $y = \frac{3}{2}x^{2/3} + 4$, $[1, 27]$

$y = \frac{x^4}{8} + \frac{1}{4x^2}$, $[1, 2]$ **8.** $y = \frac{x^5}{10} + \frac{1}{6x^3}$, $[1, 2]$

$y = \ln(\sin x)$, $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ **10.** $y = \frac{1}{2}(e^x + e^{-x})$, $[0, 2]$

Exercises 11–20, (a) graph the function, highlighting the part indicated by the given interval, (b) find a definite integral that represents the arc length of the curve over the indicated interval and observe that the integral cannot be evaluated with the techniques studied thus far, and (c) use the integration capabilities of a graphing utility to approximate the arc length.

$y = 4 - x^2$, $[0, 2]$ **12.** $y = x^2 + x - 2$, $[-2, 1]$

$y = \frac{1}{x}$, $[1, 3]$ **14.** $y = \frac{1}{x + 1}$, $[0, 1]$

15. $y = \sin x$, $[0, \pi]$ **16.** $y = \cos x$, $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

17. $x = e^{-y}$, $[0, 2]$ **18.** $y = \ln x$, $[1, 5]$

19. $y = 2 \arctan x$, $[0, 1]$ **20.** $x = \sqrt{36 - y^2}$, $[0, 3]$

pproximation **In Exercises 21 and 22, determine which value best approximates the length of the arc represented by the integral. (Make your selection on the basis of a sketch of the arc and not by performing any calculations.)**

21. $\int_0^2 \sqrt{1 + \left[\frac{d}{dx}\left(\frac{5}{x^2 + 1}\right)\right]^2}\, dx$

(a) 25 (b) 5 (c) 2 (d) −4 (e) 3

22. $\int_0^{\pi/4} \sqrt{1 + \left[\frac{d}{dx}(\tan x)\right]^2}\, dx$

(a) 3 (b) −2 (c) 4 (d) $\frac{4\pi}{3}$ (e) 1

Approximation **In Exercises 23 and 24, approximate the arc length of the graph of the function over the interval [0, 4] in four ways. (a) Use the Distance Formula to find the distance between the endpoints of the arc. (b) Use the Distance Formula to find the lengths of the four line segments connecting the points on the arc when $x = 0$, $x = 1$, $x = 2$, $x = 3$, and $x = 4$. Find the sum of the four lengths. (c) Use Simpson's Rule with $n = 10$ to approximate the integral yielding the indicated arc length. (d) Use the integration capabilities of a graphing utility to approximate the integral yielding the indicated arc length.**

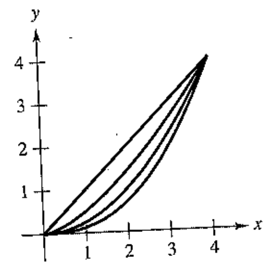
23. $f(x) = x^3$ **24.** $f(x) = (x^2 - 4)^2$

25. ***Think About It*** The figure shows the graphs of the functions $y_1 = x$, $y_2 = \frac{1}{2}x^{3/2}$, $y_3 = \frac{1}{4}x^2$, and $y_4 = \frac{1}{8}x^{5/2}$ on the interval [0, 4]. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com.*

(a) Label the functions.

(b) List the functions in order of increasing arc length.

(c) Verify your answer in part (b) by approximating each arc length accurate to three decimal places.



26. ***Think About It*** Explain why the two integrals are equal.

$$\int_1^e \sqrt{1 + \frac{1}{x^2}}\, dx = \int_0^1 \sqrt{1 + e^{2x}}\, dx$$

Use the integration capabilities of a graphing utility to verify that the integrals are equal.

27. ***Length of Pursuit*** A fleeing object leaves the origin and moves up the y-axis (see figure). At the same time, a pursuer leaves the point (1, 0) and always moves toward the fleeing object. If the pursuer's speed is twice that of the fleeing object, the equation of the path is

$$y = \frac{1}{3}(x^{3/2} - 3x^{1/2} + 2).$$

How far has the fleeing object traveled when it is caught? Show that the pursuer has traveled twice as far.

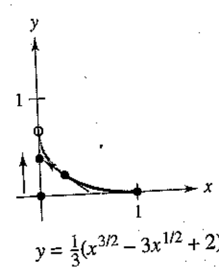


Figure for 27

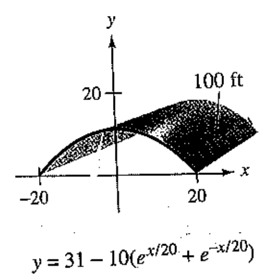


Figure for 28

28. ***Roof Area*** A barn is 100 feet long and 40 feet wide (see figure). A cross section of the roof is the inverted catenary

$$y = 31 - 10(e^{x/20} + e^{-x/20}).$$

Find the number of square feet of roofing on the barn.