Orange Book - Calculus BC Exam I #26

4/15/09

$\boxed{D}$ 1. $\int_0^1 \sqrt{x}\,(x^2+1)\,dx$

$\int_0^1 x^{5/2} + x^{1/2}\,dx$ * 10-15

$\left.\frac{2x^{7/2}}{7} + \frac{2x^{3/2}}{3}\right]_0^1$

$\frac{2}{7} + \frac{2}{3}$

$\frac{6}{21} + \frac{14}{21}$

$\boxed{\frac{20}{21}}$

$\boxed{A}$ 2. $x = e^{4t}$ $y = \sin 6t$

$\frac{dy}{dt} = \cos 6t\,(6)$ $\frac{dy}{dx} = \frac{6\cos(6t)}{4e^{4t}} = \frac{3\cos(6t)}{2e^{4t}}$

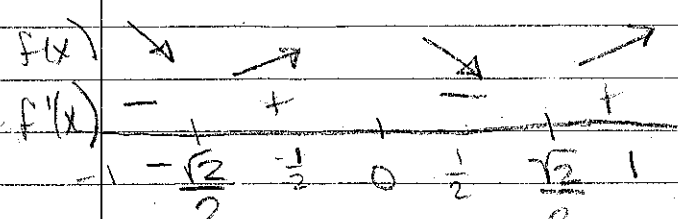
$\frac{dx}{dt} = e^{4t}(4)$

3. $f(x) = x^4 - x^2$

$\boxed{C}$ $f'(x) = 4x^3 - 2x$ $-4+2$

$0 = 2x(2x^2-1)$ $4(-\frac{1}{8}) - 2(-\frac{1}{2})$ $\frac{1}{2}$ 1

$0 = x$ $x = \pm\frac{1}{\sqrt{2}}$ $-\frac{1}{2}+1$ $4 \cdot 2$



4.B 5A

$\boxed{E}$ 6. $y = -\sqrt{x+4}$

$y' = -\frac{1}{2}(x+4)^{-1/2}$

$y' = \dfrac{-1}{2\sqrt{x+4}}$

$y' @ x=0 = -\frac{1}{4}$

$\boxed{E}$ 7. $2xy^2 + 3\ln y = x^2 - 3y^3$

$2x \cdot 2y\frac{dy}{dx} + y^2 \cdot 2 + 3\left(\frac{1}{y}\right)\frac{dy}{dx} = 2x - 9y^2\frac{dy}{dx}$

$4xy\frac{dy}{dx} + 2y^2 + \frac{3}{y}\frac{dy}{dx} = 2x - 9y^2\frac{dy}{dx}$

$4(3)(1)\frac{dy}{dx} + 2(1)^2 + \frac{3}{1}\frac{dy}{dx} = 6 - 9\frac{dy}{dx}$

$12\frac{dy}{dx} + 2 + 3\frac{dy}{dx} = 6 - 9\frac{dy}{dx}$

$24\frac{dy}{dx} = 4$

$\frac{dy}{dx} = \frac{1}{6}$

$\boxed{D}$ 8. $\displaystyle\int_1^\infty \frac{x^2}{(1+x^3)^2}\,dx$

$\displaystyle\lim_{a\to\infty}\int_1^a \frac{x^2}{(1+x^3)^2}\,dx$ $\quad u = 1+x^3$

$du = 3x^2\,dx$

$\frac{1}{3}du = x^2\,dx$

$\displaystyle\lim_{a\to\infty}\frac{1}{3}\int_1^a \frac{1}{u^2}\,du$ $\qquad \displaystyle\lim_{a\to\infty} \left.\frac{-1}{3(1+x^3)}\right]_1^a$

$\displaystyle\lim_{a\to\infty} -\frac{1}{3}\left.\frac{1}{1+x^3}\right]_1^a$ $\qquad \displaystyle\lim_{a\to\infty} \cancel{\frac{-1}{3(1+a^3)}} + \frac{1}{6} = \frac{1}{6}$

$\boxed{B}$ 9. $h'(x) = 0 \Rightarrow$ max or min on $h(x)$

$h(x)$: ↘ ↗ ↗

$h'(x)$: $-$ | $+$ | $+$

0 1 3 7

min

(A)

$\boxed{B}$ ★10. $h'(x)$ @ $x=5$ is 1 (from graph)

$\boxed{B}$ 11.

$h'(x)$: ↗ ↘ ↗ ↘ ↗

$h''(x)$: $+$ | $-$ | $+$ | $-$ | $+$

-1 2 3 4 5 7

✓

12. $\int = \dfrac{\frac{8}{25}}{1 + 3/5}$

$\boxed{A}$

$\int = \dfrac{\cancel{8}\,1}{\cancel{25}\,5} \cdot \dfrac{\cancel{5}}{8} = \dfrac{1}{5}$

$\boxed{D}$ 13. $a(t) = 2t - 3$ $t=0$ $v = -4$

$v(t) = \dfrac{2t^2}{2} - 3t + C$ furthest left = absolute min @ $t=4$

$v(t) = t^2 - 3t + C$

$-4 = C$

$v(t) = t^2 - 3t - 4$

$0 = (t-4)(t+1)$

$t = 4$ $t = -1$ → not in interval

$s(t)$: ↘ ↗

$v(t)$: $-$ | $+$

0 4 5

$f(x)$	$\nearrow$	max	$\searrow$	min	$\nearrow$	max	$\searrow$		$\searrow$	
$f'(x)$ 14.	+		−		+		−		−	
	a		d		e	f		c		b

$\boxed{E}$

$\boxed{A}$ 15. $\int_0^{\pi/2} \sqrt{(2\cos t)^2 + (-3\sin t)^2}\, dt$

$\int_0^{\pi/2} \sqrt{4\cos^2 t + 9\sin^2 t}\quad dt$

$\int_0^{\pi/2} \sqrt{4(1-\sin^2 t) + 9\sin^2 t}\ dt$

$\int_0^{\pi/2} \sqrt{4 + 5\sin^2 t}\ dt$

$\boxed{D}$ 16. $\lim_{x \to 3} \frac{e^{x^2} - e^9}{x-3} = \frac{0}{0}$

$\lim_{x \to 3} \frac{2x \cdot e^{x^2}}{1} = 6e^9$

$\boxed{E}$ 17. $f(x) = \ln(x+4)$

$f'(x) = \frac{1}{x+4}$

$f''(x) = \frac{-1}{(x+4)^2}$ $\quad -(x+4)^{-2}$

$f'''(x) = \frac{2}{(x+4)^3}$ $\quad 2(x+4)^{-3}$

$T_3 = \ln(-3+4) + \frac{1}{-3+4}(x+3) + \frac{-1}{(-3+4)^2}\frac{(x+3)^2}{2!} + \frac{2}{(-3+4)^3}\frac{(x+3)^3}{3!}$

$T_3 = 0 + (x+3) - \frac{(x+3)^2}{2} + \frac{(x+3)^3}{3}$

18. $x = \frac{4}{3}t^3 - t^2 \qquad y = t^5 + t^2 - 7t$

$\boxed{B}$

$\frac{dx}{dt} = 4t^2 - 2t \qquad \frac{dy}{dt} = 5t^4 + 2t - 7$

$\frac{dy}{dx} = \frac{5t^4 + 2t - 7}{4t^2 - 2t}$

undefined where $4t^2 - 2t = 0$

$2t(2t-1) = 0$

$t = 0 \quad t = \frac{1}{2}$

19. $\int_3^{-1} f(x)\,dx = -\int_{-1}^{3} f(x)\,dx + 3\int_{-1}^{2} f(x)$

$\boxed{A}$ $-A + B \qquad -A + B$

20. $\sum_{n=1}^{\infty} \frac{(2x-1)^n}{n \cdot 4^n} \qquad \lim_{n\to\infty} \left| \frac{(2x-1)^{n+1}}{(n+1)4^{n+1}} \cdot \frac{n \cdot 4^n}{(2x-1)^n} \right|$

$\boxed{B}$

$\lim_{n\to\infty} \left| \frac{2x-1\,(n)}{(n+1)4} \right|$

$\left| \frac{2x-1}{4} \right| \leq 1$

$|2x-1| \leq 4$

$2x-1 \leq 4 \qquad 2x-1 \geq -4$

$2x \leq 5 \qquad 2x \geq -3$

$x \leq \frac{5}{2} \qquad x \geq -\frac{3}{2}$

$-\frac{\pi}{4}, \frac{\pi}{4}$ $\quad \frac{1}{2}\int_{-\frac{\pi}{4}}^{\pi/4}$

21. $r = 3\cos 2\theta$ $\qquad \cos 2\theta = 0$

$2\theta = \pi/2, \; 3\pi/2$

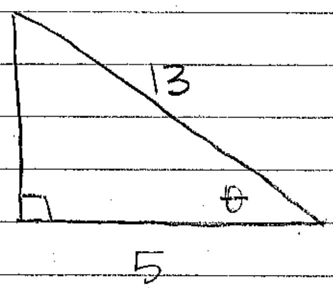
$\boxed{E}$ $\qquad \int_0^{\pi/4} (3\cos 2\theta)^2 \, d\theta$ $\qquad \theta = \pi/4, \; 3\pi/4$

h

22. $\underset{a}{|}\; - \;\underset{c}{|}\; + \;\underset{e}{|}$ $\quad f = h'(x)$

$\boxed{C}$

23.



$\boxed{D}$ $\quad 12 \quad x$

$\sin t = \frac{x}{13}$

$\cos\theta \frac{d\theta}{dt} = \frac{1}{13}\frac{dx}{dt}$

$\cos\theta\,(2) = \frac{1}{13}\frac{dx}{dt}$

169
-144
25

$\frac{5}{13}(2) = \frac{1}{13}\frac{dx}{dt}$

$10 = \frac{dx}{dt}$

$\boxed{A}$ $\quad$ 24. $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$

$h'(x) = \cos x^3$

$h'(x) = 1 - \frac{(x^3)^2}{2!} + \frac{(x^3)^4}{4!} - \frac{(x^3)^6}{6!} + \ldots$

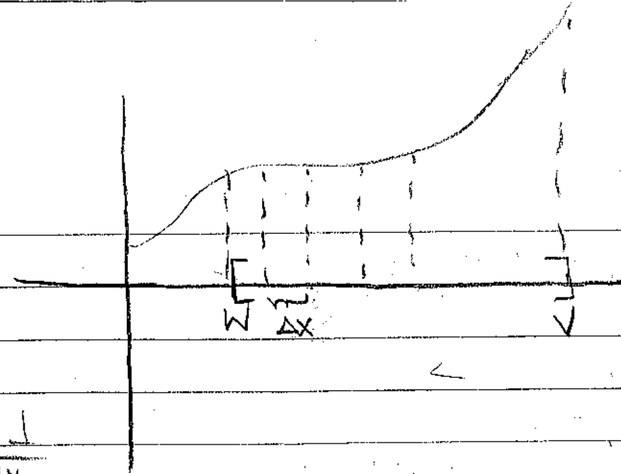
$\int h'(x) = \int 1 - \frac{x^6}{2!} + \frac{x^{12}}{4!} - \frac{x^{18}}{6!} + \ldots$

$h(x) = x - \frac{x^7}{2!\cdot 7} + \frac{x^{13}}{4!\cdot 13}$

$\downarrow$

$-\frac{1}{14}$

25. $\lim_{k \to \infty} \sum_{j=1}^{k} \frac{1}{\sqrt{x_j}} \Delta x$



$\boxed{B}$

$\frac{1}{\sqrt{x_1}} + \frac{1}{\sqrt{x_2}} + \frac{1}{\sqrt{x_3}}$

$\int_w^v \frac{1}{\sqrt{x}}\, dx$

$\int_w^v x^{-1/2}\, dx$

$2x^{1/2} \Big]_w^v$

$2\sqrt{v} - 2\sqrt{w}$

26. $\int \frac{6x-8}{(x-3)(x+2)}\, dx$

$\boxed{A}$

$\frac{6x-8}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2}$

$6x-8 = A(x+2) + B(x-3)$

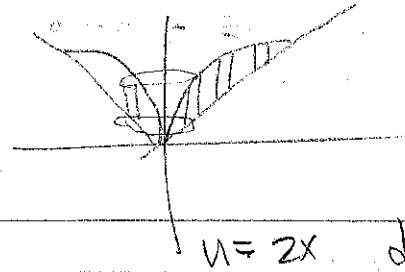
$x=-2 \qquad x=3$

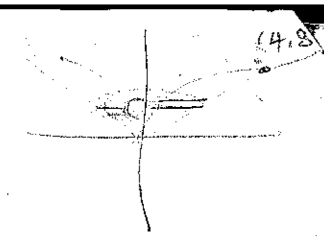
$-20 = -5B \qquad 10 = 5A$

$4 = B \qquad 2 = A$

$\int \frac{2}{x-3} + \frac{4}{x+2}\, dx$

$2\ln|x-3| + 4\ln|x+2| + C$





$\boxed{E}$ ★ 27. $\int 2x\cos x\,dx$ $u = 2x \quad dv = \cos x$

$du = 2dx \quad v = \sin x$

$2x\sin x - \int 2\sin x\,dx$

$2x\sin x + 2\cos x + C$

$\boxed{C}$ 28. C - vertical tangents at $y=0$

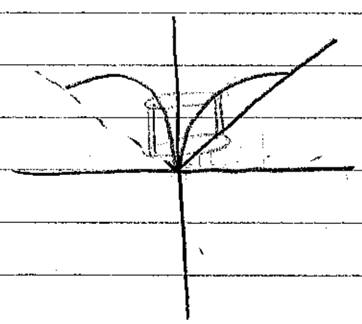
$\boxed{B}$ 29. n^n FEPL

I. diverges

II. converges to $\frac{3}{8}$

III. diverges $\frac{e^{2n}}{e^n} = e^n$

$\boxed{A}$ 30.



$2\pi \int_0^4 x(6x - x^2 - 2x)\,dx$

$2\pi \int_0^4 (4x^2 - x^3)\,dx$

$\pi \int_0^4 (8x^2 - 2x^3)\,dx$

$\boxed{E}$ 31. $f(x+h) = \ln\left(\frac{1}{e} + h\right)$

$= \ln e^{-1}$

$- \ln e$

$\lim_{x \to h} \frac{f(x+h) - f(x)}{h}$

$\frac{\ln(x+h) - \ln x}{h}$

$\frac{\ln\left(\frac{1}{e} + h\right) - \ln\frac{1}{e}}{h}$ $+1$

32. D 33. B 34. C 35. B 36. A 37. D 38. D 39. A
40. C 41. C 42. B 43. E 44. E 45. D

$\boxed{D}$ 32. $x(t) = 2\sin 4t - \cos 4t$

$v(t) = 2\cos 4t(4) - -\sin 4t(4)$
$v(t) = 8\cos 4t + 4\sin 4t$
$0 = 8\cos 4t + 4\sin 4t$
$-4\sin 4t = 8\cos 4t$
$\sin 4t = -2\cos 4t$

graph $y_1 = y_2 \Longrightarrow$ 6 times

$\boxed{B}$ 33. $h(x) = \cos 3x + \ln 4x$
$h'(x) = -\sin 3x(3) + \frac{1}{4x} \cdot 4$
$h'(x) = -3\sin 3x + \frac{1}{x}$
$h''(x) = -3\cos(3x)(3) - \frac{1}{x^2}$
$h''(x) = -9\cos(3x) - \frac{1}{x^2}$
$-9\cos(3x) - \frac{1}{x^2} = 0$ Graph $\Longrightarrow$ zero is .621
$-9\cos(3x) = \frac{1}{x^2}$

$\boxed{C}$ 34. $(-2,3)$ $(5,-7)$

$\boxed{B}$ 35. $0 \le x \le 6$ $\int_0^x (t^2-3t)\,dt \le \int_2^x t\,dt$

$$\frac{t^3}{3}-\frac{3t^2}{2}\Big]_0^x \le \frac{t^2}{2}\Big]_2^x$$

$$\frac{x^3}{3}-\frac{3x^2}{2} \le \frac{x^2}{2}-\frac{4}{2}$$

$$\frac{x^3}{3}-\frac{3x^2}{2} \le \frac{x^2}{2}-2$$

$$2x^3-9x^2 \le 3x^2-12$$

$$2x^3-12x^2+12 \le 0$$

graph

* 36. $\frac{dy}{dx} = (\ln x+2)y$ $y=2$ when $x=1$ $y=$

$$\int \frac{1}{y}\,dy = \int (\ln x+2)\,dx$$

correct answer not given

$$\ln|y| = \int \ln x\,dx + \int 2\,dx$$

$$\ln|y| = x\ln x - x + 2x + C$$

$$\ln|y| = x\ln x + x + C$$

e e

$$y = Ce^{x\ln x+x}$$

$$2 = Ce^{1\ln 1+1}$$

$$\frac{2}{e} = C$$

$$y = \frac{2}{e}e^{x(\ln x+1)}$$

$\int \ln x\,dx$

$u=\ln x$ $dv=dx$

$du=\frac{1}{x}dx$ $v=x$

$x\ln x - \int x\cdot\frac{1}{x}\,dx$

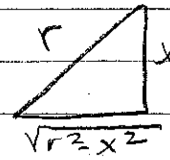
$x\ln x - \int dx$

$x\ln x - x$

$$\boxed{y = 2e^{x(\ln x+1)-1}}$$

correct!

$\boxed{D}$ 37. $\sin\theta = \dfrac{x}{r}$



$x = r\sin\theta \quad \sqrt{r^2-x^2} = \sqrt{r^2-r^2\sin^2\theta}$

$A = \frac{1}{2}\left(r + r + 2\sqrt{r^2-r^2\sin^2\theta}\right)(r\sin\theta)$

$A = \frac{1}{2} r\sin\theta\left(2r + 2r\sqrt{1-\sin^2\theta}\right)$ $(\cos^2\theta)$

$A = \frac{1}{2} r\sin\theta(2r + 2r\cos\theta)$

$A = r^2\sin\theta(1+\cos\theta)$

$A' = r^2\sin\theta(-\sin\theta) + (1+\cos\theta)(r^2\cos\theta)$

$= -r^2\sin^2\theta + r^2\cos\theta + r^2\cos^2\theta$

$0 = r^2(-\sin^2\theta + \cos\theta + \cos^2\theta)$

calculator $\Rightarrow 60°$

$\boxed{D}$ 38. $f(2)=8 \quad f(4)=5$

$m = \dfrac{5-8}{4-2} = -\dfrac{3}{2}$

Avg value $f' = \dfrac{1}{4-2}\int_2^4 f'(x)\,dx = \dfrac{1}{2}\,F(4)-F(2) = \dfrac{1}{2}(5-8)$

$= -\dfrac{3}{2}$

$\boxed{A}$ 39. $f(x) = \dfrac{2}{1+x^2} \quad [-1,2]$

$f'(c) = \dfrac{f(b)-f(a)}{b-a} \qquad \dfrac{-2(2x)}{(1+x^2)^2} = \dfrac{\frac{2}{5}-1}{2+1} = \dfrac{-\frac{3}{5}}{3} = -\dfrac{3}{15}$

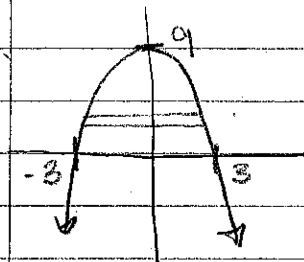
$$\frac{-4x}{(1+x^2)^2} = \frac{-3}{15}$$

$$-60x = -3(1+x^2)^2$$

$$20x = (1+x^2)^2$$

graph $x = .050$

$\boxed{C}$ 40.



$$y = 9 - x^2$$

$$y - 9 = -x^2$$

$$9 - y = x^2$$

$$\pm\sqrt{9-y} = x$$

$$\int_0^9 \left(\sqrt{9-y}\right)\cdot 3\sqrt{9-y}\,dy$$

$$\int_0^9 3(9-y)\,dy$$

41. $F(x) = \int_0^{2x^2} \cos t \, dt \quad [0, \sqrt{\pi}]$

$\boxed{C}$

$4x\cos(2x^2) \qquad \sin t\Big]_0^{2x^2}$

$4x\cos(2x^2) \qquad \sin 2x^2$

$$4x\cos(2x^2) = \frac{\sin 2\pi - \sin 0}{\sqrt{\pi}}$$

$$4x\cos(2x^2) = 0$$

Graph $\Rightarrow$ calculator = 3 times

$\boxed{B}$ 42. $g(x) = \frac{x^3}{1+x^5}$ $h(1)=2$ $h(3)=$

$$\int_1^3 \frac{x^3}{1+x^5}\,dx = .555$$

$h(3) = 2 + .555$

OMIT 43.

4

$\boxed{E}$ 44. $S = \int_0^{2\pi} \sqrt{(1-2\cos 2\theta)^2 + (4\sin 2\theta)^2}\,d\theta$

$\boxed{D}$ 45. $\frac{dy}{dx} = x+y$ $f(0)=2$

$dy = (x+y)\,dx$

$dy = 0 + 2(.5)$

$dy = 1$

$(.5, 3)$

$dy = (.5+3)(.5)$

$dy = 1.75$

$(1, 4.75)$

4