AP Calculus BC Sample Exam Questions

Multiple Choice: Section I, Part A

A calculator may not be used on questions on this part of the exam.

1. A curve is defined by the parametric equations $x(t) = 3e^{2t}$ and $y(t) = e^{3t} - 1$. What is $\frac{d^2y}{dx^2}$ in terms of t?

(A) $\frac{1}{12e^t}$

(B) $\frac{1}{9e^t}$

(C) $\frac{e^t}{2}$

(D) $\frac{3e^t}{4}$

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 2.1C: Calculate derivatives.	**EK 2.1C7: (BC)** Methods for calculating derivatives of real-valued functions can be extended to vector-valued functions, parametric functions, and functions in polar coordinates.	**MPAC 3:** Implementing algebraic/ computational processes **MPAC 2:** Connecting concepts

2.

$x_0 = 0$	$f(x_0) = 2$
$x_1 = 2$	$f(x_1) \approx 6$
$x_2 = 4$	$f(x_2) \approx 10$

Consider the differential equation $\frac{dy}{dx} = \frac{Ax^2 + 4}{y}$, where A is a constant.

Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 2$. Euler's method, starting at $x = 0$ with a step size of 2, is used to approximate $f(4)$. Steps from this approximation are shown in the table above. What is the value of A ?

(A) $\frac{1}{2}$

(B) 2

(C) 5

(D) $\frac{13}{2}$

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 2.3F: Estimate solutions to differential equations.	**EK 2.3F2: (BC)** For differential equations, Euler's method provides a procedure for approximating a solution or a point on a solution curve.	**MPAC 4:** Connecting multiple representations **MPAC 3:** Implementing algebraic/ computational processes

AP CALCULUS BC SAMPLE EXAM QUESTIONS

3. $\int \frac{12}{(x-1)(x-5)}\,dx =$

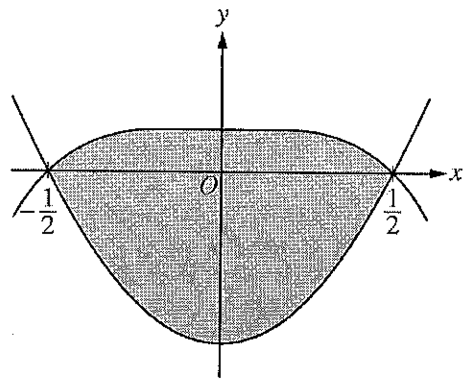
(A) $-3\ln|x-1| + 3\ln|x-5| + C$

(B) $-2\ln|x-1| + 2\ln|x-5| + C$

(C) $3\ln|x-1| - 3\ln|x-5| + C$

(D) $12\ln|x-1| + 12\ln|x-5| + C$

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 3.3B(a): Calculate antiderivatives.	**EK 3.3B5:** Techniques for finding antiderivatives include algebraic manipulation such as long division and completing the square, substitution of variables, (BC) integration by parts, and nonrepeating linear partial fractions.	**MPAC 3:** Implementing algebraic/computational processes **MPAC 5:** Building notational fluency



4. The shaded region in the figure above is bounded by the graphs of $y = x^2 - \frac{1}{4}$ and $y = \frac{1}{16} - x^4$ for $-\frac{1}{2} \le x \le \frac{1}{2}$. Which of the following expressions gives the perimeter of the region?

(A) $2\int_0^{1/2} \sqrt{4x^2 + 16x^6}\, dx$

(B) $2\int_0^{1/2} \sqrt{1 + 4x^2 + 16x^6}\, dx$

(C) $2\int_0^{1/2} \sqrt{1 + 4x^2}\, dx + 2\int_0^{1/2} \sqrt{1 + 16x^6}\, dx$

(D) $2\int_0^{1/2} \sqrt{1 + \left(x^2 - \frac{1}{4}\right)^2}\, dx + 2\int_0^{1/2} \sqrt{1 + \left(\frac{1}{16} - x^4\right)^2}\, dx$

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 3.4D: Apply definite integrals to problems involving area, volume, (BC) and length of a curve.	**EK 3.4D3: (BC)** The length of a planar curve defined by a function or by a parametrically defined curve can be calculated using a definite integral.	**MPAC 2:** Connecting concepts **MPAC 4:** Connecting multiple representations

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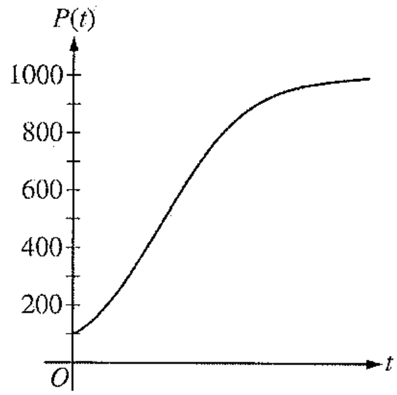
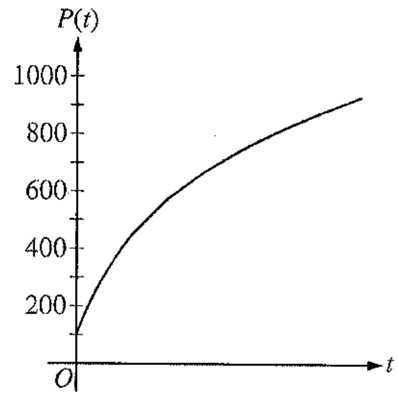
5. The number of fish in a lake is modeled by the function P that satisfies the differential equation $\frac{dP}{dt} = 0.003P(1000 - P)$, where t is the time in years. Which of the following could be the graph of $y = P(t)$?

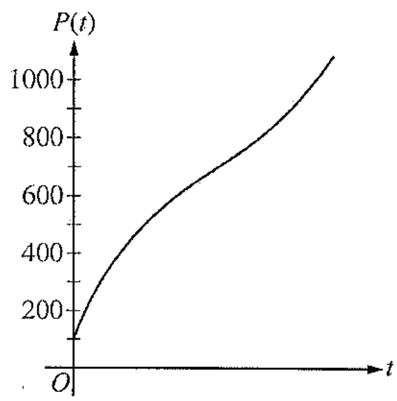
(A)



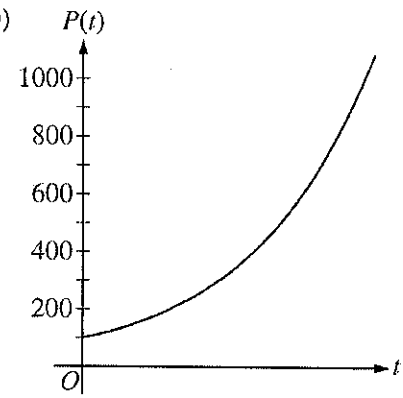
(B)



(C)



(D)



Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 3.5B: Interpret, create and solve differential equations from problems in context.	**EK 3.5B2: (BC)** The model for logistic growth that arises from the statement "The rate of change of a quantity is jointly proportional to the size of the quantity and the difference between the quantity and the carrying capacity" is $\frac{dy}{dt} = ky(a - y)$.	**MPAC 2:** Connecting concepts **MPAC 4:** Connecting multiple representations

6. Which of the following series is absolutely convergent?

(A) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{2n}$

(B) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\sqrt{n}}$

(C) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n+1}$

(D) $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{1}{2}\right)^{n}$

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 4.1A: Determine whether a series converges or diverges.	**EK 4.1A4:** A series may be absolutely convergent, conditionally convergent, or divergent.	**MPAC 1:** Reasoning with definitions and theorems **MPAC 2:** Connecting concepts

7. Which of the following series cannot be shown to converge using the limit comparison test with the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$?

(A) $\sum_{n=1}^{\infty} \frac{4}{3n^2 - n}$

(B) $\sum_{n=1}^{\infty} \frac{15}{\sqrt{n^4 + 5}}$

(C) $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right)$

(D) $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 4.1A: Determine whether a series converges or diverges.	**EK 4.1A6:** In addition to examining the limit of the sequence of partial sums of the series, methods for determining whether a series of numbers converges or diverges are the *n*th term test, the comparison test, the limit comparison test, the integral test, the ratio test, and the alternating series test.	**MPAC 2:** Connecting concepts **MPAC 3:** Implementing algebraic/ computational processes

8. The third-degree Taylor polynomial for the function f about $x = 0$ is $T(x) = 3 - 4x + 2x^2 - 3x^3$. Which of the following tables gives the values of f and its first three derivatives at $x = 0$?

(a)

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$
0	3	−8	6	−12

(b)

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$
0	3	−4	2	−3

(c)

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$
0	3	−4	4	−18

(d)

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$
0	3	−4	4	−9

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 4.2A: Construct and use Taylor polynomials.	**EK 4.2A1:** The coefficient of the nth-degree term in a Taylor polynomial centered at $x = a$ for the function f is $\frac{f^{(n)}(a)}{n!}$.	**MPAC 1:** Reasoning with definitions and theorems **MPAC 4:** Connecting multiple representations

9. What is the interval of convergence for the power series $\sum_{n=1}^{\infty}(-1)^n \frac{n+2}{n \cdot 3^n}(x-4)^n$?

(A) $-3 < x < 3$

(B) $-3 < x \leq 3$

(C) $1 < x < 7$

(D) $1 < x \leq 7$

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 4.2C: Determine the radius and interval of convergence of a power series.	**EK 4.2C2:** The ratio test can be used to determine the radius of convergence of a power series.	**MPAC 3:** Implementing algebraic/computational processes
LO 4.1A: Determine whether a series converges or diverges.	**EK 4.1A6:** In addition to examining the limit of the sequence of partial sums of the series, methods for determining whether a series of numbers converges or diverges are the *n*th term test, the comparison test, the limit comparison test, the integral test, the ratio test, and the alternating series test.	**MPAC 1:** Reasoning with definitions and theorems

Multiple Choice: Section I, Part B

A graphing calculator is required for some questions on this part of the exam.

10. For time $t \geq 0$ seconds, the position of an object traveling along a curve in the xy-plane is given by the parametric equations $x(t)$ and $y(t)$, where $\frac{dx}{dt} = t^2 + 3$ and $\frac{dy}{dt} = t^3 + t$. At what time t is the speed of the object 10 units per second?

(A) 1.675

(B) 1.813

(C) 4.217

(D) 10.191

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 2.3C: Solve problems involving related rates, optimization, rectilinear motion, **(BC)** and planar motion.	**EK 2.3C4: (BC)** Derivatives can be used to determine velocity, speed, and acceleration for a particle moving along curves given by parametric or vector-valued functions.	**MPAC 2:** Connecting concepts **MPAC 3:** Implementing algebraic/ computational processes

11. A particle moving in the *xy*-plane has velocity vector given by $v(t) = \left\langle e^{\sin t}, 5t^2 \right\rangle$ for time $t \geq 0$. What is the magnitude of the displacement of the particle between time $t = 1$ and $t = 2$?

(A) 3.778

(B) 11.954

(C) 11.992

(D) 15.001

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 3.4C: Apply definite integrals to problems involving motion.	**EK 3.4C2: (BC)** The definite integral can be used to determine displacement, distance, and position of a particle moving along a curve given by parametric or vector-valued functions.	**MPAC 1:** Reasoning with definitions and theorems **MPAC 3:** Implementing algebraic/computational processes

12. Consider the series $\sum_{n=0}^{\infty}(-1)^n a_n$, where $a_n > 0$ for all n. Which of the following conditions guarantees that the series converges?

(A) $\lim_{n\to\infty} a_n = 0$

(B) $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} < 1$

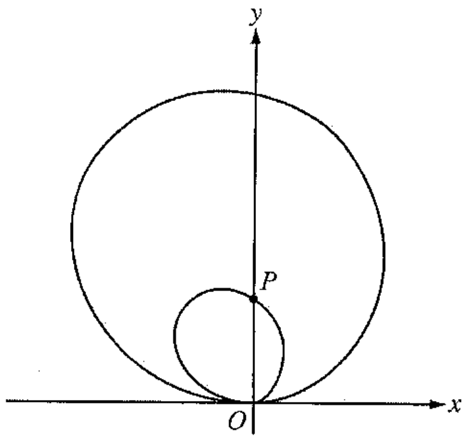
(C) $a_{n+1} < a_n$ for all n

(D) $\int_0^{\infty} f(x)\,dx$ converges, where $f(n) = a_n$ for all n

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 4.1A: Determine whether a series converges or diverges.	**EK 4.1A6:** In addition to examining the limit of the sequence of partial sums of the series, methods for determining whether a series of numbers converges or diverges are the *n*th term test, the comparison test, the limit comparison test, the integral test, the ratio test, and the alternating series test.	**MPAC 1:** Reasoning with definitions and theorems **MPAC 5:** Building notational fluency
LO 4.1A: Determine whether a series converges or diverges.	**EK 4.1A5:** If a series converges absolutely, then it converges.	

Free Response: Section II, Part A

A graphing calculator is required for problems on this part of the exam.



1. Let r be the function given by $r(\theta) = 3\theta \sin \theta$ for $0 \le \theta \le 2\pi$. The graph of r in polar coordinates consists of two loops, as shown in the figure above. Point P is on the graph of r and the y-axis.

   (A) Find the rate of change of the x-coordinate with respect to θ at the point P.

   (B) Find the area of the region between the inner and outer loops of the graph.

   (C) The function r satisfies $\dfrac{dr}{d\theta} = 3\sin\theta + 3\theta\cos\theta$. For $0 \le \theta \le 2\pi$, find the value of θ that gives the point on the graph that is farthest from the origin. Justify your answer.

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 2.2A: Use derivatives to analyze properties of a function.	**EK 2.2A4: (BC)** For a curve given by a polar equation $r = f(\theta)$, derivatives of r, x, and y with respect to θ and first and second derivatives of y with respect to x can provide information about the curve.	**MPAC 1:** Reasoning with definitions and theorems **MPAC 2:** Connecting concepts
LO 2.3C: Solve problems involving related rates, optimization, rectilinear motion, **(BC)** and planar motion.	**EK 2.3C3:** The derivative can be used to solve optimization problems, that is, finding a maximum or minimum value of a function over a given interval.	**MPAC 3:** Implementing algebraic/computational processes **MPAC 4:** Connecting multiple representations
LO 3.4D: Apply definite integrals to problems involving area, volume, **(BC)** and length of a curve.	**EK 3.4D1:** Areas of certain regions in the plane can be calculated with definite integrals. **(BC)** Areas bounded by polar curves can be calculated with definite integrals.	**MPAC 5:** Building notational fluency **MPAC 6:** Communicating

Free Response: Section II, Part B

No calculator is allowed for problems on this part of the exam.

2. Consider the function f given by $f(x) = xe^{-2x}$ for all $x \geq 0$.

(A) Find $\lim_{x \to \infty} f(x)$.

(B) Find the maximum value of f for $x \geq 0$. Justify your answer.

(C) Evaluate $\int_0^\infty f(x)\,dx$, or show that the integral diverges.

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 1.1D: Deduce and interpret behavior of functions using limits.	**EK 1.1D2:** Relative magnitudes of functions and their rates of change can be compared using limits.	**MPAC 1:** Reasoning with definitions and theorems **MPAC 2:** Connecting concepts **MPAC 3:** Implementing algebraic/ computational processes **MPAC 4:** Building notational fluency **MPAC 6:** Communicating
LO 2.2A: Use derivatives to analyze properties of a function.	**EK 2.2A1:** First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.	
LO 3.2D: (BC) Evaluate an improper integral or show that an improper integral diverges.	**EK 3.2D2: (BC)** Improper integrals can be determined using limits of definite integrals.	
LO 3.3B(b): Evaluate definite integrals.	**EK 3.3B5:** Techniques for finding antiderivatives include algebraic manipulation such as long division and completing the square, substitution of variables, **(BC)** integration by parts, and nonrepeating linear partial fractions.	

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3. The function f is defined by the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(x-2)^n}{3^n(n+1)} = 1 + \frac{x-2}{3 \cdot 2} + \frac{(x-2)^2}{3^2 \cdot 3} + \frac{(x-2)^3}{3^3 \cdot 4} + \dots + \frac{(x-2)^n}{3^n(n+1)} + \dots$$

for all real numbers x for which the series converges.

(A) Determine the interval of convergence of the power series for f. Show the work that leads to your answer.

(B) Find the value of $f''(2)$.

(C) Use the first three nonzero terms of the power series for f to approximate $f(1)$. Use the alternating series error bound to show that this approximation differs from $f(1)$ by less than $\frac{1}{100}$.

Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus
LO 4.1A: Determine whether a series converges or diverges.	**EK 4.1A3:** Common series of numbers include geometric series, the harmonic series, and *p*-series.	**MPAC 1:** Reasoning with definitions and theorems **MPAC 2:** Connecting concepts **MPAC 3:** Implementing algebraic/computational processes **MPAC 5:** Building notational fluency **MPAC 6:** Communicating
LO 4.1A: Determine whether a series converges or diverges.	**EK 4.1A4:** A series may be absolutely convergent, conditionally convergent, or divergent.	
LO 4.1B: Determine or estimate the sum of a series.	**EK 4.1B2:** If an alternating series converges by the alternating series test, then the alternating series error bound can be used to estimate how close a partial sum is to the value of the infinite series.	
LO 4.2A: Construct and use Taylor polynomials.	**EK 4.2A1:** The coefficient of the nth-degree term in a Taylor polynomial centered at $x = a$ for the function f is $\frac{f^{(n)}(a)}{n!}$.	
LO 4.2C: Determine the radius and interval of convergence of a power series.	**EK 4.2C1:** If a power series converges, it either converges at a single point or has an interval of convergence.	
LO 4.2C: Determine the radius and interval of convergence of a power series.	**EK 4.2C2:** The ratio test can be used to determine the radius of convergence of a power series.	
LO 4.2C: Determine the radius and interval of convergence of a power series.	**EK 4.2C3:** If a power series has a positive radius of convergence, then the power series is the Taylor series of the function to which it converges over the open interval.	