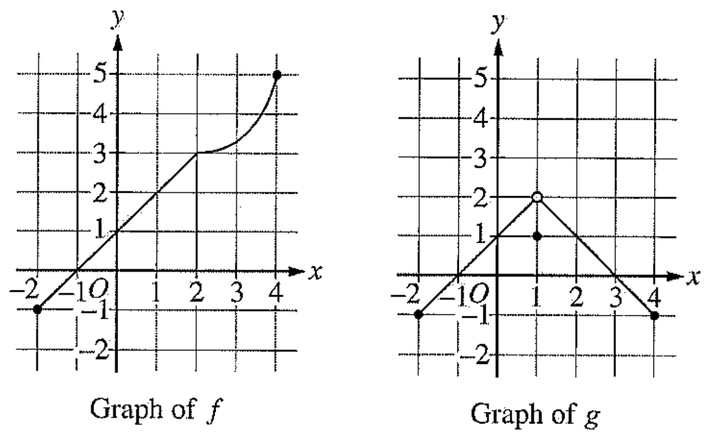
AP Calculus AB Sample Exam Questions

Multiple Choice: Section I, Part A

A calculator may not be used on questions on this part of the exam.



1. The graphs of the functions f and g are shown above. The value of $\lim_{x \to 1} f(g(x))$ is

(A) 1

(B) 2

(C) 3

(D) nonexistent

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 1.1C: Determine limits of functions.	**EK 1.1C1:** Limits of sums, differences, products, quotients, and composite functions can be found using the basic theorems of limits and algebraic rules.	**MPAC 4:** Connecting multiple representations **MPAC 2:** Connecting concepts

2. $\lim_{x \to 0} \frac{7x - \sin x}{x^2 + \sin(3x)} =$

(A) 6

(B) 2

(C) 1

(D) 0

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 1.1C: Determine limits of functions.	**EK 1.1C3:** Limits of the indeterminate forms $\frac{0}{0}$ and $\frac{\infty}{\infty}$ may be evaluated using L'Hospital's Rule.	**MPAC 3:** Implementing algebraic/computational processes **MPAC 5:** Building notational fluency

3. If $f(x) = \sin(\ln(2x))$, then $f'(x) =$

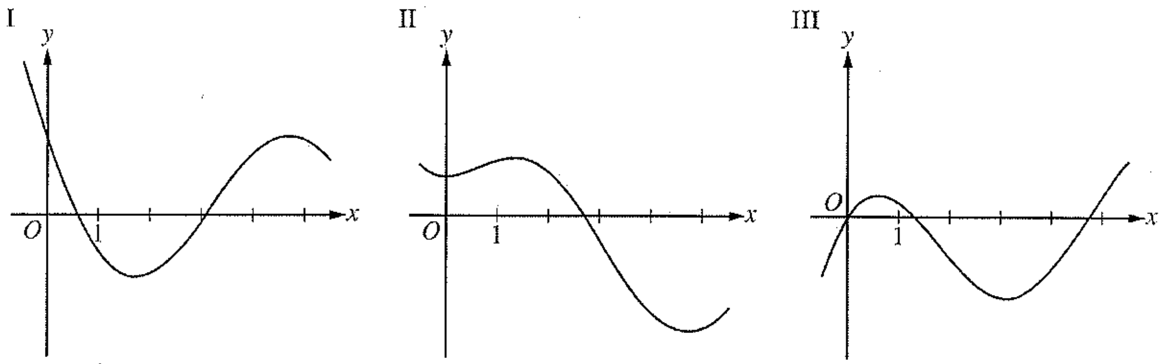
(A) $\dfrac{\sin(\ln(2x))}{2x}$

(B) $\dfrac{\cos(\ln(2x))}{x}$

(C) $\dfrac{\cos(\ln(2x))}{2x}$

(D) $\cos\left(\dfrac{1}{2x}\right)$

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 2.1C: Calculate derivatives.	**EK 2.1C4:** The chain rule provides a way to differentiate composite functions.	**MPAC 3:** Implementing algebraic/computational processes **MPAC 5:** Building notational fluency



4. Three graphs labeled I, II, and III are shown above. One is the graph of f, one is the graph of f', and one is the graph of f''. Which of the following correctly identifies each of the three graphs?

	f	f'	f''
(A)	I	II	III
(B)	II	I	III
(C)	II	III	I
(D)	III	I	II

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 2.2A: Use derivatives to analyze properties of a function.	**EK 2.2A3:** Key features of the graphs of f, f', and f'' are related to one another.	**MPAC 2:** Connecting concepts **MPAC 4:** Connecting multiple representations

5. The local linear approximation to the function g at $x=\frac{1}{2}$ is $y=4x+1$. What is the value of $g\left(\frac{1}{2}\right)+g'\left(\frac{1}{2}\right)$?

(A) 4

(B) 5

(C) 6

(D) 7

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 2.3B: Solve problems involving the slope of a tangent line.	**EK 2.3B2:** The tangent line is the graph of a locally linear approximation of the function near the point of tangency.	**MPAC 2:** Connecting concepts **MPAC 1:** Reasoning with definitions and theorems

6. For time $t \geq 0$, the velocity of a particle moving along the x-axis is given by $v(t) = (t-5)(t-2)^2$. At what values of t is the acceleration of the particle equal to 0?

(A) 2 only

(B) 4 only

(C) 2 and 4

(D) 2 and 5

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 2.3C: Solve problems involving related rates, optimization, rectilinear motion, (BC) and planar motion.	**EK 2.3C1:** The derivative can be used to solve rectilinear motion problems involving position, speed, velocity, and acceleration.	**MPAC 2:** Connecting concepts **MPAC 3:** Implementing algebraic/computational processes
LO 2.1C: Calculate derivatives.	**EK 2.1C3:** Sums, differences, products, and quotients of functions can be differentiated using derivative rules.	

7. The cost, in dollars, to shred the confidential documents of a company is modeled by C, a differentiable function of the weight of documents in pounds. Of the following, which is the best interpretation of $C'(500) = 80$?

(A) The cost to shred 500 pounds of documents is \$80.

(B) The average cost to shred documents is $\frac{80}{500}$ dollar per pound.

(C) Increasing the weight of documents by 500 pounds will increase the cost to shred the documents by approximately \$80.

(D) The cost to shred documents is increasing at a rate of \$80 per pound when the weight of the documents is 500 pounds.

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 2.3A: Interpret the meaning of a derivative within a problem.	**EK 2.3A1:** The unit for $f'(x)$ is the unit for f divided by the unit for x.	**MPAC 2:** Connecting concepts **MPAC 5:** Building notational fluency

8. Which of the following integral expressions is equal to $\lim_{n\to\infty}\sum_{k=1}^{n}\left(\sqrt{1+\frac{3k}{n}}\cdot\frac{1}{n}\right)$?

(A) $\int_0^1 \sqrt{1+3x}\,dx$

(B) $\int_0^3 \sqrt{1+x}\,dx$

(C) $\int_1^4 \sqrt{x}\,dx$

(D) $\frac{1}{3}\int_0^3 \sqrt{x}\,dx$

Learning Objective	**Essential Knowledge**	**Mathematical Practice for AP Calculus**
LO 3.2A(b): Express the limit of a Riemann sum in integral notation.	**EK 3.2A2:** The definite integral of a continuous function f over the interval $[a, b]$, denoted by $\int_a^b f(x)\,dx$, is the limit of Riemann sums as the widths of the subintervals approach 0. That is, $\int_a^b f(x)\,dx = \lim_{\max \Delta x_i \to 0}\sum_{i=1}^{n} f(x_i^*)\Delta x_i$ where x_i^* is a value in the ith subinterval, Δx_i is the width of the ith subinterval, n is the number of subintervals, and $\max \Delta x_i$ is the width of the largest subinterval. Another form of the definition is $\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\Delta x_i$, where $\Delta x_i = \frac{b-a}{n}$ and x_i^* is a value in the ith subinterval.	**MPAC 1:** Reasoning with definitions and theorems **MPAC 5:** Building notational fluency

9. $f(x) = \begin{cases} x & \text{for } x < 2 \\ 3 & \text{for } x \geq 2 \end{cases}$

If f is the function defined above, then $\int_{-1}^{4} f(x)\,dx$ is

(A) $\frac{9}{2}$

(B) $\frac{15}{2}$

(C) $\frac{17}{2}$

(D) undefined

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 3.2C: Calculate a definite integral using areas and properties of definite integrals.	**EK 3.2C3:** The definition of the definite integral may be extended to functions with removable or jump discontinuities.	**MPAC 2:** Connecting concepts **MPAC 3:** Implementing algebraic/computational processes

10. $\int e^x \cos\left(e^x + 1\right) dx =$

(A) $\sin\left(e^x + 1\right) + C$

(B) $e^x \sin\left(e^x + 1\right) + C$

(C) $e^x \sin\left(e^x + x\right) + C$

(D) $\frac{1}{2}\cos^2\left(e^x + 1\right) + C$

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 3.3B(a): Calculate antiderivatives.	**EK 3.3B5:** Techniques for finding antiderivatives include algebraic manipulation such as long division and completing the square, substitution of variables, **(BC)** integration by parts, and nonrepeating linear partial fractions.	**MPAC 3:** Implementing algebraic/computational processes **MPAC 5:** Building notational fluency

11. At time t, a population of bacteria grows at the rate of $5e^{0.2t}+4t$ grams per day, where t is measured in days. By how many grams has the population grown from time $t=0$ days to $t=10$ days?

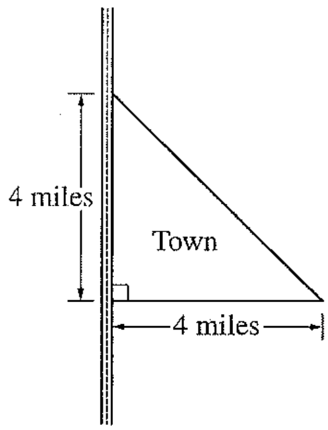
(A) $5e^2+40$

(B) $5e^2+195$

(C) $25e^2+175$

(D) $25e^2+375$

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 3.4A: Interpret the meaning of a definite integral within a problem.	**EK 3.4A2:** The definite integral of the rate of change of a quantity over an interval gives the net change of that quantity over that interval.	**MPAC 2:** Connecting concepts **MPAC 3:** Implementing algebraic/computational processes



12. The right triangle shown in the figure above represents the boundary of a town that is bordered by a highway. The population density of the town at a distance of x miles from the highway is modeled by $D(x)=\sqrt{x+1}$, where $D(x)$ is measured in thousands of people per square mile. According to the model, which of the following expressions gives the total population, in thousands, of the town?

(A) $\int_0^4 \sqrt{x+1}\,dx$

(B) $\int_0^4 8\sqrt{x+1}\,dx$

(C) $\int_0^4 x\sqrt{x+1}\,dx$

(D) $\int_0^4 (4-x)\sqrt{x+1}\,dx$

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 3.4A: Interpret the meaning of a definite integral within a problem.	**EK 3.4A3:** The limit of an approximating Riemann sum can be interpreted as a definite integral.	**MPAC 2:** Connecting concepts **MPAC 5:** Building notational fluency

13. Which of the following is the solution to the differential equation $\frac{dy}{dx} = y\sec^2 x$ with the initial condition $y\left(\frac{\pi}{4}\right) = -1$?

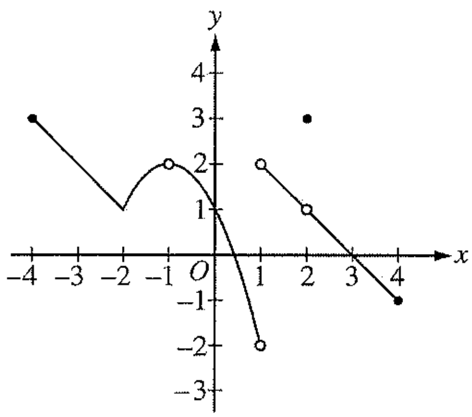
(A) $y = -e^{\tan x}$

(B) $y = -e^{(-1+\tan x)}$

(C) $y = -e^{\left(\sec^3 x - 2\sqrt{2}\right)/3}$

(D) $y = -\sqrt{2\tan x - 1}$

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 3.5A: Analyze differential equations to obtain general and specific solutions.	**EK 3.5A2:** Some differential equations can be solved by separation of variables.	**MPAC 3:** Implementing algebraic/computational processes **MPAC 2:** Connecting concepts



Graph of f

14. The graph of the function f is shown in the figure above. For how many values of x in the open interval $(-4, 4)$ is f discontinuous?

(A) one

(B) two

(C) three

(D) four

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 1.2A: Analyze functions for intervals of continuity or points of discontinuity.	**EK 1.2A3:** Types of discontinuities include removable discontinuities, jump discontinuities, and discontinuities due to vertical asymptotes.	**MPAC 2:** Connecting concepts **MPAC 4:** Connecting multiple representations

Return to Table of Contents

15.

x	0	1	2
$f(x)$	5	2	−7
$f'(x)$	−2	−5	−14

The table above gives selected values of a differentiable and decreasing function f and its derivative. If g is the inverse function of f, what is the value of $g'(2)$?

(A) $-\frac{1}{5}$

(B) $-\frac{1}{14}$

(C) $\frac{1}{5}$

(D) 5

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 2.1C: Calculate derivatives.	**EK 2.1C6:** The chain rule can be used to find the derivative of an inverse function, provided the derivative of that function exists.	**MPAC 3:** Implementing algebraic/computational processes **MPAC 4:** Connecting multiple representations

Multiple Choice: Section I, Part B

A graphing calculator is required for some questions on this part of the exam.

16. The derivative of the function f is given by $f'(x) = -\frac{x}{3} + \cos\left(x^2\right)$. At what values of x does f have a relative minimum on the interval $0 < x < 3$?

(A) 1.094 and 2.608

(B) 1.798

(C) 2.372

(D) 2.493

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 2.2A: Use derivatives to analyze properties of a function.	**EK 2.2A1:** First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.	**MPAC 2:** Connecting concepts **MPAC 3:** Implementing algebraic/computational processes

Return to Table of Contents

17. The second derivative of a function g is given by $g''(x) = 2^{-x^2} + \cos x + x$. For $-5 < x < 5$, on what open intervals is the graph of g concave up?

(A) $-5 < x < -1.016$ only

(B) $-1.016 < x < 5$ only

(C) $0.463 < x < 2.100$ only

(D) $-5 < x < 0.463$ and $2.100 < x < 5$

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 2.2A: Use derivatives to analyze properties of a function.	**EK 2.2A1:** First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.	**MPAC 2:** Connecting concepts **MPAC 3:** Implementing algebraic/computational processes

18. The temperature, in degrees Fahrenheit $(°F)$, of water in a pond is modeled by the function H given by $H(t) = 55 - 9\cos\left(\frac{2\pi}{365}(t + 10)\right)$, where t is the number of days since January 1 $(t = 0)$. What is the instantaneous rate of change of the temperature of the water at time $t = 90$ days?

(A) 0.114°F/day

(B) 0.153°F/day

(C) 50.252°F/day

(D) 56.350°F/day

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 2.3D: Solve problems involving rates of change in applied contexts.	**EK 2.3D1:** The derivative can be used to express information about rates of change in applied contexts.	**MPAC 2:** Connecting concepts **MPAC 3:** Implementing algebraic/ computational processes

19.

x	0	2	4	8
$f(x)$	3	4	9	13
$f'(x)$	0	1	1	2

The table above gives values of a differentiable function f and its derivative at selected values of x. If h is the function given by $h(x) = f(2x)$, which of the following statements must be true?

(I) h is increasing on $2 < x < 4$.

(II) There exists c, where $0 < c < 4$, such that $h(c) = 12$.

(III) There exists c, where $0 < c < 2$, such that $h'(c) = 3$.

(A) II only

(B) I and III only

(C) II and III only

(D) I, II, and III

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 2.4A: Apply the Mean Value Theorem to describe the behavior of a function over an interval.	**EK 2.4A1:** If a function f is continuous over the interval $[a, b]$ and differentiable over the interval (a, b), the Mean Value Theorem guarantees a point within that open interval where the instantaneous rate of change equals the average rate of change over the interval.	**MPAC 1:** Reasoning with definitions and theorems **MPAC 4:** Connecting multiple representations
LO 1.2B: Determine the applicability of important calculus theorems using continuity.	**EK 1.2B1:** Continuity is an essential condition for theorems such as the Intermediate Value Theorem, the Extreme Value Theorem, and the Mean Value Theorem.	

20. Let h be the function defined by $h(x)=\frac{1}{\sqrt{x^5+1}}$. If g is an antiderivative of h and $g(2)=3$, what is the value of $g(4)$?

(A) -0.020

(B) 0.152

(C) 3.031

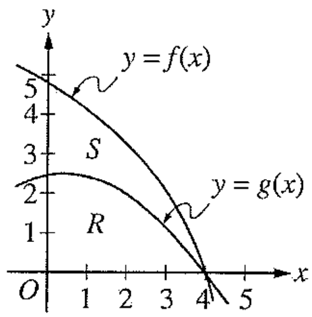
(D) 3.152

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 3.3B(b): Evaluate definite integrals.	**EK 3.3B2:** If f is continuous on the interval $[a, b]$ and F is an antiderivative of f, then $\int_a^b f(x)dx=F(b)-F(a)$.	**MPAC 1:** Reasoning with definitions and theorems **MPAC 2:** Connecting concepts

Return to Table of Contents

Free Response: Section II, Part A

A graphing calculator is required for problems on this part of the exam.



1. Let R be the region in the first quadrant bounded by the graph of g, and let S be the region in the first quadrant between the graphs of f and g, as shown in the figure above. The region in the first quadrant bounded by the graph of f and the coordinate axes has area 12.142. The function g is given by $g(x) = \left(\sqrt{x+6}\right)\cos\left(\frac{\pi x}{8}\right)$, and the function f is not explicitly given. The graphs of f and g intersect at the point $(4, 0)$.

   (A) Find the area of S.

   (B) A solid is generated when S is revolved about the horizontal line $y = 5$. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

   (C) Region R is the base of an art sculpture. At all points in R at a distance x from the y-axis, the height of the sculpture is given by $h(x) = 4 - x$. Find the volume of the art sculpture.

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 3.2C: Calculate a definite integral using areas and properties of definite integrals.	**EK 3.2C2:** Properties of definite integrals include the integral of a constant times a function, the integral of the sum of two functions, reversal of limits of integration, and the integral of a function over adjacent intervals.	**MPAC 1:** Reasoning with definitions and theorems **MPAC 2:** Connecting concepts
LO 3.4D: Apply definite integrals to problems involving area, volume, (BC) and length of a curve.	**EK 3.4D1:** Areas of certain regions in the plane can be calculated with definite integrals. (BC) Areas bounded by polar curves can be calculated with definite integrals.	**MPAC 3:** Implementing algebraic/computational processes **MPAC 4:** Connecting multiple representations
LO 3.4D: Apply definite integrals to problems involving area, volume, (BC) and length of a curve.	**EK 3.4D2:** Volumes of solids with known cross sections, including discs and washers, can be calculated with definite integrals.	**MPAC 5:** Building notational fluency **MPAC 6:** Communicating

Free Response: Section II, Part B

2.

t (minutes)	0	3	5	6	9
$r(t)$ (rotations per minute)	72	95	112	77	50

Rochelle rode a stationary bicycle. The number of rotations per minute of the wheel of the stationary bicycle at time t minutes during Rochelle's ride is modeled by a differentiable function r for $0 \le t \le 9$ minutes. Values of $r(t)$ for selected values of t are shown in the table above.

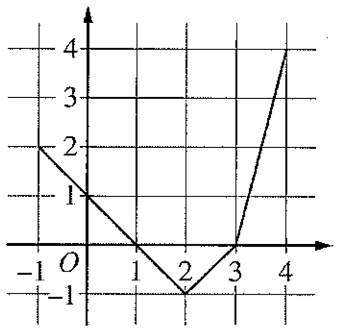
(A) Estimate $r'(4)$. Show the computations that lead to your answer. Indicate units of measure.

(B) Is there a time t, for $3 \le t \le 5$, at which $r(t)$ is 106 rotations per minute? Justify your answer.

(C) Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\int_0^9 r(t)\,dt$. Using correct units, explain the meaning of $\int_0^9 r(t)\,dt$ in the context of the problem.

(D) Sarah also rode a stationary bicycle. The number of rotations per minute of the wheel of the stationary bicycle at time t minutes during Sarah's ride is modeled by the function s, defined by $s(t) = 40 + 20\pi \sin\left(\frac{\pi t}{18}\right)$ for $0 \le t \le 9$ minutes. Find the average number of rotations per minute of the wheel of the stationary bicycle for $0 \le t \le 9$ minutes.

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 1.2B: Determine the applicability of important calculus theorems using continuity.	**EK 1.2B1:** Continuity is an essential condition for theorems such as the Intermediate Value Theorem, the Extreme Value Theorem, and the Mean Value Theorem.	**MPAC 1:** Reasoning with definitions and theorems **MPAC 2:** Connecting concepts **MPAC 3:** Implementing algebraic/computational processes **MPAC 4:** Connecting multiple representations **MPAC 5:** Building notational fluency **MPAC 6:** Communicating
LO 2.1B: Estimate derivatives.	**EK 2.1B1:** The derivative at a point can be estimated from information given in tables or graphs.	
LO 3.2B: Approximate a definite integral.	**EK 3.2B2:** Definite integrals can be approximated using a left Riemann sum, a right Riemann sum, a midpoint Riemann sum, or a trapezoidal sum; approximations can be computed using either uniform or nonuniform partitions.	
LO 3.3B(b): Evaluate definite integrals.	**EK 3.3B2:** If f is continuous on the interval $[a, b]$ and F is an antiderivative of f, then $\int_a^b f(x)\,dx = F(b) - F(a)$.	
LO 3.4A: Interpret the meaning of a definite integral within a problem.	**EK 3.4A2:** The definite integral of the rate of change of a quantity over an interval gives the net change of that quantity over that interval.	
LO 3.4B: Apply definite integrals to problems involving the average value of a function.	**EK 3.4B1:** The average value of a function f over an interval $[a, b]$ is $\frac{1}{b-a}\int_a^b f(x)\,dx$.	
LO 3.4E: Use the definite integral to solve problems in various contexts.	**EK 3.4E1:** The definite integral can be used to express information about accumulation and net change in many applied contexts.	



Graph of f

3. Let f be a continuous function defined on the closed interval $-1 \le x \le 4$. The graph of f, consisting of three line segments, is shown above. Let g be the function defined by $g(x) = 5 + \int_2^x f(t)\,dt$ for $-1 \le x \le 4$.

   (A) Find $g(4)$.

   (B) On what intervals is g increasing? Justify your answer.

   (C) On the closed interval $-1 \le x \le 4$, find the absolute minimum value of g and find the absolute maximum value of g. Justify your answers.

   (D) Let $h(x) = x \cdot g(x)$. Find $h'(2)$.

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 2.1C: Calculate derivatives.	**EK 2.1C3:** Sums, differences, products, and quotients of functions can be differentiated using derivative rules.	**MPAC 1:** Reasoning with definitions and theorems **MPAC 2:** Connecting concepts **MPAC 3:** Implementing algebraic/computational processes **MPAC 4:** Connecting multiple representations **MPAC 5:** Building notational fluency **MPAC 6:** Communicating
LO 2.2A: Use derivatives to analyze properties of a function.	**EK 2.2A1:** First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.	
LO 3.2C: Calculate a definite integral using areas and properties of definite integrals.	**EK 3.2C1:** In some cases, a definite integral can be evaluated by using geometry and the connection between the definite integral and area.	
LO 3.3A: Analyze functions defined by an integral.	**EK 3.3A3:** Graphical, numerical, analytical, and verbal representations of a function f provide information about the function g defined as $g(x)=\int_a^x f(t)\,dt$.	

Answers and Rubrics (AB)

Answers to Multiple-Choice Questions

1	C
2	B
3	B
4	C
5	D
6	C
7	D
8	A
9	B
10	A
11	C
12	D
13	B
14	C
15	A
16	C
17	B
18	B
19	C
20	D