AP CALCULUS AB PRACTICE TEST 1
Section I, Part A: Multiple-Choice Questions
Time: 55 minutes
Number of Questions: 28

A calculator may not be used on this part of the examination.

1. What is $\int \frac{x-3}{x}\,dx$?

   (A) $1-3\ln x+C$
   (B) $x-3\ln x+C$
   (C) $1+\frac{3}{x^2}+C$
   (D) $\frac{x^2-3x}{x^2}+C$
   (E) $\frac{x^2}{2}-3\ln x+C$

2. What is the value of $\lim_{x\to -1}\frac{x^2-3x-4}{x^2-1}$?

   (A) $\frac{5}{2}$
   (B) 1
   (C) 0
   (D) $-\frac{5}{2}$
   (E) The limit does not exist.

3. What is the equation of the tangent to $f(x)=3x-5\cos 2x$ at $x=0$?

   (A) $x=-5x-3$
   (B) $y=-5x+3$
   (C) $y=3x+5$
   (D) $y=3x-5$
   (E) $y=x-5$

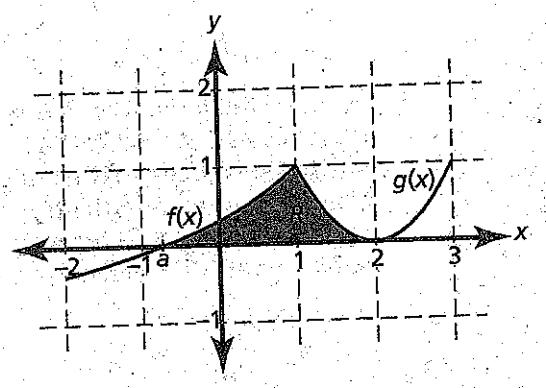
4. A particle moves along the y-axis so that its position at any time t, for $0\le t\le 5$, is given by $y(t)=t^4-18t^2$. In which interval(s) is the particle speeding up?

   (A) $0<t<\sqrt{3}$
   (B) $0<t<\sqrt{3}$ and $3<t<5$
   (C) $3<t<5$
   (D) $\sqrt{3}<t<3$ and $3<t<5$
   (E) $\sqrt{3}<t<3$

5. Which of the following statements is (are) false for $f(x)=e^x\sin x$?

   I. $\lim_{x\to 0} f(x)=0$
   II. $\lim_{x\to 0} f'(x)=1$
   III. $\lim_{x\to 0} f''(x)=2$

   (A) I only
   (B) II only
   (C) III only
   (D) II and III only
   (E) None of the statements is false.



6. The region R, bounded by $f(x)$, $g(x)$, and the x-axis, is shown in the diagram above. Which one of the following integrals represents the volume of the solid generated when R is rotated about the line $y=1$?

   (A) $\pi\int_a^1[1-f(x)]^2\,dx+\pi\int_1^2[1-g(x)]^2\,dx$
   (B) $\pi\int_a^1\{1^2-[1-f(x)]^2\}\,dx+\pi\int_1^2\{1^2-[1-g(x)]^2\}\,dx$
   (C) $\pi\int_a^1\{1-[f(x)]^2\}\,dx+\pi\int_1^2\{1-[g(x)]^2\}\,dx$
   (D) $\pi\int_a^1[1-f(x^2)]\,dx+\pi\int_1^2[1-g(x^2)]\,dx$
   (E) none of these

7. Let $f(x) = (3 + 2x - x^2)^3$ be defined for the closed interval $-2 \le x \le 3$. If M is the y-coordinate of the absolute maximum and m is the y-coordinate of the absolute minimum, what is $|M + m|$?
(A) 189
(B) 125
(C) 64
(D) 61
(E) none of these

8. Find the equation of the curve that passes through the point (1, 2) and has a slope of $\left(3 + \frac{1}{x}\right)y$ at any point (x, y) on the curve.
(A) $2xe^{3x-3}$
(B) $2xe^{3x+3}$
(C) $2xe^{3}$
(D) $2e^{3x-3}$
(E) none of these

9. A continuous function $h(t)$ is defined in the closed interval $10 \le t \le 16$ with values given in the table below. Using the data, find the trapezoidal approximation with three subintervals of unequal length to estimate $\int_{10}^{16} h(t)\,dt$.

t	h(t)
10	10
12	20
15	50
16	80

(A) $\frac{359}{3}$
(B) 130
(C) 200
(D) 270
(E) 718

10. Find the x-coordinate of the point on $f(x) = \frac{4}{\sqrt{x}}$ that is closest to the origin.
(A) 1
(B) 2
(C) $\sqrt{2}$
(D) $2\sqrt{2}$
(E) $\sqrt[3]{2}$

11. Evaluate $\lim_{x \to \infty} \frac{3 - \sqrt{x^2 - 1}}{2x + 5}$.
(A) $-\frac{1}{2}$
(B) 0
(C) $\frac{3}{5}$
(D) $\frac{3}{2}$
(E) The limit does not exist.

12. If $\tan y + x^3 = y^2 + 1$ and $\frac{dx}{dt} = -2$, what is the value of $\frac{dy}{dt}$ at the point (1, 0)?
(A) -6
(B) -2.5
(C) 0
(D) $\frac{1}{2(\cos 1)^2}$
(E) 6

13. A particle moves in a line with velocity $v(t) = 3t^2 - e^t$. What is the average velocity of the particle in the closed interval $0 \le t \le 2$?
(A) $\frac{8 - e^2}{2}$
(B) $\frac{9 - e^2}{2}$
(C) $\frac{11 - e^2}{2}$
(D) $\frac{13 - e^2}{2}$
(E) $13 - e^2$

14. What is the value of $k + c$ if
$$f(x) = \begin{cases} 2kx^2 - x, & x > 3 \\ x^3 + cx, & x \le 3 \end{cases}$$
is everywhere differentiable?
(A) $\frac{5}{4}$
(B) 3
(C) 8
(D) 11
(E) 24

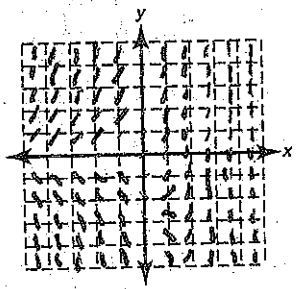
15. A particle moves along the x-axis with a velocity given by $v(t) = t - \sqrt[3]{t}$ for $0 \le t \le 8$. If the particle is 4 units to the left of the origin at $t = 0$, where is the particle at $t = 8$?
(A) 24 units to the right of the origin
(B) 20 units to the right of the origin
(C) 16 units to the right of the origin
(D) $1\frac{3}{4}$ units to the left of the origin
(E) $3\frac{1}{12}$ units to the left of the origin

16. $\int_{-2}^{5} |x+1|\,dx =$
(A) 17
(B) 17.5
(C) 18.5
(D) 19
(E) 19.5

17. $\int \frac{x+2}{x^2+1}\,dx =$
(A) $\ln(x^2+1) + C$
(B) $\left(\frac{x^2}{2} + 2x\right)\ln(x^2+1) + C$
(C) $\frac{1}{2}\ln(x^2+1) + C$
(D) $\frac{1}{2}\ln(x^2+1) + 2\tan^{-1}(x^2) + C$
(E) $\frac{1}{2}\ln(x^2+1) + 2\tan^{-1}(x) + C$

18. The tangent line to the graph of $g(x)$ at the point (3, 5) has a slope of –2. Use the equation of the tangent to estimate $g(2.98)$.
(A) 2.50
(B) 4.98
(C) 5.02
(D) 5.04
(E) 7.02

19. Which of the following is the slope field for $\frac{dy}{dx} = \frac{e^x}{y}$?

(A)


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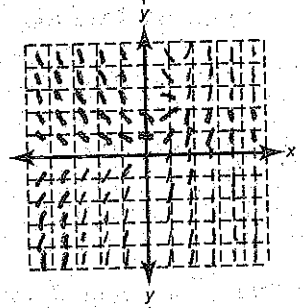

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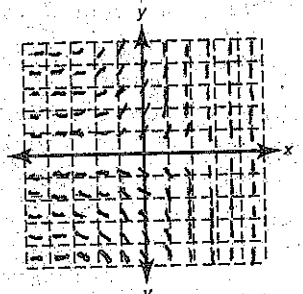

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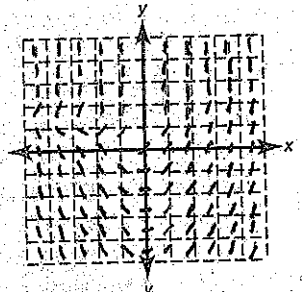
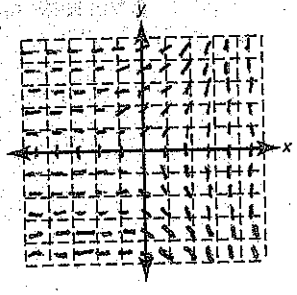


(E)


20. Consider the piecewise function,

$$g(x)=\begin{cases} e^{\left(\frac{x}{10}+C\right)}, & x<10 \\ 3, & x=10 \\ \log(x)+1, & x>10 \end{cases}.$$

Find the value of C so that $\lim_{x\to 10} g(x)$ exists.

A) 3
B) 2
C) 1
D) $\ln 2 - 1$
E) The limit does not exist.

21. If $y = 4^{x^2}$, what is $y'(1)$?

(A) 0
(B) $\ln 4$
(C) $2 \ln 4$
(D) $1 + 2 \ln 4$
(E) $8 \ln 4$

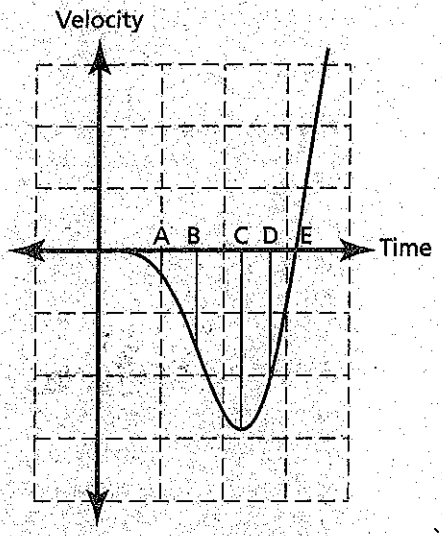
22. What is the value of $g(2)$ if

$$g(x)=3+\frac{d}{dx}\left[\int_{x}^{x^2}\left(1+t^2\right)dt\right]?$$

(A) 8
(B) 20
(C) 23
(D) 24
(E) 71

23. Which of the following statements is true for $f(x)=\frac{1+e^x}{e^x-1}$?

(A) $f(x)$ has a relative maximum at $x = 1$.
(B) $f(x)$ has a y-intercept at $x = 0$.
(C) $f(x)$ has a root of 0.
(D) $f(x)$ is decreasing for all x, $x \neq 0$.
(E) $f(x)$ has a vertical asymptote at $x = 1$.



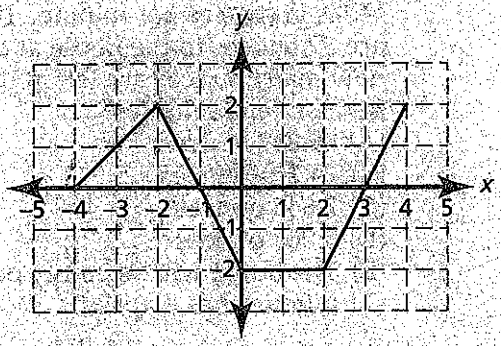
24. A particle moves along the x-axis. At which time on the velocity versus time graph given above is the particle farthest left of its starting point?

(A) A
(B) B
(C) C
(D) D
(E) E

25. If $\sin xy = x + y$, what is $\frac{dy}{dx}$?

(A) $\frac{y\cos xy+1}{1-x\cos xy}$
(B) $\frac{y\cos xy-1}{1-x\cos xy}$
(C) $\frac{\cos xy-1}{1-x\cos xy}$
(D) $\frac{1}{\cos xy-1}$
(E) $\frac{\cos x-1}{1-\cos y}$

The graph of $f(x)$ consists of four line segments as shown below. Let g be the function given by $g(x) = \int_{-4}^{x} f(t)\,dt$. Use this information for Problems 26–28.



26. What is $g(0)$?
(A) 4
(B) 2
(C) 0
(D) –2
(E) –4

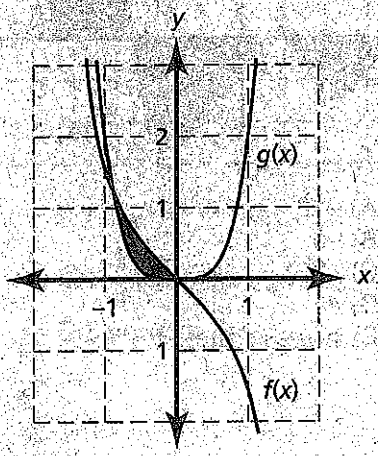
27. What is the equation of the tangent to $g(x)$ at the point $[3, g(3)]$?
(A) $y = 0$
(B) $y = 1$
(C) $y = x - 3$
(D) $y = x + 3$
(E) $y = -3$

28. Which of the following is false for $g(x)$?
(A) $g(x)$ has a relative maximum at $x = -1$.
(B) $g(x)$ has a relative minimum at $x = 3$.
(C) $g(x)$ has a relative maximum at $x = -2$.
(D) $g(x)$ is decreasing in the interval $2 < x < 3$.
(E) $g(x)$ is increasing in the interval $-2 < x < -1$.

Section I, Part B: Multiple-Choice Questions
Time: 50 minutes
Number of Questions: 17

A calculator may be used on this part of the examination.

29. What is the area of the first closed region to the left of the y-axis, bounded by the curves $f(x) = -\tan x$ and $g(x) = 2x^4$?
(A) 1.206
(B) 0.931
(C) 0.691
(D) 0.452
(E) 0.240



30. What is the average rate of change of $f(x) = \frac{e^{\frac{1}{x}}}{x^2}$ in the interval $-4 \le x \le -1$?
(A) 0.106
(B) 0.137
(C) 0.319
(D) 0.411
(E) 1.233

31. Consider the integral expression $\int_0^{\frac{\pi}{2}} \sin(2x)e^{\cos(2x)}\,dx$. If $u = \cos 2x$ then which integral below is equivalent to the given integral?

(A) $-\frac{1}{2}\int_0^{\pi} e^u\,du$

(B) $-2\int_0^{\pi} e^u\,du$

(C) $-\frac{1}{2}\int_{-1}^{1} e^u\,du$

(D) $\frac{1}{2}\int_{-1}^{1} e^u\,du$

(E) $2\int_{-1}^{1} e^u\,du$

32. Let $f(x) = \frac{1}{x}$ and $k > 1$. If the area between the x-axis and the graph of $f(x)$ in the closed interval $k \le x \le k+1$ is 0.125 where $k > 1$, then what is the value of k?

(A) 0.133
(B) 1.133
(C) 1.334
(D) 2.998
(E) 7.510

33. A solid has its base in the xy-plane, bounded by the x-axis, the y-axis, and the function $y = 3 - x^5$. If cross sections taken perpendicular to the x-axis are semicircles whose diameters are in the xy-plane, what is the volume of this solid?

(A) 3.335
(B) 4.247
(C) 5.239
(D) 6.671
(E) 13.342

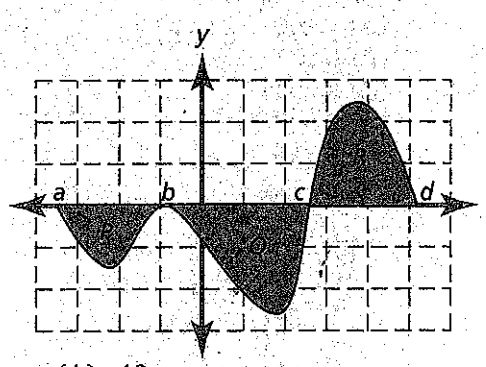
34. Shampoo drips from a crack in the side of a plastic bottle at a rate modeled by $Y(t) = \frac{t}{\sqrt{1+t^{\frac{3}{2}}}}$, where $Y(t)$ is in ounces per minute. If there are 32 ounces in the bottle at $t = 0$, how many ounces are left in the bottle after 5 minutes?

(A) 26.937 ounces
(B) 24.355 ounces
(C) 7.645 ounces
(D) 5.063 ounces
(E) The bottle will be empty before 5 minutes has elapsed.

35. Consider the function $f(x) = x^3 + 2$ in the closed interval $0 < a \le c \le 2$. If the value guaranteed by the Mean Value Theorem in the closed interval is $c = 1.720$, then what is the value of a?

(A) 1.260
(B) 1.424
(C) 1.602
(D) 1.680
(E) none of these

36. The sketch of $f(x)$ is shown below, with regions bounded by $f(x)$ and the x-axis indicated by P, Q, and R. If $\int_a^d f(x)\,dx = -7$, $\int_b^d f(x)\,dx = -2$ and $\int_c^a f(x)\,dx = 17$, what is $\int_b^c f(x)\,dx$?



(A) –12
(B) –6
(C) –3
(D) 4
(E) none of these

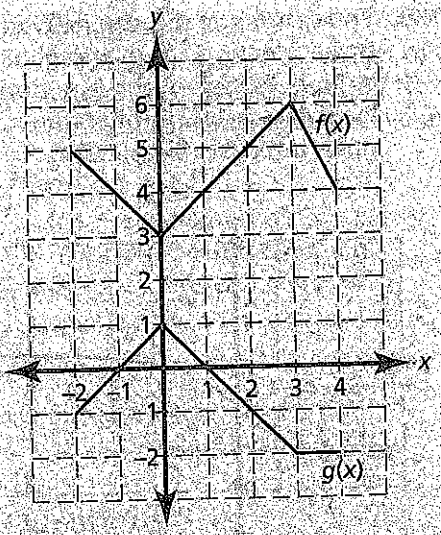
37. Let $h(x) = xg(x)$, where $g(x) = f^{-1}(x)$. Use the table of values below to find $h'(5)$.

x	$f(x)$	$f'(x)$
2	4	–1
3	5	2
5	1	3

(A) $\frac{1}{2}$
(B) 2.5
(C) 3
(D) $4\frac{2}{3}$
(E) 5.5

38. Let $f(x) = \sin x$ and $g(x) = p \ln x$ in the closed interval $0 \le x \le \frac{\pi}{2}$. For what value of p will the tangents to the curves at their points of intersection be perpendicular?
(A) –0.447
(B) 0.410
(C) 1.260
(D) 1.303
(E) none of these

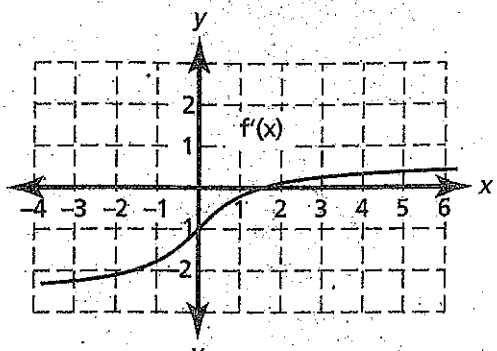
39. The height of a conical sand pile is always twice the radius. If sand is being added to the pile at a rate of 30π cm³/min, how fast is the height of the pile increasing when the circumference of the base of the sand pile is 120π cm?
($V_{cone} = \frac{\pi}{3} r^2 h$)
(A) $\frac{1}{120\pi}$ cm/min
(B) $\frac{1}{120}$ cm/min
(C) $\frac{2}{15}$ cm/min
(D) $\frac{1}{4}$ cm/min
(E) none of these

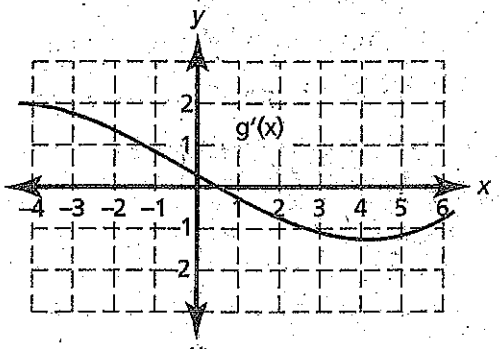


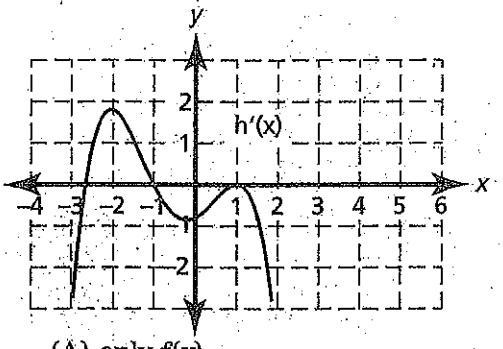
40. The graphs of $f(x)$ and $g(x)$ are shown above. If $h(x) = \frac{g(2x)}{f(x)}$, use the graphs to find $h'(1)$.
(A) $-\frac{7}{4}$
(B) $-\frac{9}{16}$
(C) $-\frac{7}{16}$
(D) $-\frac{5}{16}$
(E) $-\frac{3}{16}$

41. The number of home fires each day in a certain city increases as the temperature drops. The rate of home fires is modeled by $F(t) = 4\cos\left(\frac{t}{58} - 2\right) + 6$, for $0 \le t \le 365$ days, where midnight on January 1st corresponds to $t = 0$. Which of the following is *closest* to the approximate number of fires in the first quarter of the year?
(A) 910
(B) 660
(C) 540
(D) 330
(E) 240

42. The graphs of the derivatives of three functions, f, g, and h, are given below. Which of the functions has at least one point of inflection in the open interval $-3 < x < 2$?







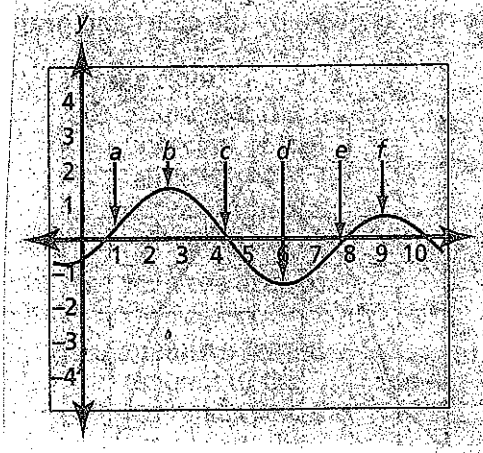
(A) only $f(x)$
(B) only $g(x)$
(C) only $h(x)$
(D) only $f(x)$ and $g(x)$
(E) only $g(x)$ and $h(x)$

x	f	g	f'	g'
1	3	4	$\frac{2}{3}$	$-\frac{5}{2}$
2	4	2	$\frac{4}{3}$	$-\frac{3}{2}$
4	8	1	$\frac{8}{3}$	$\frac{1}{2}$

43. If $f(x)$ and $g(x)$ are differentiable functions with values as given in the chart above, and $k(x) = f\left(g(x^2)\right)$, what is $k'(2)$?

(A) $\frac{1}{3}$
(B) $\frac{2}{3}$
(C) $\frac{4}{3}$
(D) $\frac{16}{3}$
(E) none of these

44. The price of a newly issued stock varies sinusoidally during the first 10 days after its initial offering and is modeled by $P(t) = \log(2t+1)\sin t + 20$, where t is in days. To the nearest cent, what is the price of the stock when the price of the stock is decreasing most rapidly in the interval $0 \le t \le 10$?
(A) \$7.98
(B) \$9.49
(C) \$19.91
(D) \$20.12
(E) \$21.22



45. The graph of $g'(x)$ is shown on the graph to the left. For which of the stated interval(s) is the function $g(x)$ both increasing and concave up?
(A) $a < x < b$
(B) $e < x < f$
(C) $a < x < b$ and $e < x < f$
(D) $a < x < c$ and $e < x < h$
(E) $a < x < b$ and $d < x < f$