Alternating Series

p.595 #9-36 mult. of 3 $\boxed{\text{41-49 odd}}$

9. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ $\lim_{n\to\infty} \frac{1}{n} = 0$ ✓ $a_{n+1} \le a_n$ ✓

convergent

12. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$ $\lim_{n\to\infty} \frac{1}{\ln(n+1)} = 0$ $a_{n+1} \le a_n$ ✓

convergent

15. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$ ✓ $a_{n+1} \le a_n$ ✓

convergent

18. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\ln(n+1)}{n+1}$ $\lim_{n\to\infty} \frac{\ln(n+1)}{n+1}$ $\frac{\infty}{\infty}$ $a_{n+1} \le a_n$ ✓

$\frac{\ln(n+2)}{n+2} \le \frac{\ln(n+1)}{n+1}$

$\lim_{n\to\infty} \frac{1}{n+1} = 0$ ✓

21. $\sum_{n=1}^{\infty} \cos n\pi$ diverges by n^{th} term test

24. $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}$ $\lim_{n\to\infty} \frac{1}{(2n+1)!} = 0$ ✓ $a_{n+1} \le a_n$ ✓

27. $\sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{e^n - e^{-n}}$ $\lim_{n\to\infty} \frac{2}{e^n - \frac{1}{e^n}}$ $a_{n+1} \le a_n$ ✓

Converges $\frac{2}{\frac{e^{2n}-1}{e^n}}$

$\lim_{n\to\infty} \frac{2e^n}{e^{2n}-1} = \lim_{n\to\infty} \frac{2e^n}{2e^{2n}} = \lim_{n\to\infty} \frac{1}{e^n} = 0$

30. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}4}{\ln(n+1)}$ $\lim_{n\to\infty} \frac{4}{\ln(n+1)} = 0$ $a_{n+1} \le a_n$ ✓

convergent

$\frac{4}{\ln 2} - \frac{4}{\ln 3} + \frac{4}{\ln 4} - \frac{4}{\ln 5} + \frac{4}{\ln 6} - \frac{4}{\ln 7}$

$5.771 - 3.641 + 2.885 - 2.485 + 2.232 - 2.056 = 2.706$

$S_6 = 2.706$

$|S - S_6| \le a_7$

$|S - 2.706| \le \frac{4}{\ln 8}$

$|S - 2.706| \le 1.9236$

$S - 2.706 \le 1.9236$ and $S - 2.706 \ge -1.9236$

$S \le 4.6296$ $S \ge .7824$

$\boxed{.7824 \le S \le 4.6296}$

33. error must be $\le a_{n+1}$

$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$ $\frac{1}{(n+1)!} \le .001$

$1, \frac{1}{2!}, \frac{1}{3!}, \frac{1}{4!}, \frac{1}{5!}, \frac{1}{6!}, \frac{1}{7!}$

$1/2 \quad 1/6 \quad 1/24 \quad 1/120 \quad 1/720 \quad .0001$

$\boxed{n=6}$ $1 - 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120} + \frac{1}{720} = \boxed{.368 \approx \frac{1}{e}}$

36. $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} = \cos 1$ $\boxed{\frac{-1^0}{0!} = \frac{1}{1}}$

$\frac{1}{(2n+2)!} \le .001$

$n=0$ $n=1$ $n=2$ $n=3$

$\frac{1}{2!}, \frac{1}{4!}, \frac{1}{6!}, \frac{1}{8!}, \frac{1}{10!}$

$\frac{1}{720}$ $.00002$

$\boxed{n=3}$

$1 - \frac{1}{2} + \frac{1}{24} - \frac{1}{720} = \boxed{.540 \approx \cos 1}$

41. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+1)^2}$ $\frac{1}{4} - \frac{1}{9} + \frac{1}{16} -$

$\lim_{n\to\infty} \frac{1}{(n+1)^2} = 0$ $a_{n+1} \le a_n$ converges

$\sum_{n=1}^{\infty} \left|\frac{(-1)^{n+1}}{(n+1)^2}\right| = \sum_{n=1}^{\infty} \frac{1}{(n+1)^2}$ $u = n+1$

$\lim_{a\to\infty} \int_1^a u^{-2}\,du$

$\lim_{a\to\infty} \left.\frac{-1}{(n+1)}\right]_1^a$

$\lim_{a\to\infty} \frac{-1}{a+1} + \frac{1}{2}$

$\frac{1}{2}$

absolutely convergent

43. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ alternating ✓ $\frac{1}{1} - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{2} + \frac{1}{\sqrt{5}}$

$\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0 \qquad a_{n+1} \leq a_n$

converges

$\sum_{n=1}^{\infty} \left| \frac{(-1)^{n+1}}{\sqrt{n}} \right| = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ divergent p-series

$\boxed{\text{conditionally convergent}}$

45. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^2}{(n+1)^2}$

diverges by n^{th} term test

47. $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

$\lim_{n\to\infty} \frac{1}{\ln n} = 0$ ✓ $\qquad a_{n+1} \leq a_n$ ✓

converges

$\sum_{n=2}^{\infty} \left| \frac{(-1)^n}{\ln n} \right| = \sum_{n=2}^{\infty} \frac{1}{\ln n}$ compare to $\sum_{n=2}^{\infty} \frac{1}{n}$

$\frac{1}{\ln 2}, \frac{1}{\ln 3}, \frac{1}{\ln 4} \qquad 1, \frac{1}{2}, \frac{1}{3}$

1.44 .910 .721

$\frac{1}{\ln n} > \frac{1}{n}$ divergent harmonic

$\therefore \frac{1}{\ln n}$ diverges

$\boxed{\text{conditionally convergent}}$

49. $\sum_{n=2}^{\infty} \frac{(-1)^n n}{n^3-1}$ $\lim_{n\to\infty} \frac{n}{n^3-1} = 0$ $a_{n+1} \leq a_n$

$\sum_{n=2}^{\infty} \frac{n}{n^3-1}$ compare to $\frac{1}{n^2}$

$\lim_{n\to\infty} \frac{\frac{n}{n^3-1}}{\frac{1}{n^2}}$ $\boxed{\text{limit comparison}}$

$\lim_{n\to\infty} \frac{n}{n^3-1} \cdot \frac{n^2}{1} = 1$

$\boxed{\text{absolutely convergent}}$