CALCULUS AB
SECTION I, Part A
Time—55 minutes
Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

1. $\lim_{x\to\infty} \dfrac{(2x-1)(3-x)}{(x-1)(x+3)}$ is

(A) -3 (B) -2 (C) 2 (D) 3 (E) nonexistent

2. $\int \dfrac{1}{x^2}\,dx =$

(A) $\ln x^2 + C$ (B) $-\ln x^2 + C$ (C) $x^{-1} + C$ (D) $-x^{-1} + C$ (E) $-2x^{-3} + C$

3. If $f(x) = (x-1)\left(x^2+2\right)^3$, then $f'(x) =$

(A) $6x\left(x^2+2\right)^2$

(B) $6x(x-1)\left(x^2+2\right)^2$

(C) $\left(x^2+2\right)^2\left(x^2+3x-1\right)$

(D) $\left(x^2+2\right)^2\left(7x^2-6x+2\right)$

(E) $-3(x-1)\left(x^2+2\right)^2$

4. $\int (\sin(2x) + \cos(2x))\,dx =$

(A) $\frac{1}{2}\cos(2x) + \frac{1}{2}\sin(2x) + C$

(B) $-\frac{1}{2}\cos(2x) + \frac{1}{2}\sin(2x) + C$

(C) $2\cos(2x) + 2\sin(2x) + C$

(D) $2\cos(2x) - 2\sin(2x) + C$

(E) $-2\cos(2x) + 2\sin(2x) + C$

5. $\lim_{x\to 0} \dfrac{5x^4 + 8x^2}{3x^4 - 16x^2}$ is

(A) $-\frac{1}{2}$ (B) 0 (C) 1 (D) $\frac{5}{3}$ (E) nonexistent

6. Let f be the function defined above. Which of the following statements about f are true?

$$f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$$

I. f has a limit at $x = 2$.
II. f is continuous at $x = 2$.
III. f is differentiable at $x = 2$.

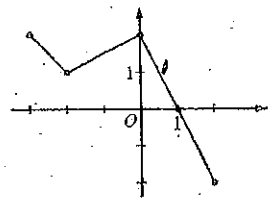
(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I, II, and III

7. A particle moves along the x-axis with velocity given by $v(t) = 3t^2 + 6t$ for time $t \geq 0$. If the particle is at position $x = 2$ at time $t = 0$, what is the position of the particle at time $t = 1$?

(A) 4 (B) 6 (C) 9 (D) 11 (E) 12

8. If $f(x) = \cos(3x)$, then $f'\left(\frac{\pi}{9}\right) =$

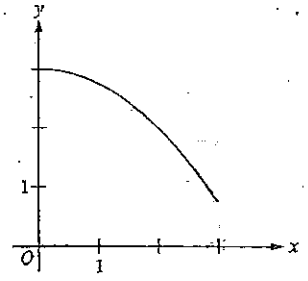
(A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $-\frac{\sqrt{3}}{2}$ (D) $-\frac{3}{2}$ (E) $-\frac{3\sqrt{3}}{2}$



Graph of f

9. The graph of the piecewise linear function f is shown in the figure above. If $g(x) = \int_{-2}^{x} f(t)\,dt$, which of the following values is greatest?

(A) $g(-3)$ (B) $g(-2)$ (C) $g(0)$ (D) $g(1)$ (E) $g(2)$



Graph of f

10. The graph of the function f is shown above for $0 \leq x \leq 3$. Of the following, which has the least value?

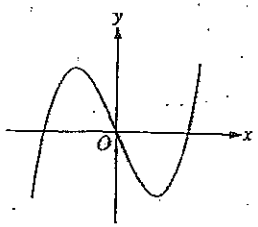
(A) $\int_1^3 f(x)\,dx$

(B) Left Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

(C) Right Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

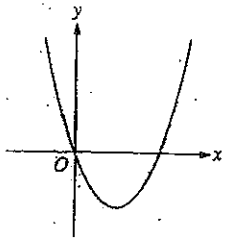
(D) Midpoint Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

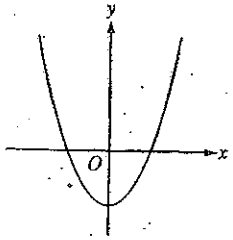
(E) Trapezoidal sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length



Graph of f

11. The graph of a function f is shown above. Which of the following could be the graph of f', the derivative of f?

(A)


(B)


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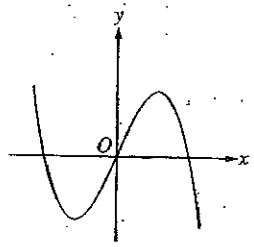

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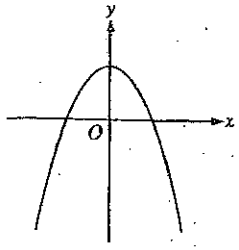
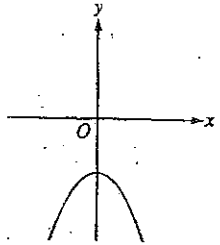


(E)


12. If $f(x) = e^{(2/x)}$, then $f'(x) =$

(A) $2e^{(2/x)}\ln x$ (B) $e^{(2/x)}$ (C) $e^{(-2/x^2)}$ (D) $-\frac{2}{x^2}e^{(2/x)}$ (E) $-2x^2e^{(2/x)}$

13. If $f(x) = x^2 + 2x$, then $\frac{d}{dx}\{f(\ln x)\} =$

(A) $\frac{2\ln x + 2}{x}$ (B) $2x\ln x + 2x$ (C) $2\ln x + 2$ (D) $2\ln x + \frac{2}{x}$ (E) $\frac{2x+2}{x}$

x	0	1	2	3
$f''(x)$	5	0	−7	4

14. The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true?

(A) f is increasing on the interval $(0, 2)$.

(B) f is decreasing on the interval $(0, 2)$.

(C) f has a local maximum at $x = 1$.

(D) The graph of f has a point of inflection at $x = 1$.

(E) The graph of f changes concavity in the interval $(0, 2)$.

15. $\int \frac{x}{x^2-4}\,dx =$

(A) $\frac{-1}{4(x^2-4)^2} + C$

(B) $\frac{1}{2(x^2-4)} + C$

(C) $\frac{1}{2}\ln|x^2-4| + C$

(D) $2\ln|x^2-4| + C$

(E) $\frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$

16. If $\sin(xy) = x$, then $\frac{dy}{dx} =$

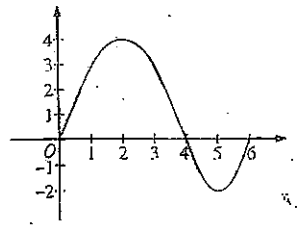
(A) $\frac{1}{\cos(xy)}$

(B) $\frac{1}{x\cos(xy)}$

(C) $\frac{1-\cos(xy)}{\cos(xy)}$

(D) $\frac{1-y\cos(xy)}{x\cos(xy)}$

(E) $\frac{y(1-\cos(xy))}{x}$



Graph of f

17. The graph of the function f shown above has horizontal tangents at $x = 2$ and $x = 5$. Let g be the functic defined by $g(x) = \int_0^x f(t)\,dt$. For what values of x does the graph of g have a point of inflection?

(A) 2 only (B) 4 only (C) 2 and 5 only (D) 2, 4, and 5 (E) 0, 4, and 6

18. In the xy-plane, the line $x + y = k$, where k is a constant, is tangent to the graph of $y = x^2 + 3x + 1$. Wh the value of k?

(A) −3 (B) −2 (C) −1 (D) 0 (E) 1

19. What are all horizontal asymptotes of the graph of $y = \frac{5+2^x}{1-2^x}$ in the xy-plane?

(A) $y = -1$ only

(B) $y = 0$ only

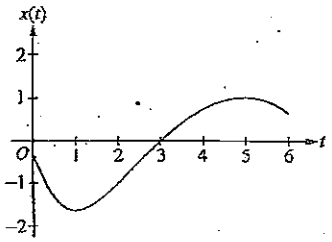
(C) $y = 5$ only

(D) $y = -1$ and $y = 0$

(E) $y = -1$ and $y = 5$

20. Let f be a function with a second derivative given by $f''(x) = x^2(x-3)(x-6)$. What are the x-coordinates of the points of inflection of the graph of f?

(A) 0 only (B) 3 only (C) 0 and 6 only (D) 3 and 6 only (E) 0, 3, and 6



21. A particle moves along a straight line. The graph of the particle's position $x(t)$ at time t is shown above for $0 < t < 6$. The graph has horizontal tangents at $t = 1$ and $t = 5$ and a point of inflection at $t = 2$. For what values of t is the velocity of the particle increasing?

(A) $0 < t < 2$

(B) $1 < t < 5$

(C) $2 < t < 6$

(D) $3 < t < 5$ only

(E) $1 < t < 2$ and $5 < t < 6$

22. A rumor spreads among a population of N people at a rate proportional to the product of the number of people who have heard the rumor and the number of people who have not heard the rumor. If p denotes the number of people who have heard the rumor, which of the following differential equations could be used to model this situation with respect to time t, where k is a positive constant?

(A) $\frac{dp}{dt} = kp$

(B) $\frac{dp}{dt} = kp(N-p)$

(C) $\frac{dp}{dt} = kp(p-N)$

(D) $\frac{dp}{dt} = kt(N-t)$

(E) $\frac{dp}{dt} = kt(t-N)$

23. Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{x^2}{y}$ with the initial condition $y(3) = -2$?

(A) $y = 2e^{-9+x^3/3}$

(B) $y = -2e^{-9+x^3/3}$

(C) $y = \sqrt{\frac{2x^3}{3}}$

(D) $y = \sqrt{\frac{2x^3}{3} - 14}$

(E) $y = -\sqrt{\frac{2x^3}{3} - 14}$

24. The function f is twice differentiable with $f(2) = 1$, $f'(2) = 4$, and $f''(2) = 3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of f at $x = 2$?

(A) 0.4 (B) 0.6 (C) 0.7 (D) 1.3 (E) 1.4

$$f(x) = \begin{cases} cx + d & \text{for } x \le 2 \\ x^2 - cx & \text{for } x > 2 \end{cases}$$

25. Let f be the function defined above, where c and d are constants. If f is differentiable at $x = 2$, what is the value of $c + d$?

(A) -4 (B) -2 (C) 0 (D) 2 (E) 4

26. What is the slope of the line tangent to the curve $y = \arctan(4x)$ at the point at which $x = \frac{1}{4}$?

(A) 2 (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) -2

27. Shown above is a slope field for which of the following differential equations?

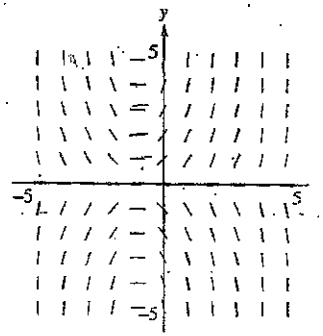
(A) $\frac{dy}{dx} = xy$

(B) $\frac{dy}{dx} = xy - y$

(C) $\frac{dy}{dx} = xy + y$

(D) $\frac{dy}{dx} = xy + x$

(E) $\frac{dy}{dx} = (x+1)^3$

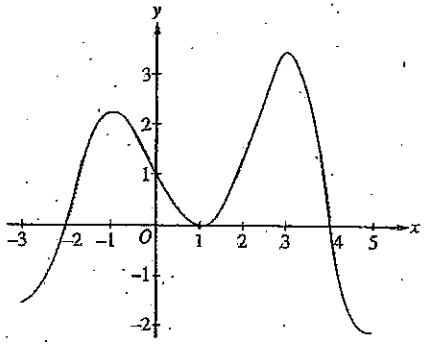


82. A particle moves along a straight line with velocity given by $v(t) = 7 - (1.01)^{-t^2}$ at time $t \geq 0$. What is the acceleration of the particle at time $t = 3$?

(A) −0.914 (B) 0.055 (C) 5.486 (D) 6.086 (E) 18.087

83. What is the area enclosed by the curves $y = x^3 - 8x^2 + 18x - 5$ and $y = x + 5$?

(A) 10.667 (B) 11.833 (C) 14.583 (D) 21.333 (E) 32



Graph of f'

84. The graph of the derivative of a function f is shown in the figure above. The graph has horizontal tangent lines at $x = -1$, $x = 1$, and $x = 3$. At which of the following values of x does f have a relative maximum?

(A) −2 only (B) 1 only (C) 4 only (D) −1 and 3 only (E) −2, 1, and 4

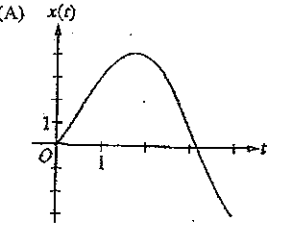
x	−4	−3	−2	−1
$f(x)$	0.75	−1.5	−2.25	−1.5
$f'(x)$	−3	−1.5	0	1.5

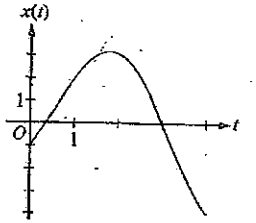
85. The table above gives values of a function f and its derivative at selected values of x. If f' is continuous on the interval $[-4, -1]$, what is the value of $\int_{-4}^{-1} f'(x)\,dx$?

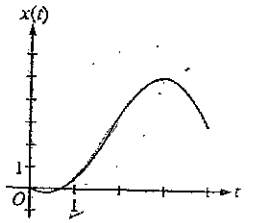
(A) −4.5 (B) −2.25 (C) 0 (D) 2.25 (E) 4.5

t	0	1	2	3	4
$v(t)$	−1	2	3	0	−4

86. The table gives selected values of the velocity, $v(t)$, of a particle moving along the x-axis. At time $t = 0$, the particle is at the origin. Which of the following could be the graph of the position, $x(t)$, of the particle for $0 \leq t \leq 4$?

(A)


(B)


(C)


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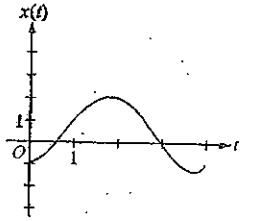

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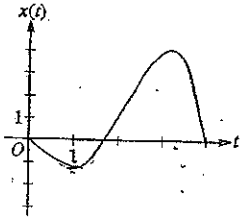


87. An object traveling in a straight line has position $x(t)$ at time t. If the initial position is $x(0) = 2$ and the velocity of the object is $v(t) = \sqrt[3]{1 + t^2}$, what is the position of the object at time $t = 3$?

(A) 0.431 (B) 2.154 (C) 4.512 (D) 6.512 (E) 17.408

88. The radius of a sphere is decreasing at a rate of 2 centimeters per second. At the instant when the radius of the sphere is 3 centimeters, what is the rate of change, in square centimeters per second, of the surface area of the sphere? (The surface area S of a sphere with radius r is $S = 4\pi r^2$.)

(A) -108π (B) -72π (C) -48π (D) -24π (E) -16π

28. Let f be a differentiable function such that $f(3) = 15$, $f(6) = 3$, $f'(3) = -8$, and $f'(6) = -2$. The function g is differentiable and $g(x) = f^{-1}(x)$ for all x. What is the value of $g'(3)$?

(A) $-\frac{1}{2}$

(B) $-\frac{1}{8}$

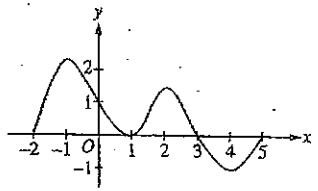
(C) $\frac{1}{6}$

(D) $\frac{1}{3}$

(E) The value of $g'(3)$ cannot be determined from the information given.

CALCULUS AB
SECTION I, Part B
Time—50 minutes
Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.



Graph of f'

76. The graph of f', the derivative of f, is shown above for $-2 \le x \le 5$. On what intervals is f increasing?

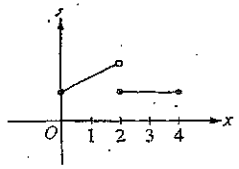
(A) $[-2, 1]$ only

(B) $[-2, 3]$

(C) $[3, 5]$ only

(D) $[0, 1.5]$ and $[3, 5]$

(E) $[-2, -1]$, $[1, 2]$, and $[4, 5]$



Graph of f

77. The figure above shows the graph of a function f with domain $0 \le x \le 4$. Which of the following statements are true?

I. $\lim_{x \to 2^-} f(x)$ exists.

II. $\lim_{x \to 2^+} f(x)$ exists.

III. $\lim_{x \to 2} f(x)$ exists.

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

78. The first derivative of the function f is defined by $f'(x) = \sin\left(x^3 - x\right)$ for $0 \le x \le 2$. On what intervals is f increasing?

(A) $1 \le x \le 1.445$ only

(B) $1 \le x \le 1.691$

(C) $1.445 \le x \le 1.875$

(D) $0.577 \le x \le 1.445$ and $1.875 \le x \le 2$

(E) $0 \le x \le 1$ and $1.691 \le x \le 2$

79. If $\int_{-5}^{2} f(x)\,dx = -17$ and $\int_{5}^{2} f(x)\,dx = -4$, what is the value of $\int_{-5}^{5} f(x)\,dx$?

(A) −21 (B) −13 (C) 0 (D) 13 (E) 21

80. The derivative of the function f is given by $f'(x) = x^2 \cos\left(x^2\right)$. How many points of inflection does the graph of f have on the open interval $(-2, 2)$?

(A) One (B) Two (C) Three (D) Four (E) Five

81. If $G(x)$ is an antiderivative for $f(x)$ and $G(2) = -7$, then $G(4) =$

(A) $f'(4)$

(B) $-7 + f'(4)$

(C) $\int_2^4 f(t)\,dt$

(D) $\int_2^4 \{-7 + f(t)\}\,dt$

(E) $-7 + \int_2^4 f(t)\,dt$

89. The function f is continuous for $-2 \le x \le 2$ and $f(-2) = f(2) = 0$. If there is no c, where $-2 < c < 2$, for which $f'(c) = 0$, which of the following statements must be true?

(A) For $-2 < k < 2$, $f'(k) > 0$.

(B) For $-2 < k < 2$, $f'(k) < 0$.

(C) For $-2 < k < 2$, $f'(k)$ exists.

(D) For $-2 < k < 2$, $f'(k)$ exists, but f' is not continuous.

(E) For some k, where $-2 < k < 2$, $f'(k)$ does not exist.

90. The function f is continuous on the closed interval $[2, 4]$ and twice differentiable on the open interval $(2, 4)$. If $f'(3) = 2$ and $f''(x) < 0$ on the open interval $(2, 4)$, which of the following could be a table of values for f?

(A)

x	$f(x)$
2	2.5
3	5
4	6.5

(B)

x	$f(x)$
2	2.5
3	5
4	7

(C)

x	$f(x)$
2	3
3	5
4	6.5

(D)

x	$f(x)$
2	3
3	5
4	7

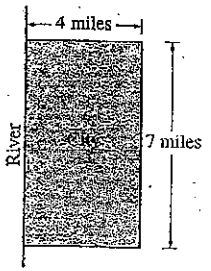
(E)

x	$f(x)$
2	3.5
3	5
4	7.5

91. What is the average value of $y = \dfrac{\cos x}{x^2 + x + 2}$ on the closed interval $[-1, 3]$?

(A) −0.085 (B) 0.090 (C) 0.183 (D) 0.244 (E) 0.732

92. A city located beside a river has a rectangular boundary as shown in the figure above. The population density of the city at any point along a strip x miles from the river's edge is $f(x)$ persons per square mile. Which of the following expressions gives the population of the city?



(A) $\int_0^4 f(x)\,dx$

(B) $7\int_0^4 f(x)\,dx$

(C) $28\int_0^4 f(x)\,dx$

(D) $\int_0^7 f(x)\,dx$

(E) $4\int_0^7 f(x)\,dx$