EXERCISES FOR SECTION 4.4

Graphical Reasoning **In Exercises 1–4, use a graphing utility to graph the integrand. Use the graph to determine whether the definite integral is positive, negative, or zero.**

1. $\displaystyle\int_0^{\pi} \frac{4}{x^2+1}\,dx$
2. $\displaystyle\int_0^{\pi} \cos x\,dx$
3. $\displaystyle\int_{-2}^{2} x\sqrt{x^2+1}\,dx$
4. $\displaystyle\int_{-2}^{2} x\sqrt{2-x}\,dx$

In Exercises 5–26, evaluate the definite integral of the algebraic function. Use a graphing utility to verify your result.

5. $\displaystyle\int_0^1 2x\,dx$
6. $\displaystyle\int_2^7 3\,dv$
7. $\displaystyle\int_{-1}^0 (x-2)\,dx$
8. $\displaystyle\int_2^5 (-3v+4)\,dv$
9. $\displaystyle\int_{-1}^1 (t^2-2)\,dt$
10. $\displaystyle\int_1^3 (3x^2+5x-4)\,dx$
11. $\displaystyle\int_0^1 (2t-1)^2\,dt$
12. $\displaystyle\int_{-1}^1 (t^3-9t)\,dt$
13. $\displaystyle\int_1^2 \left(\frac{3}{x^2}-1\right)dx$
14. $\displaystyle\int_{-2}^{-1} \left(u-\frac{1}{u^2}\right)du$
15. $\displaystyle\int_1^4 \frac{u-2}{\sqrt{u}}\,du$
16. $\displaystyle\int_{-3}^3 v^{1/3}\,dv$
17. $\displaystyle\int_{-1}^1 \left(\sqrt[3]{t}-2\right)dt$
18. $\displaystyle\int_1^8 \sqrt{\frac{2}{x}}\,dx$
19. $\displaystyle\int_0^1 \frac{x-\sqrt{x}}{3}\,dx$
20. $\displaystyle\int_0^2 (2-t)\sqrt{t}\,dt$
21. $\displaystyle\int_{-1}^0 \left(t^{1/3}-t^{2/3}\right)dt$
22. $\displaystyle\int_{-8}^{-1} \frac{x-x^2}{2\sqrt[3]{x}}\,dx$
23. $\displaystyle\int_0^3 |2x-3|\,dx$
24. $\displaystyle\int_1^4 (3-|x-3|)\,dx$
25. $\displaystyle\int_0^3 |x^2-4|\,dx$
26. $\displaystyle\int_0^4 |x^2-4x+3|\,dx$

In Exercises 27–32, evaluate the definite integral of the trigonometric function. Use a graphing utility to verify your result.

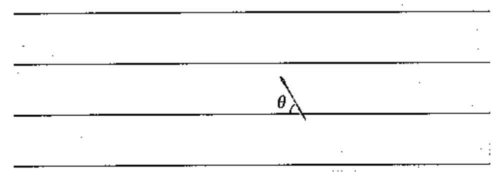
27. $\displaystyle\int_0^{\pi} (1+\sin x)\,dx$
28. $\displaystyle\int_0^{\pi/4} \frac{1-\sin^2\theta}{\cos^2\theta}\,d\theta$
29. $\displaystyle\int_{-\pi/6}^{\pi/6} \sec^2 x\,dx$
30. $\displaystyle\int_{\pi/4}^{\pi/2} (2-\csc^2 x)\,dx$
31. $\displaystyle\int_{-\pi/3}^{\pi/3} 4\sec\theta\tan\theta\,d\theta$
32. $\displaystyle\int_{-\pi/2}^{\pi/2} (2t+\cos t)\,dt$

33. ***Depreciation*** A company purchases a new machine for which the rate of depreciation is $dV/dt = 10{,}000(t-6)$, $0 \le t \le 5$, where V is the value of the machine after t years. Set up and evaluate the definite integral that yields the total loss of value of the machine over the first 3 years.

34. ***Buffon's Needle Experiment*** A horizontal plane is ruled with parallel lines 2 inches apart. If a 2-inch needle is tossed randomly onto the plane, the probability that the needle will touch a line is

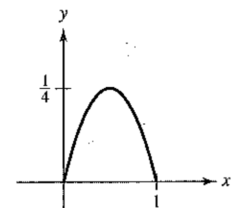
$$P = \frac{2}{\pi}\int_0^{\pi/2} \sin\theta\,d\theta$$

where θ is the acute angle between the needle and any one of the parallel lines. Find this probability.

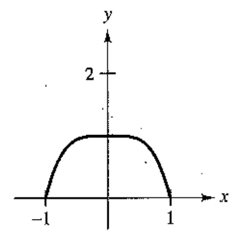


In Exercises 35–40, determine the area of the indicated region.

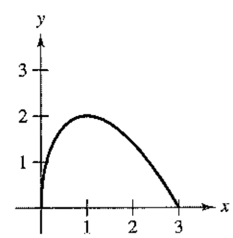
35. $y = x - x^2$



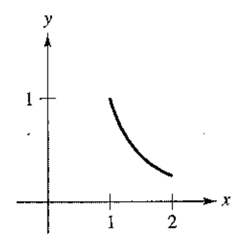
36. $y = 1 - x^4$



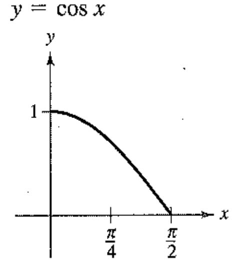
37. $y = (3-x)\sqrt{x}$



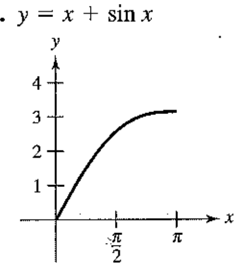
38. $y = \dfrac{1}{x^2}$



39. $y = \cos x$



40. $y = x + \sin x$



In Exercises 41–44, find the area of the region bounded by the graphs of the equations.

41. $y = 3x^2 + 1,\quad x = 0,\quad x = 2,\quad y = 0$
42. $y = 1 + \sqrt[3]{x},\quad x = 0,\quad x = 8,\quad y = 0$
43. $y = x^3 + x,\quad x = 2,\quad y = 0$
44. $y = -x^2 + 3x,\quad y = 0$

In Exercises 45–48, find the value(s) of c guaranteed by the Mean Value Theorem for Integrals for the function over the indicated interval.

Function	*Interval*
45. $f(x) = x - 2\sqrt{x}$	$[0, 2]$
46. $f(x) = \dfrac{9}{x^3}$	$[1, 3]$
47. $f(x) = 2\sec^2 x$	$[-\pi/4, \pi/4]$
48. $f(x) = \cos x$	$[-\pi/3, \pi/3]$

In Exercises 49–52, find the average value of the function over the interval and all values of x in the interval for which the function equals its average value.

Function	*Interval*
49. $f(x) = 4 - x^2$	$[-2, 2]$
50. $f(x) = \dfrac{4(x^2 + 1)}{x^2}$	$[1, 3]$
51. $f(x) = \sin x$	$[0, \pi]$
52. $f(x) = \cos x$	$[0, \pi/2]$

Getting at the Concept

53. State the Fundamental Theorem of Calculus.

54. The graph of f is given in the figure.

(a) Evaluate $\displaystyle\int_1^7 f(x)\,dx$.

(b) Determine the average value of f on the interval $[1, 7]$.

(c) Determine the answers to parts (a) and (b) if the graph is translated two units upward.

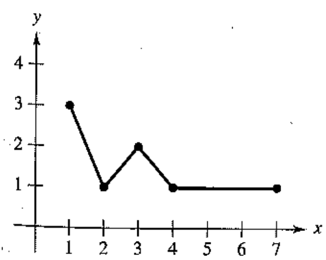


Figure for 54

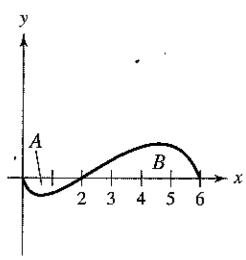


Figure for 55–60

In Exercises 55–60, use the graph of f shown in the figure. The shaded region A has an area of 1.5, and $\int_0^6 f(x)\,dx = 3.5$. Use this information to fill in the blanks.

55. $\displaystyle\int_0^2 f(x)\,dx =$ ______

56. $\displaystyle\int_2^6 f(x)\,dx =$ ______

57. $\displaystyle\int_0^6 |f(x)|\,dx =$ ______

58. $\displaystyle\int_0^2 -2f(x)\,dx =$ ______

59. $\displaystyle\int_0^6 [2 + f(x)]\,dx =$ ______

60. The average value of f over the interval $[0, 6]$ is ______.

61. ***Force*** The force F (in newtons) of a hydraulic cylinder in a press is proportional to the square of $\sec x$, where x is the distance (in meters) that the cylinder is extended in its cycle. The domain of F is $[0, \pi/3]$, and $F(0) = 500$.

(a) Find F as a function of x.

(b) Find the average force exerted by the press over the interval $[0, \pi/3]$.

62. ***Blood Flow*** The velocity v of the flow of blood at a distance r from the central axis of an artery of radius R is

$$v = k(R^2 - r^2)$$

where k is the constant of proportionality. Find the average rate of flow of blood along a radius of the artery. (Use 0 and R as the limits of integration.)

63. ***Respiratory Cycle*** The volume V in liters of air in the lungs during a 5-second respiratory cycle is approximated by the model

$$V = 0.1729t + 0.1522t^2 - 0.0374t^3$$

where t is the time in seconds. Approximate the average volume of air in the lungs during one cycle.

64. ***Average Profit*** A company introduces a new product, and the profit in thousands of dollars over the first 6 months is approximated by the model

$$P = 5(\sqrt{t} + 30), \qquad t = 1, 2, 3, 4, 5, 6.$$

(a) Use the model to complete the table and use the entries to calculate (arithmetically) the average profit over the first 6 months.

t	1	2	3	4	5	6
P						

(b) Find the average value of the profit function by integration and compare the result with that in part (a). (Integrate over the interval $[0.5, 6.5]$.)

(c) What, if any, is the advantage of using the approximation of the average given by the definite integral? (Note that the integral approximation utilizes all real values of t in the interval rather than just integers.)

65. ***Average Sales*** A company fit a model to the monthly sales data of a seasonal product. The model is

$$S(t) = \frac{t}{4} + 1.8 + 0.5\sin\left(\frac{\pi t}{6}\right), \qquad 0 \le t \le 24$$

where S is sales (in thousands) and t is time in months.

(a) Use a graphing utility to graph $f(t) = 0.5\sin(\pi t/6)$ for $0 \le t \le 24$. Use the graph to explain why the average value of $f(t)$ is 0 over the interval.

(b) Use a graphing utility to graph $S(t)$ and the line $g(t) = t/4 + 1.8$ in the same viewing window. Use the graph and the result of part (a) to explain why g is called the *trend line*.